## Introduction to cellular systems IoT, a.a. 2018/2019 Un. of Rome "La Sapienza"

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Frequency reuse: cellular systems

## Multi-cell systems

- The radio resource is to be divided among base stations
- The amount of radio resource (bandwidth) is very limited and it is not possible to dedicate it exclusively to a physical channel of a particular cell
- In the division of the radio resource among cells the resource is reused several times in cells that are sufficiently distant so that the mutual interference becomes strongly attenuated (remember path loss)
- The reuse of frequencies is a critical aspect in the design of cellular systems as it determines on one hand the number of channels to assign to each cell and on the other hand the channel quality
- We will devote much attention to the problem!
$\rightarrow 1 \mathrm{BS}$ per cell $\Rightarrow$ Cell: Portion of territory covered by one radio station $\Rightarrow$ One or more carriers (frequencies; channels) per cell
$\rightarrow$ Mobile users full-duplex connected with BS
$\rightarrow 1$ MSC controls many BSs
$\rightarrow$ MSC connected to PSTN



## terrestrialzone

 larger number of BSs


19 Base Station
12 carriers
4 carriers/cell


Worst case:
4 calls (all users in same cell)
Best case:
76 calls (4 users per cell, 19 cells)
Average case >> 12
Low transmit power
Key advantages:
-Increased capacity (freq. reuse)

- Decreased tx power

Gellular coverage (microcells)


Usage of same spectrum
(12 carriers)
(4 carriers/cell)

Disadvantage:
mobility management
additional infrastructure costs

- Increased via frequency reuse
- Frequency reuse depends on interference
- need to sufficiently separate cells
$\checkmark$ reuse pattern $=$ cluster size $(7 \rightarrow 4 \rightarrow 3)$ : discussed later
- Cellular system capacity: depends on
- overall number of carriers
$\checkmark$ Larger spectrum occupation
- frequency reuse pattern
- Cell size
$\checkmark$ Smaller cell (cell $\rightarrow$ microcell $\rightarrow$ picocell) $=$ greater capacity
$\checkmark$ Smaller cell = lower transmission power
$\checkmark$ Smaller cell $=$ increased handover management burden



## PART 2 <br> Cellular Coverage Concepts

Lecture 2.2
Clusters and CCI

- Reuse distance:
- Key concept
- In the real world depends on $\checkmark$ Territorial patterns (hills, etc)
$\checkmark$ Transmitted power
- and other propagation issues such as antenna directivity, height of transmission antenna, etc
- Simplified hexagonal cells model:
- reuse distance depends on reuse pattern (cluster size)
- Possible clusters:

$$
\checkmark 3,4,7,9,12,13,16,19, \ldots
$$



Cluster size




- General formula
- Valid for hexagonal geometry


## $D=R \sqrt{3 K}$

- $\mathrm{D}=$ reuse distance
- $\mathrm{R}=$ cell radius
- K=cluster size
- $\mathrm{q}=\mathrm{D} / \mathrm{R}=$ frequency reuse factor

| $\mathbf{K}$ | $\mathbf{q}=\mathbf{D} / \mathbf{R}$ |
| :---: | :---: |
| 3 | 3,00 |
| 4 | 3,46 |
| 7 | 4,58 |
| 9 | 5,20 |
| 12 | 6,00 |
| 13 | 6,24 |



- Distance between two cell centers:

$$
\begin{aligned}
& \text { Wh/ }-\left(\mathrm{u}_{1}, \mathrm{v}_{1}\right) \longleftrightarrow\left(\mathrm{u}_{2}, \mathrm{v}_{2}\right) \\
& D=\sqrt{\left.\left[u_{2}-u_{1}\right) \cos 30^{\circ}\right]^{2}+\left[\left(v_{2}-v_{1}\right)+\left(u_{2}-u_{1}\right) \sin 30^{\circ}\right]^{2}}
\end{aligned}
$$

- Simplifies to:

$$
D=\sqrt{\left(u_{2}-u_{1}\right)^{2}+\left(v_{2}-v_{1}\right)^{2}+\left(u_{2}-u_{1}\right)\left(v_{2}-v_{1}\right)}
$$

