Formal Methods in Software Development

The Fairness Problem LTL and CTL MC with Fairness

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Lesson 5a:

The Fairness problem

Fairness

Often, system models **abstract from details** such as, for example, **scheduler policies**

Interleaving semantics does not rule out unrealistic behaviour, for example, those in which **some processes do not make any progress**

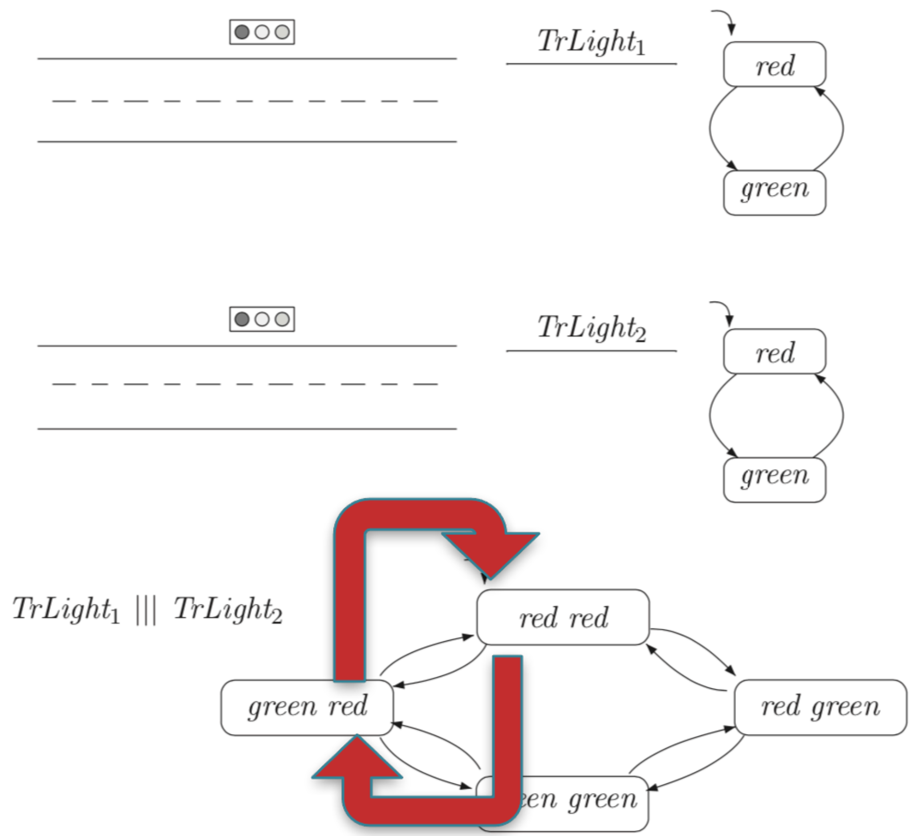
Two possible approaches:

1. Embody a **fair process scheduling in the model**, as in the case of Peterson mutual exclusion algorithm.

2. Assume **some fairness properties**, and perform model checking under such assumptions.

In the following, we follow the second approach, which is more abstract.

Example: Interleaving Semantics



Two independent traffic lights (lesson **1**):

Interleaving semantics allows infinite executions in which **only the first traffic light commute**: {red_1, red_2}{green_1, red_2} {red_1, red_2}{green_1, red_2} {red_1, red_2}{green_1, red_2}…

Example: (un)fair Schedulers

Let us consider, the **mutual exclusion protocol** (shared variables) and the following **starvation freedom** property:

"Once access is requested, a process does not have to wait infinitely long before acquiring access to its critical section"

This is violated, just because, **abstracting from the scheduling policy**, in the model there exists an execution that **assignes** the **critical resource** always **to the same process**.

Also the property:

"Each of the processes is infinitely often in its critical section"

is violated also by the **Peterson protocol**, as it **does not exclude** that **a process would never** (or finitely often**)** **request** to enter its critical section.