- Distance of cell (i,j) from ( 0,0 ):

$$
\begin{aligned}
& D=\sqrt{i^{2}+j^{2}+i j} \sqrt{3} R \\
& D_{R}=\sqrt{i^{2}+j^{2}+i j}
\end{aligned}
$$

- Cluster: easy to see that

$$
K=D_{R}^{2}=i^{2}+j^{2}+i j
$$

- hence:

$$
D=R \sqrt{3 K}
$$

- Distance between two cell centers:

$$
-\left(\mathrm{u}_{1}, \mathrm{v}_{1}\right) \leftarrow \rightarrow\left(\mathrm{u}_{2}, \mathrm{v}_{2}\right)
$$

$$
D=\sqrt{\left.\left[\left(u_{2}-u_{1}\right) \cos 30^{\circ}\right]^{2}+\left(v_{2}-v_{1}\right)+\left(u_{2}-u_{1}\right) \sin 30^{\circ}\right]^{2}}
$$

- Simplifies to:

$$
D=\sqrt{\left(u_{2}-u_{1}\right)^{2}+\left(v_{2}-v_{1}\right)^{2}+\left(u_{2}-u_{1}\right)\left(v_{2}-v_{1}\right)}
$$

- Distance of cell $(\mathrm{i}, \mathrm{j})$ from $(0,0)$ :

$$
\begin{aligned}
& D=\sqrt{i^{2}+j^{2}+i j} \sqrt{3} R \\
& D_{R}=\sqrt{i^{2}+j^{2}+i j}
\end{aligned}
$$

- Cluster: easy to see that

$$
K=D_{R}^{2}=i^{2}+j^{2}+i j
$$

- hence:

$$
D=R \sqrt{3 K}
$$

## Proof



- Distance between two cell centers:
$-\left(\mathrm{u}_{1}, \mathrm{v}_{1}\right) \leftrightarrow \rightarrow\left(\mathrm{u}_{2}, \mathrm{v}_{2}\right)$
$D=\sqrt{\left[\left(u_{2}-u_{1}\right) \cos 30^{\circ}\right]^{2}+\left[\left(v_{2}-v_{1}\right)+u_{2}-u_{1}\right) \sin 30}$
- Simplifies to:

$$
D=\sqrt{\left(u_{2}-u_{1}\right)^{2}+\left(v_{2}-v_{1}\right)^{2}+\left(u_{2}-u_{1}\right)\left(v_{2}-v_{1}\right)}
$$

- Distance of cell $(i, j)$ from $(0,0)$ :

$$
\begin{aligned}
& D=\sqrt{i^{2}+j^{2}+i j} \sqrt{3} R \\
& D_{R}=\sqrt{i^{2}+j^{2}+i j}
\end{aligned}
$$

- Cluster: easy to see that

$$
K=D_{R}^{2}=i^{2}+j^{2}+i j
$$

- hence:
$D=R \sqrt{3 K}$
- Distance between two cell centers:

$$
-\left(\mathrm{u}_{1}, \mathrm{v}_{1}\right) \leftarrow \rightarrow\left(\mathrm{u}_{2}, \mathrm{v}_{2}\right)
$$

$$
D=\sqrt{\left[\left(u_{2}-u_{1}\right) \cos 30^{\circ}\right]^{2}+\left[\left(v_{2}-v_{1}\right)+\left(u_{2}-u_{1}\right) \sin 30^{\circ}\right]}
$$

- Simplifies to:

$$
D=\sqrt{\left(u_{2}-u_{1}\right)^{2}+\left(v_{2}-v_{1}\right)^{2}+\left(u_{2}-u_{1}\right)\left(v_{2}-v_{1}\right)}
$$

- Distance of cell $(\mathrm{i}, \mathrm{j})$ from $(0,0)$ :

$$
D=\sqrt{i^{2}+j^{2}+i} \sqrt{3} R \quad \begin{gathered}
\text { Distance } \\
\text { Between centers }
\end{gathered}
$$