Fairness notions

There are several notions of fairness:

- **Unconditional Fairness**: "every process gets its turn infinitely often" (without conditions, aka **impartiality**)
- **Strong Fairness**: "every process that is **enabled infinitely often** gets its turn infinitely often" (aka **compassion**)
- **Weak Fairness**: "every process that is **continuously enabled from a certain point on** gets its turn infinitely often" (aka **justice**)

Many other fairness notions have been introduced in literature and **there is no clear consensus** about which notion should be used in some scenario. It depends on the application.

We will see in the following **some roadmap**.

Fairness def. (action based)

Definition: Given a transition system without terminal states $\mathcal{T} = (S, Act, \rightarrow, I, AP, L)$ a set of actions $A \subseteq$ Act, and an infinite path $\pi = s_0\alpha_0 s_1\alpha_1 s_2\alpha_2\ldots$ we say that:

- π is **unconditionally *A*-fair** whenever for infinite many indices i, $\alpha_i \in A$ (*similar* to LTL formula $\approx$ **G F** A)

- π is **strongly *A*-fair** whenever if for **infinite many indices** i, $\alpha_i \in enabled(s_i) \cap A \neq \varnothing$ then for infinite many indices j we have $\alpha_j \in A$ (*similar* to LTL formula $\approx$ **G F** A$\rightarrow$ **G F** A)

- π is **weakly *A*-fair** whenever if exists i_0, such that **for all indices $i \geq i_0$** (= *almost always*) $\alpha_i \in enabled(s_i) \cap A \neq \varnothing$ then for infinite many indices j we have $\alpha_j \in A$ (*similar* to LTL formula $\approx$ **F G** A$\rightarrow$ **G F** A).

[Observe that LTL formulas are **state-based**]

Ex: Shared variables program

$$\begin{aligned} \textbf{proc}\ \mathsf{Inc} &= \textbf{while}\ \langle x \geqslant 0\ \textbf{do}\ x := x+1 \rangle\ \textbf{od} \\ \textbf{proc}\ \mathsf{Reset} &= x := -1 \end{aligned}$$

This process terminates only if **unconditional fairness** is assumed. If the process Inc or the process Reset can execute infinitely often, the concurrent program does not terminate.

[Brackets ⟨…⟩ means "atomic actions"]

Which notion of fairness we should use? No answer!

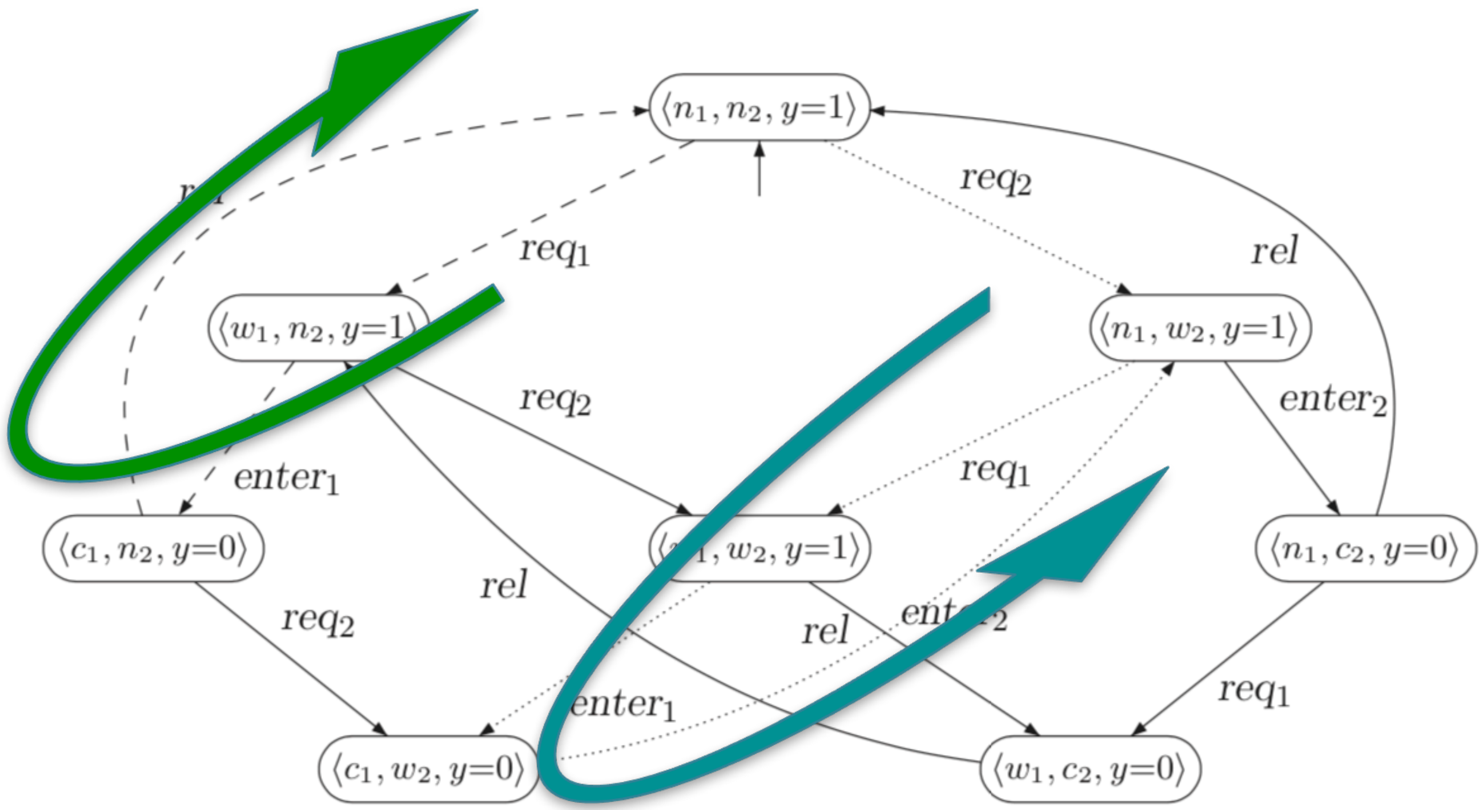
Keep in mind: if the fairness constraints are **too strong, relevant computation can be ruled out**. By contrast, if the fairness constraints are **too weak**, we refute a property because we consider **unrealistic behaviour** of a system.

Uncond. Fairnes A $\Longrightarrow$ Strong Fairness A $\Longrightarrow$ Weak Fairness A

Mutual Exclusion Reloaded

The dashed execution fragment **is strongly fair** (**premises are vacously true**), but **not unconditionally fair** for { $enter_2$ }.