If $R$ is the radius of a hexagon, half the distance between two adjacent hexagonal cells is

$$
\sqrt{(R)^{2}-\left(\frac{R}{2}\right)^{2}}=\sqrt{\frac{3}{4} R^{2}}=R \frac{\sqrt{3}}{2}
$$

So the distance between two adjacent cells is 2 times this amount!
$\left.\begin{array}{rl}\text { Proof }\end{array} \begin{array}{rl}\text { Distance between two } \\ \text { centers: }\end{array}\right)$

$$
D=\sqrt{\left[\left(u_{2}-u_{1}\right) \cos 30^{\circ}\right]^{2}+\left[\left(v_{2}-v_{1}\right)+\left(u_{2}-u_{1}\right) \sin 30^{\circ}\right]^{2}}
$$

- Simplifies to:

$$
D=\sqrt{\left(u_{2}-u_{1}\right)^{2}+\left(v_{2}-v_{1}\right)^{2}+\left(u_{2}-u_{1}\right)\left(v_{2}-v_{1}\right)}
$$

- Distance of cell ( $\mathrm{i}, \mathrm{j}$ ) from (0,0):

$$
D=\sqrt{i^{2}+j^{2}+i j} \sqrt{3} R
$$

$D_{R}=\sqrt{i^{2}+j^{2}+i j}$ (we are defining $D_{R}$ )

- Cluster: possible to see that

$$
D=R \sqrt{3 K}
$$

- Let us focus on a cell using a set of carriers $A$
- We cover the area with "clusters of cells" providing the pattern for frequency reuse
- Let $D$ be the fixed distance between the centers of interfering cells in adjacent clusters
- We can approximate the area of each cluster with the area of a hexagon whose radius is given by

$$
D / \sqrt{3}
$$

## $$
K=D_{R}^{2}=i^{2}+j^{2}+i j
$$ <br> Proof

$$
K=D_{R}^{2}=i^{2}+j^{2}+i j
$$

- Let us focus on a cell using a set of carriers $A$
- We cover the area with "clusters of cells" providing the pattern for frequency reuse
- Let $D$ be the fixed distance between the centers of interfering cells in adjacent clusters
- We can approximate the area of each cluster with the area of a hexagon whose radius is given by

$$
D / \sqrt{3}
$$

## Proof

- We can approximate the area of each cluster with the area of a hexagon whose radius is given by $D / \sqrt{3}$
- if the radius of a hexagonal cell is r , the distance between the centers of two adjacent hexagons is $d=\sqrt{3} r$
- The distance between the centers of adjacent clusters is defined as D
- Therefore, the radius $r$ of the hexagon containing the cluster is $D / \sqrt{3}$


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- The area occupied by a cluster A is then given by:

$$
\frac{3}{2}\left(\frac{D}{\sqrt{3}}\right)^{2} \sqrt{3}
$$

- How many hexagons of area

$$
\frac{3}{2}(R)^{2} \sqrt{3}
$$

may be in an area equal to $\left.\frac{3}{2}\left(\frac{D}{\sqrt{3}}\right)^{2} \sqrt{3}\right)$ ?

- Answer:

$$
K=\frac{A_{\text {custer }}}{A_{\text {cella }}}=\frac{\frac{3}{2}\left(\frac{D}{\sqrt{3}}\right)^{2} \sqrt{3}}{\frac{3}{2}(R)^{2} \sqrt{3}}=\left(\frac{D}{R \sqrt{3}}\right)^{2}=\left(D_{R}\right)^{2}
$$

$K=\left(\frac{D}{R \sqrt{3}}\right)^{2}=\frac{D^{2}}{3 R^{2}}$

$$
D=\sqrt{3 K R^{2}}=R \sqrt{3 K}
$$

- The area occupied by a cluster A is then given by:

$$
\frac{3}{2}\left(\frac{D}{\sqrt{3}}\right)^{2} \sqrt{3}
$$

- How many hexagons of area

$$
\frac{3}{2}(R)^{2} \sqrt{3}
$$

may be in an area equal to $\frac{3}{2}\left(\frac{D}{\sqrt{3}}\right)^{2} \sqrt{3}$ ?