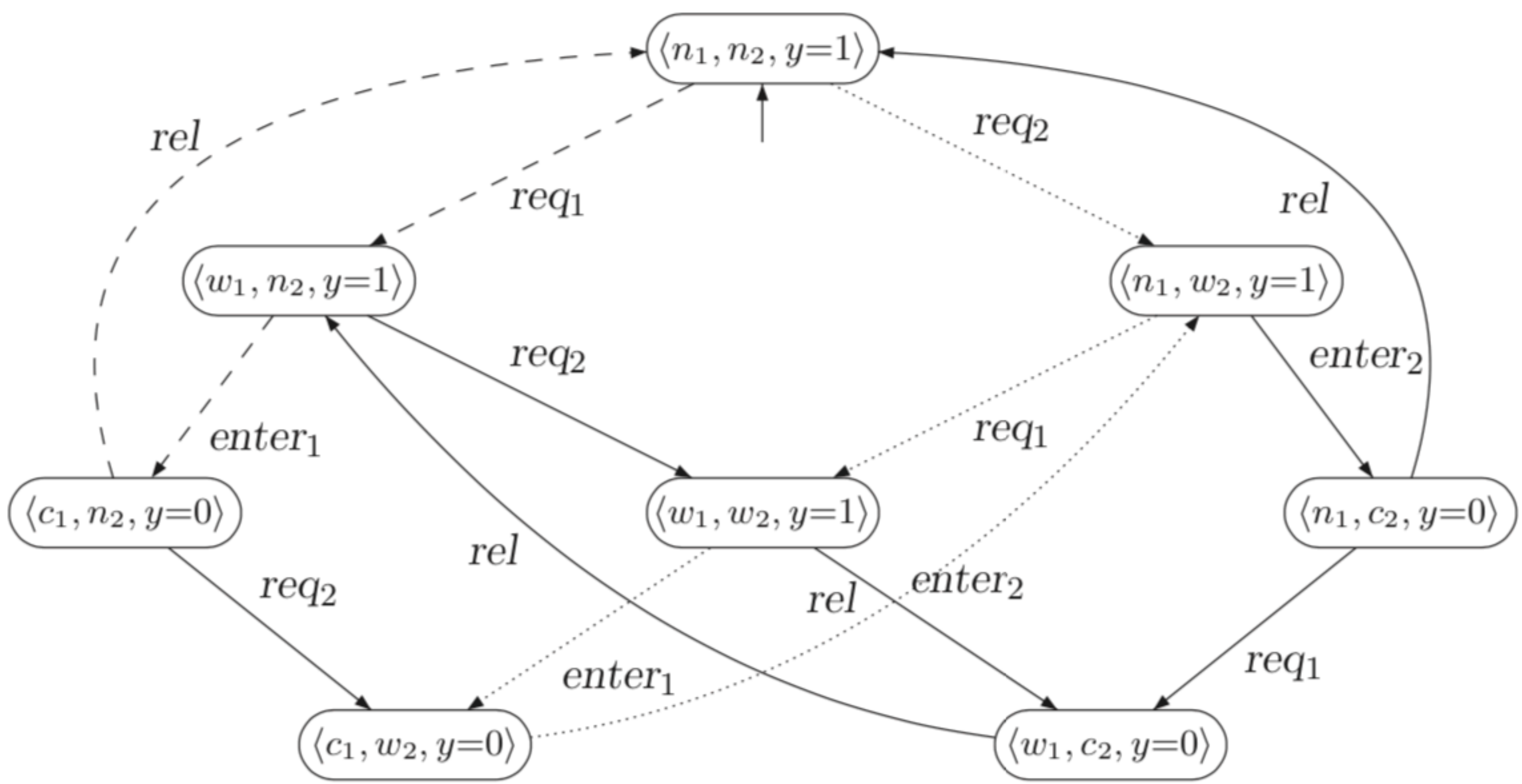
The dotted **is weakly fair, but not strongly fair** for { $enter_2$ }. Process 1 requests access infinitely often, **but not continuously** ($enter_2$ is not enabled in $\langle c_1, w_2, y{=}0\rangle$)



Example: How to model Fairness

Be careful in defining fairness assumption!

The **strong fairness** assumption **{{enter$_1$, enter$_2$}}** ensure only that **one of the two process enter its critical section infinitely often**. Maybe that **{{enter$_1$}, {enter$_2$}}** is what one wants!



Fairness: Linear Time Properties

Definition: Let P $\subseteq (2^{AP})^\omega$ be an LT property over AP and let $\mathcal{F}$ be a fairness assumption over A. A transition system $\mathcal{M}$ **fairly satisfies *P***, notation $\mathcal{M} \vDash_{\mathcal{F}} P$, if and only if fairTraces($\mathcal{M}$)$\subseteq P$.

If all executions of $\mathcal{M}$ satisfies $\mathcal{F}$, then $\mathcal{M} \vDash_{\mathcal{F}} P$ iff $\mathcal{M} \vDash P$.

More in general, we have that $\mathcal{M} \vDash P \Longrightarrow \mathcal{M} \vDash_{\mathcal{F}} P$ (**fair executions are a subset of all executions**).

As said before, we also have:

$$\mathcal{M} \vDash_{\text{weak } \mathcal{F}} P \Longrightarrow \mathcal{M} \vDash_{\text{strong } \mathcal{F}} P \Longrightarrow \mathcal{M} \vDash_{\text{uncond } \mathcal{F}} P$$

Example [Independent Traffic Lights] The fair assumption:

$\{\{\text{switchToGreen}_1, \text{switchToRed}_1\}, \{\text{switchToGreen}_2, \text{switchToRed}_2\}\}$

rules out unrealistic behaviour, no matter if this is interpreted as strong, weak or unconditional fairness constraint.

$\text{TrLight}_1 \parallel \text{TrLight}_2 \vDash_{\mathcal{F}}$ **F G** green ≡"*each traffic light is green infin. often*"

Example: Mutual Exclusion

Let us consider again the semaphore based mutual exclusion protocol. Let us define the following fairness constraints:

$$\mathcal{F}_{\text{weak}} = \{\{req_1\},\{req_2\}\} \quad \mathcal{F}_{\text{strong}} = \{\{enter_1\},\{enter_2\}\} \quad \mathcal{F}_{\text{uncond}} = \varnothing$$

The strong fairness assumption $\mathcal{F}_{\text{strong}}$ **does not forbid a process to never release its critical section**.

The weak fairness assumption $\mathcal{F}_{\text{weak}}$ **implies that a process requires to enter critical section infinitely often** (and hence it has to leave infinitely often its critical section because ***req_1* is enabled when *c_2* holds**!)

Also **Peterson's protocol** ensure that process will enter its critical section if it requires it infinitely often. But **it does not ensure that processes leave their critical section**. To ensure this, we should impose the weak fairness assumption $\mathcal{F}_{\text{weak}} = \{\{req_1\},\{req_2\}\}$.

Weak or Strong Fairness?

Rule of Thumb:

Strong fairness is appropriate to obtain an adequate resolution of **contentions between processes** or **communication**.

Weak fairness suffices for sets of actions that represent the concurrent execution of **independent actions** (**interleaving**)

Concurrency = interleaving + (strong or weak) fairness:

Let us assume we have n processes represented by transition systems $\mathcal{M}_i = (S_i, A_i, \rightarrow_i, AP_i, L_i)$ and consider the parallel composition:

$$\mathcal{M} = \mathcal{M}_1 \parallel \mathcal{M}_2 \parallel \ldots \parallel \mathcal{M}_n$$

and let us suppose that each pair $1 \leq i < j \leq n$ of processes synchronize on a set of actions $H_{i,j}$.

Fair Synchronization

The **strong fairness assumption** $\{A_1, A_2, \ldots, A_n\}$ means that **each process makes some progress infinitely often** (provided that infinitely often a process has an enabled action to execute). This assumption is satisfied, however, only with internal actions and no sync!

- $\{\{\alpha\} \mid \alpha \in H_{i,j}\ 0 < i < j \leq n\}$ forces **every synchronization action** to be performed infinitely often
- $\{H_{i,j} \mid 0 < i < j \leq n\}$ forces **every pair of processes to synchronize infinitely often**, maybe on the same action
- $\{\bigcup_{0<i<j\leq n} H_{i,j}\}$ just requires **that there are infinite synchronization actions**, regardless of which are processes involved

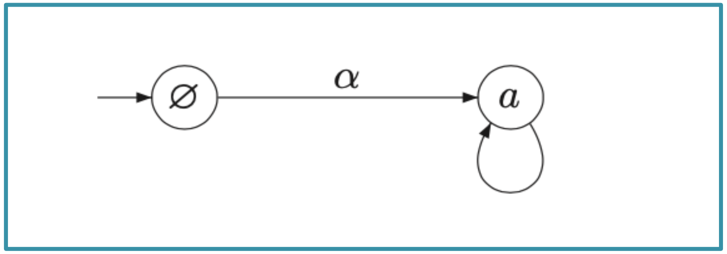
For **internal actions**, the **weak fairness assumption**: $\{A_1 \backslash H_1, \ldots, A_n \backslash H_n\}$, where $H_i = \bigcup_{i \neq j} H_j$, is appropriate, **since internal actions are continously ready to be executed**.