## Since:

$$
\begin{aligned}
& D=\sqrt{i^{2}+j^{2}+i j} \sqrt{3} R \\
& D_{R}=\sqrt{i^{2}+j^{2}+i j}
\end{aligned}
$$

- Answer:

$$
\begin{gathered}
K=\frac{A_{\text {cluster }}}{A_{\text {cella }}}=\frac{\frac{3}{2}\left(\frac{D}{\sqrt{3}}\right)^{2} \sqrt{3}}{\frac{3}{2}(R)^{2} \sqrt{3}}=\left(\frac{D}{R \sqrt{3}}\right)^{2} \\
\frac{D^{2}}{R^{2}} \quad \square D=\sqrt{3 K R^{2}}=R \sqrt{3 K}
\end{gathered}
$$



| $\mathbf{i}$ | $\mathbf{j}$ | $\mathbf{K}=\mathbf{i i}+\mathbf{j j}+\mathbf{i j}$ | $\mathbf{q = D / R}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 1,73 |
| 1 | 1 | 3 | 3,00 |
| 2 | 0 | 4 | 3,46 |
| 2 | 1 | 7 | 4,58 |
| 2 | 2 | 12 | 6,00 |
| 3 | 0 | 9 | 5,20 |
| 3 | 1 | 13 | 6,24 |
| 3 | 2 | 19 | 7,55 |
| 3 | 3 | 27 | 9,00 |
| 4 | 0 | 16 | 6,93 |
| 4 | 1 | 21 | 7,94 |
| 4 | 2 | 28 | 9,17 |
| 4 | 3 | 37 | 10,54 |
| 4 | 4 | 48 | 12,00 |
| 5 | 0 | 25 | 8,66 |
| 5 | 1 | 31 | 9,64 |

Feasible cluster sizes: $1,3,4,7,9,12,13,16, \ldots$

## Co-Channer Interference

- Co-Channel Interference (CCI)
- sum of interference from remote cells $\frac{S}{N}=\frac{\text { signal power }(\mathrm{S})}{\text { noise power }\left(\mathrm{N}_{\mathrm{S}}\right)+\text { interfering signal power (I) }}$

$$
\frac{S}{I}=\frac{\text { signal power }(\mathrm{S})}{\text { interfering signal power }(\mathrm{I})}
$$

$$
\frac{S}{N} \approx \frac{S}{I} \quad \text { as } \mathrm{N}_{\mathrm{s}} \text { small }
$$

- Assumptions
- $\mathrm{N}_{\mathrm{I}}=6$ interfering cells
$\checkmark \mathrm{N}_{\mathrm{I}}=6$ : first ring interferers only
$\checkmark$ we neglect second-ring interferers
- Key simplification
- Signal for MS at distance R
- Signal from BS interferers at distance $D$
- Negligible Noise Ns

$$
\checkmark S / N \sim S / I
$$

- d-n propagation law $\checkmark \eta=4$ (in general)
- Same parameters for all BSs
$\checkmark$ Same Ptx, antenna gains, etc

$$
\frac{S}{N} \approx \frac{S}{I}=\frac{\operatorname{cost} \cdot R^{-\eta}}{\Gamma^{N_{I}}}=\begin{gathered}
\text { By using the assumptions of } \\
\text { same cost and same } \mathrm{D}:
\end{gathered}
$$

Results depend

$$
=\frac{1}{N_{I}}\left(\frac{R}{D}\right)^{-\eta}=\frac{1}{N_{I}}\left(\frac{D}{R}\right)^{\eta}=\frac{1}{N_{I}} q^{\eta} \quad \begin{gathered}
\text { on ratio } \mathrm{q}=\mathrm{D} / \mathrm{R} \\
\text { (q=frequency reuse factor) }
\end{gathered}
$$

Alternative expression: recalling that $D=R \sqrt{3 K}$

$$
\begin{aligned}
& \frac{S}{N} \approx \frac{S}{I}=\frac{1}{N_{I}}\left(\frac{R}{R \sqrt{3 K}}\right)^{-\eta}=\frac{1}{N_{I}}(3 K)^{\eta / 2}=\frac{(3 K)^{\eta / 2}}{6} \\
& \mathrm{~N}_{\mathrm{I}}=6, \eta=4 \rightarrow \frac{S}{I}=\frac{(3 K)^{2}}{6}=\frac{3}{2} K^{2}
\end{aligned}
$$