Fairness and Safety Properties

In all our examples, we deal with liveness properties. This is not incidental. **Fairness is** (**almost**) **irrilevant** with respect to **safety properties**.

Definition: Given a transition system $\mathcal{M}$ and a set of actions of $\mathcal{M}$ a fairness policy $\mathcal{F}$ is realizable if **for all reachable state *s*** of $\mathcal{M}$, **the set of fair path** starting in s **is not empty**.

Example:



The unconditional fairness assumption $\{\{\alpha\}\}$ **is not realizable**, just because α can be executed just once, and therefore there is no path in which α appears infinitely often.

Fairness and Safety Properties

Theorem. Let $\mathcal{M}$ be a transition system with AP as set of atomic proposition and $\mathcal{F}$ a realizable fairness policy, and P_{safe} be a safety property over AP. Then

$$\mathcal{M} \vDash_{\mathcal{F}} P_{\text{safe}} \Leftrightarrow \mathcal{M} \vDash P_{\text{safe}}$$

Proof: ($\Rightarrow$) this is true for any linear property (previous slides).

($\Leftarrow$) By contradiction, let us suppose that $\mathcal{M} \vDash_{\mathcal{F}} P_{\text{safe}}$ but not $\mathcal{M} \vDash P_{\text{safe}.}$ Then there exists an execution $\pi \notin P_{\text{safe}}$ and π is not fair. π is ruled out by a finite bad prefix π^* that ends in a state s. Since $\mathcal{F}$ is realizable, there exists a fair path π' starting in s. But clearly, $\pi^* \pi'$ is a faire path that does not satisfy P_{safe} against the hypothesis that $\mathcal{M} \vDash_{\mathcal{F}} P_{\text{safe}}$. $\square$

Lesson 5b:

Fairness
in
LTL Model Checking

Fairness is expressible in LTL

The three notions of fairness we have considered **can be expressed by LTL formulas** of the shape:

Unconditional fairness: $\mathbf{G}\,\mathbf{F}\,\varphi$

Strong fairness: $\mathbf{G}\,\mathbf{F}\,\varphi \rightarrow \mathbf{G}\,\mathbf{F}\,\psi$

Weak fairness: $\mathbf{F}\,\mathbf{G}\,\varphi \rightarrow \mathbf{G}\,\mathbf{F}\,\psi$

The only problem is that LTL formulas are built on atomic propositions that **label states**: therefore φ and ψ depend on the state labeling and they single out **set of states** of a transition system $\mathcal{M}$, i.e. $\{ s \mid \mathcal{M}, s \vDash \varphi\}$.

By contrast, so far we have defined fairness assumptions as set of actions, however…

Example: Mutual Exclusion

The strong (**action based**) fairness assumption $\mathcal{F}_{\text{strong}} = \{\{enter_1\}, \{enter_2\}\}$ can be represented by the (**state based**) LTL formula:

$$\text{sfair}_1 = \mathbf{G}\,\mathbf{F}\,(wait_1 \wedge \neg crit_2) \rightarrow \mathbf{G}\,\mathbf{F}\,crit_1$$

Observe that $enter_1$ can be executed only if P_1 is in the state $wait_1$ and P_2 is not in its critical section.

The assumption sfair_2 can be defined analogously.

$\mathcal{F}_{\text{strong}}$ does not forbid a process to never leave its critical section. The (**action based**) weak assumption = $\{\{req_1\}, \{req_2\}\}$ can be encoded as the (**state based**) LTL formula

$$\text{wfair}_1 = \mathbf{F}\,\mathbf{G}\,noncrit_1 \rightarrow \mathbf{G}\,\mathbf{F}\,wait_1$$

Observe that the action req_1 is executable only if P_1 is in the state $noncrit_1$.

The assumption wfair_2 can be defined analogously.

Action vs State based Fairness 1

Kripke structures **have no action labels**. One can always keep **information into states**.

Let $\mathcal{M}$ be a transition system $(S, A, \rightarrow, I, AP, L)$. We can define the system $\mathcal{M}'=(S', A', \rightarrow', I', AP', L')$ where:

- $A' = A \cup \{ \text{begin} \}$
- $I' = I \times \{ \text{begin} \}$
- $S' = I' \cup (S \times A)$
- If $s_0 \rightarrow_\alpha s$ ($s_0 \in I$), then $(s_0, \text{begin}) \rightarrow'_\alpha (s, \alpha)$.
If $s \rightarrow_\alpha t$ then for all β, $(s, \beta) \in S'$. $(s, \beta) \rightarrow'_\alpha (t, \alpha)$
- $AP' = AP \cup \{ \text{enabled}(\alpha), \text{taken}(\alpha) \mid \alpha \in A \}$
- $L'(s, \alpha) = L(s) \cup \{ \text{taken}(\alpha) \} \cup \{ \text{enabled}(\beta) \mid \beta \in \text{Act}(s) \}$
and $L'(s_0, \text{begin}) = L(s_0) \cup \{ \text{enabled}(\beta) \mid \beta \in \text{Act}(s_0) \}$

Action vs State based Fairness 2

Theorem. traces($\mathcal{M}$) = traces($\mathcal{M}'$). Moreover, strong fairness for a set of actions $F \subseteq A$ can be described by the LTL formula:

$$strongFair_F \equiv \mathbf{G}\ \mathbf{F}\ \text{enabled}(F) \rightarrow \text{taken}(F)$$

[similar for weak fairness and unconditional fairness]

Theorem. Let $\mathcal{M}$ be a transition system without terminal states and let φ be a LTL formula and let $\mathcal{F}$ be a fairness assumption that can be modeled by a LTL formula ψ. Then:

$$\mathcal{M} \models_{\mathcal{F}} \varphi \text{ if and only if } \mathcal{M} \models \text{fair} \rightarrow \varphi$$

Proof. $\mathcal{M} \models$ fair $\rightarrow \varphi$ if and only if $\neg$fair or fair $\wedge\, \varphi$. $\neg$fair is satisfied on all non fair path, whereas fair $\wedge\, \varphi$ holds on all fair paths satisfying fair. $\square$

Be careful about complexity

Best LTL model checking algorithms ***are exponential on the size of the formula*** φ to be verified.

If fairness constraints are modeled by complex LTL formula ψ, the **computational cost** to solve the model checking problem

$$\mathcal{M} \vDash \text{fair} \rightarrow \varphi$$

could be huge!

For example, if fairness constraints are described by n set of actions $A_1, \ldots, A_n$ the formula is

$$\bigwedge_{i=1,\ \ldots,\ n} \mathbf{G}\ \mathbf{F}\ \text{enabled}(A_i) \rightarrow \text{taken}(A_i)$$

Lesson 5c:

Fairness in CTL & CTL Model Checking with fairness constraints

Strong Fairness in CTL

We will consider **strong fairness constraints** of the form:

$$sfair = \bigwedge_{1 \le i \le k} \mathbf{G}\,\mathbf{F}\,\varphi_i \rightarrow \mathbf{G}\,\mathbf{F}\,\psi_i$$

Where φ_i and ψ_i are **CTL formulas** (without fairness). Observe that being CTL formulas, **φ_i and ψ_i identify a set of states** of a Kripke structure $\mathcal{M}$: Sat(φ_i) = $\{ s \mid \mathcal{M}, s \vDash \varphi_i \}$.

On the other hand, given a path $\pi = s_0 s_1 s_2 \ldots$, we have:

$$\pi \vDash_{\mathrm{LTL}} \mathbf{G}\,\mathbf{F}\,\varphi_i \rightarrow \mathbf{G}\,\mathbf{F}\,\psi_i$$

if for all $1 \le i \le k$, there exists j such that $s_j \vDash_{\mathrm{CTL}} \varphi_i$ for **finitely many** indices j [$\neg\, \mathbf{G}\,\mathbf{F}\,\varphi_i$], or $s_j \vDash_{\mathrm{CTL}} \psi_i$ for **infinitely many** indices [$\mathbf{G}\,\mathbf{F}\,\varphi_i \wedge \mathbf{G}\,\mathbf{F}\,\psi_i$] (remember that $a \rightarrow b \equiv \neg a \vee b$).