USAGE: Given an S/I target, cluster size K is obtained

## Examples

## $\frac{S}{N} \approx \frac{S}{I}=\frac{(3 K)^{n / 2}}{6}$

- target conditions:
$-\mathrm{S} / \mathrm{I}=9 \mathrm{~dB}$
- $\eta=4$
- Solution:
$\frac{S}{I}=10^{0.9}=7.94 \approx 8$
$\frac{S}{I}=\left.\frac{(3 K)^{\eta / 2}}{6}\right|_{\eta=4} \Rightarrow K=\sqrt{\frac{2}{3} \cdot \frac{S}{I}}$
$K \geq 2.3 \quad \Rightarrow \quad K=3$
- target conditions:
$-\mathrm{S} / \mathrm{I}=18 \mathrm{~dB}$
$-\eta=4.2$
- Solution:

$$
\begin{aligned}
& \frac{S}{I}[d B]=5 \eta \log (3 K)-10 \log 6 \\
& \log (3 K)=\frac{18+7.78}{21}=1.23 \\
& K \geq \frac{10^{1.23}}{3}=5.63 \Rightarrow K=7
\end{aligned}
$$

| $\mathbf{K}$ | $\mathbf{q}=\mathbf{D} / \mathbf{R}$ | $\mathbf{S} / \mathbf{I}$ | $\mathbf{S} / \mathbf{l d B}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{3}$ | 3,00 | 13,5 | 11,3 |
| $\mathbf{4}$ | 3,46 | 24,0 | 13,8 |
| 7 | 4,58 | 73,5 | 18,7 |
| 9 | 5,20 | 121,5 | 20,8 |
| 12 | 6,00 | 216,0 | 23,3 |
| 13 | 6,24 | 253,5 | 24,0 |
| 16 | 6,93 | 384,0 | 25,8 |
| 19 | 7,55 | 541,5 | 27,3 |
| 21 | 7,94 | 661,5 | 28,2 |
| 25 | 8,66 | 937,5 | 29,7 |



- First tier of interferers are at distance D; secondtier at approximation distance 2D; third tier at distance 3D etc.
- Often interferers have a significant impact only if they belong to the first tier
- General formula:

$$
S I R=\frac{R^{-\eta}}{N_{11}(D)^{-\eta}+N_{12}(2 D)^{-\eta}+N_{13}(3 D)^{-\eta}}
$$

- NIi=number of interefers belonging to the i-th tier


Same antennas and same power

$$
\begin{aligned}
\operatorname{SIR} & =\frac{P_{t} \cdot G \cdot d^{-\eta}}{\sum_{i=1}^{6} P_{t} \cdot G \cdot d_{i}^{-\eta}}= \\
& =\frac{d^{-\eta}}{\sum_{i=1}^{6} d_{i}^{-\eta}}
\end{aligned}
$$

- Worst case: $\mathrm{d}=\mathrm{r}$
- Approximation: $d_{i}=D-R$

$$
S I R=\frac{R^{-\eta}}{6(D-R)^{-\eta}}
$$

- Directional antennas
- Cell divided into sectors
- Each sector uses different frequencies
- To avoid interference at sector borders
- PROS:
- CCI reduction
- CONS:
- Increased handover rate

- Less effective "trunking" leads to performnce impairments


## SAPIENZACCI realuction var sectorization three sectors case

- Inferference from 2 cells, only
- Instead of 6 cells

With usual approxs
(specifically, Dint ~ D)

$$
\begin{aligned}
& {\left[\frac{S}{I}\right]_{120^{\circ}}=\frac{R^{-\eta}}{2 D^{-\eta}}=3 \cdot\left[\frac{S}{I}\right]_{o m n i}} \\
& {\left[\frac{S}{I}\right]_{120^{\circ}} d B=\left[\frac{S}{I}\right]_{\text {omni }} d B+4.77}
\end{aligned}
$$



Conclusion: 3 sectors $=4.77 \mathrm{~dB}$ improvement

- $60^{\circ}$ Directional antennas
- CCI reduction:
- 1 interfereer only
$-6 \times \mathrm{S} / \mathrm{I}$ in the omni case
- Improvement: 7.78 dB


## SAPIENZA A6sectors

Only BS
which disturbs receptions / transmissions to / from MU in the reference cell



Cellular systems: Planning

- Regardless of the manner with which the resource is divided the number of channels that we can assign to each cell is limited
- Apart for special cases (which we will see, as those of dynamic allocation) the number of channels is fixed
- The number of simultaneous conversations per cell is limited and it is therefore possible that upon arrival of a new call that requires to establish a circuit (eg. Voice) there are no more available channels in the radio access network (resulting in call blocking)
- To evaluate the performance in terms of call blocking probability we need to characterize the traffic
- Process of arrivals (voice calls are well modeled by the Poisson process)
- Rate of arrivals
- Average call duration
- In the case of a queue system that represents the management of calls in a cell
- The probability that the number of arrivals $N(t, t+\tau)$ in a time interval between t and $\mathrm{t}+\tau$ is equal to k is given by:

$$
P[N(t, t+\tau)=k]=\frac{(\lambda \tau)^{k}}{k!} e^{-\lambda \tau}
$$

Poisson arrival process

## In steady state $E[A(T)]=A$

$$
A=\lambda X
$$

Average traffic (active calls)
in an interval of size $T$
$\lambda$ Arrival rate of calls(call/s)
$X$ average duration of calls (s)

## $A$ is adimensional

Traffic is measured in Erlang

- To model the arrival of calls in a cell with $n$ channels we can use a queueing system with $n$ serving units and a queue length equal to zero
- If the process of arrivals is Poisson, the probability of rejecting a call is given $y$ the Erlang B equation:

$$
B(n, A)=\frac{\frac{A^{n}}{n!}}{\sum_{k=0}^{n} \frac{A^{k}}{k!}}
$$

- where $A=\lambda T$ (in Erlang), $\lambda$ is the average arrival rate (call/s), $T$ is the average call duration
- NOTE: it works for any call duration distribution
- The blocked traffic which is not served is:

$$
A_{p}=A \cdot B(n, A)
$$

- Carried traffic:

$$
A_{s}=A \cdot(1-B(n, A))=A-A_{p}
$$

- Channel utilization coefficient is given by:

$$
\rho=\frac{A_{s}}{n}=\frac{A \cdot(1-B(n, A))}{n}, \quad 0 \leq \rho \leq 1
$$

- Fundamental formula for telephone networks planning
- Ao=offered traffic in Erlangs
$\rightarrow$ Efficient recursive computation available

$$
E_{1, C}\left(A_{o}\right)=\frac{A_{o} E_{1, C-1}\left(A_{o}\right)}{C+A_{o} E_{1, C-1}\left(A_{o}\right)}
$$

$$
\Pi_{\text {block }}=\frac{\frac{A_{o}^{C}}{C!}}{\sum_{j=0}^{C} \frac{A_{o}^{j}}{j!}}=E_{1, C}\left(A_{o}\right)
$$

- Target: support users with a given Grade Of Service (GOS)
- GOS expressed in terms of upper-bound for the blocking probability
$\checkmark$ GOS example: subscribers should find a line available in the $99 \%$ of the cases, i.e, they should be blocked in no more than $1 \%$ of the attempts
- Given:
$\checkmark$ Offered load $\mathrm{A}_{0}$
$\checkmark$ Target GOS Brarget
- C (number of channels) is obtained from numerical inversion of

$$
B_{\text {target }}=E_{1, C}\left(A_{o}\right)
$$

Offered load (erl) Carried load (erl)


Blocked traffic
efficiency: $\quad \rho=\frac{A_{c}}{C}=\frac{A_{o}\left(1-E_{1, C}\left(A_{o}\right)\right)}{C}$

$$
\approx \frac{A_{o}}{C} \text { if small blocking }
$$

Fundamental property: for same GOS, efficiency increases as C grows!!


GOS $=1 \%$ maximum blocking.
Resulting system dimensioning and efficiency:

| 40 erl | C > $=53$ | $\rho=74.9 \%$ |
| :---: | :---: | :---: |
| 60 erl | C > $=75$ | $\rho=79.3 \%$ |
| 80 erl | $C>=96$ | $\rho=82.6 \%$ |
| 100 erl | C > $=117$ | $\rho=84.6 \%$ |



FIGURE 4.21 Trunking efficiency plots.

## SAPIENZA Erlang B calculation- tables

Example: How many channels are required to support 100 users with a GOS of $2 \%$ if the average traffic per user is 30 mE ?
$100 \times 30 \mathrm{mE}=3$ Erlangs 3 Erlangs @ $2 \%$ GOS = 8 channels

| Trunks | 0.01 | 0.015 | (0.02) | 0.03 |
| :---: | :---: | :---: | :---: | :---: |
| P(B) $=$ |  |  |  |  |
| 1 | 0.010 | 0.015 | 0.020 | 0.031 |
| 2 | 0.153 | 0.190 | 0.223 | 0.282 |
| 8 | 0.465 | 0.636 | 0.603 | 0.715 |
| 4 | 0.870 | 0.992 | 1.092 | 1.259 |
| 5 | 1362 | 1.524 | 1.657 | 1.877 |
| 6 | 1.918 | 2.114 | 2.277 | 2.544 |
| 7 | 2.508 | 2.743 | 3.986 | 3.250 |
| 8 | 3129 | 3.405 | (3.627) | 3.987 |
|  | 3.783 | 4.095 | 4.345 | 4.748 |
| 10 | 4.462 | 4.808 | 5.084 | 5.529 |

ErlangB Online calculator:
http://mmc,et,tudelft.n//~frits/Erlang,htm

Cell size (radius R) may be determined on the basis of traffic considerations

Given a provider with 50 channels available, how many users can be supported If each user makes an average of 4 calls/hour, each call lasting on average 2 minutes?

- First step:
- Given num channels and GOS
- $C=50$ available channels in a cell
- Blocking probability $<=\mathbf{2 \%}$
- Evaluate maximum cell (offered) load
- From Erlang-B inversion(tables) $\mathrm{A}=40.25 \mathrm{erl}$
- Second step
- Given traffic generated by each user
- Each user: 4 calls/busy-hour
- Each call: 2 min on average
- $A_{i}=4 \times 2 / 60=0.1333 \mathrm{erl} / \mathrm{user}$
- Evaluate max num of users in cell
- $\mathrm{M}=40.25 / 0.1333 \sim 302$

Second question: if the user density Is 500 users $/ \mathrm{km} 2$ how should we Set the cell radius?

$$
\delta=\frac{M}{\pi R^{2}} \Rightarrow R=\sqrt{\frac{M}{\pi \delta}}
$$

$\rightarrow$ Third tep:
$\Rightarrow$ Given ensity of users
$\rightarrow \delta=5$ users $/ \mathrm{km}^{2}$
$\Rightarrow$ Evaluate o $\|$ radius
$\Rightarrow \mathrm{R} \sim 438 \mathrm{~m}$
It is preferrable To approximate

With hexagons

- Three service providers are planning to provide cellular service for an urban area. The target GOS is $2 \%$ blocking. Users make 3 calls/busy-hour, each lasting 3 minutes in average ( $\mathrm{A}_{\mathrm{i}}=3 / 20=0.15$ )
- Question: how many users can support each provider?
- Provider A configuration: 20 cells, each with 40 channels
- Provider B configuration: 30 cells, each with 30 channels
- Provider C configuration: 40 cells, each with 20 channels
$\rightarrow$ Provider A: $\quad \rightarrow$ Provider B:
$\Rightarrow 40$ channels/cell
$\Rightarrow$ at $2 \%$ : $\mathrm{A}_{0}=30.99 \mathrm{er} / \mathrm{cell}$
$\Rightarrow 619.8$ erl-total ( 20 cells)
$\Rightarrow M=4132$ overall users
$\Rightarrow 30$ channels/cell
$\Rightarrow$ at 2\%: $\mathrm{A}_{0}=21.93$ er $/ \mathrm{cell} \quad \Rightarrow$ at $2 \%: \mathrm{A}_{0}=13.18$ er $/ \mathrm{cell}$
$\Rightarrow 657.9$ erl-total ( 30 cells) $\Rightarrow 527.2$ erl-total ( 40 cells)
$\Rightarrow M=4386$ overall users
$\rightarrow$ Provider C:
$\Rightarrow 20$ channels/cell
$\Rightarrow$ at $2 \%: \mathrm{A}_{\mathrm{o}}=13.18$ erl/cell
$\Rightarrow 527.2$ erl-total ( 40 cells)
$\Rightarrow M=3515$ overall users

Compare case A with C! The reason is the lower efficiency of 20 channels versus 40

- Three service providers are planning to provide cellular service for an urban area. The target GOS is $2 \%$ blocking. Users make 3 calls/busy-hour, each lasting 3 minutes in average ( $\mathrm{A}_{\mathrm{i}}=3 / 20=0.15$ )
- Question: how many users can support each provid
- Provider A configuration: 20 cells, each with 40 channels
- Provider B configuration: 30 cells, each with 30 channels
- Provider C configuration: 40 cells, each with 20 channeßr with a few channels
$\Rightarrow 619.8$ erl-total ( 20 cells) $\quad \Rightarrow 657.9$ erl-total ( 30 cells)
$\Rightarrow M=4132$ overall users
$\rightarrow$ Provider B:
$\Rightarrow 30$ channels/cell
$\Rightarrow$ at $2 \%$ : $A_{0}=21.93 \mathrm{erl} / \mathrm{cell}$
$\Rightarrow M=4386$ overall users
(for example, because we have selected a different K)
$\rightarrow \mathrm{Pr}$
- Assume cluster K=7
- Omnidirectional antennas: $\mathrm{S} / \mathrm{I}=18.7 \mathrm{~dB}$
- 120 sectors: $\mathrm{S} / \mathrm{I}=23.4 \mathrm{~dB}$
- 600 sectors:

$$
\mathrm{S} / \mathrm{I}=26.4 \mathrm{~dB}
$$

- Sectorization yields to better S/I
- BUT: the price to pay is a much lower trunking efficiency!
- With 60 channels/cell, GOS=1\%,
- Omni: 60 channels $\quad A_{0}=1 \times 46.95=46.95 \mathrm{erl}$

$$
\rho=77.46 \%
$$

- 120: 60/3=20 channels
$\mathrm{A}_{0}=3 \times 12.03=36.09 \mathrm{erl}$

$$
\rho=59.54 \%
$$

-60 : $60 / 6=10$ channels $\quad A_{0}=6 \times 4.46=26.76 \mathrm{erl}$ $\rho=44.15 \%$

- Assume cluster K=7
- Omnidirectional antennas: $\quad \mathrm{S} / \mathrm{I}=18.7 \mathrm{~dB}$
- 1200 sectors: $\quad \mathrm{S} / \mathrm{I}=23.4 \mathrm{~dB}$ $\mathrm{S} / \mathrm{I}=26.4 \mathrm{~dB}$


## On the other hand <br> Lower CCI can allow <br> To select a smaller K

- 60 sectors:
- Sectorization yields to better S/I
- BUT: the price to pay is a much lower trunking efficiency!
- With 60 channels/cell, GOS=1\%,
- Omni: 60 channels $\quad A_{0}=1 \times 46.95=46.95 \mathrm{erl}$

$$
\rho=77.46 \%
$$

- 1200: 60/3=20 channels
$\mathrm{A}_{0}=3 \times 12.03=36.09 \mathrm{erl}$
- 60ㅇ 60/6=10 channels
$\mathrm{A}_{0}=6 \times 4.46=26.76 \mathrm{erl}$ $\rho=44.15 \%$


Shaped by terrain, shadowing, etc
Cell border: local threshold, beyond which neighboring BS signal is received stronger than current one