A path π is fair $\mathcal{M}$, if $\pi \vDash_{\mathrm{LTL}} \mathbf{G}\,\mathbf{F}\,\varphi_i \rightarrow \mathbf{G}\,\mathbf{F}\,\psi_i$. We denote with:

- fairPaths(s) the set of fair paths starting in a state s,
- fairPaths($\mathcal{M}$) the set of fair paths starting in an initial state s_0 of $\mathcal{M}$.

Fairness is not expressible in CTL

Formulas of the form **G F** $\varphi \rightarrow$ **G F** ψ are not in CTL, because:

1. The formula **G F** φ has two consecutive temporal operators
2. The boolean connective $\rightarrow$ is applied to two path formulas

In CTL we **must change the semantics** of **E** and **A** stipulating that they quantify over **fair paths**.

We define a new $\vDash_F$ semantic satisfaction judgement:

1.	$M, s \vDash_F p$	$\Leftrightarrow$	there exists a fair path starting from s and $p \in L(s)$.
5.	$M, s \vDash_F \mathbf{E}(g_1)$	$\Leftrightarrow$	there exists a fair path π starting from s such that $\pi \vDash_F g_1$.
6.	$M, s \vDash_F \mathbf{A}(g_1)$	$\Leftrightarrow$	for all fair paths π starting from s, $\pi \vDash_F g_1$.

Observe that **1**. influence indirectly also the semantics of temporal operators **X** or **U**!!!

CTL model checking with fairness

Theorem. The CTL model checking problem **with fairness** can be reduced to:

1. The CTL model checking problem **without fairness,** and
2. The problem of computing $\mathrm{Sat}_{\mathbf{fair}}(\mathbf{E\,G}\ a)$ for some $a \in AP$.

Proof: This approach is quite straightforward for the propositional logic fragment, for example $\mathrm{Sat}_{\mathrm{fair}}(a)$ if $a \in L(s)$ and there exists a fair path starting in s, that is $\mathcal{M}, s \vDash_{\mathrm{fair}} \mathbf{E\,G}$ true.

Similarly, of course, for $\mathcal{M}, s \vDash_{\mathrm{fair}} \mathbf{E\,X}\,f$: there must be a fair path starting in s such that $s_1 \vDash f$ and $\mathcal{M}, s_1 \vDash_{\mathrm{fair}} \mathbf{E\,G}$ true.

As for $\mathcal{M}, s \vDash_{\mathrm{fair}} \mathbf{E}\,[f_1\ \mathbf{U}\ f_2]$ there must be a fair path starting in s such that there exist $\mathcal{M}, s_n \vDash_{\mathrm{fair}} f_2$ and $\mathcal{M}, s_i \vDash_{\mathrm{fair}} f_1$, for all $1 \leq i \leq n$ and $s_n \vDash_{\mathrm{fair}} \mathbf{E\,G}$ true (**observe that only the infinite suffix is relevant for fairness**).

Obviously, in the iterative CTL algorithm, $\mathcal{M}, s \vDash_{\mathrm{fair}} \mathbf{E\,G}\,f$ is applied when f has been processed, and hence the problem is to check $\mathcal{M}, s \vDash \mathbf{E\,G}\ a_f$ with a_f atomic proposition. □

Summing up…

Let a_{fair} be a fresh atomic proposition such that:

$a_{\text{fair}} \in L(s)$ if and only if $s \in \text{Sat}_{\textbf{fair}}(\mathbf{E}\,\mathbf{G}\ \text{true}) \equiv \mathcal{M}, s \models_{\text{fair}} \mathbf{E}\,\mathbf{G}\ \text{true}$

Then:

$$\text{Sat}_{\textbf{fair}}(\mathbf{E}\,\mathbf{X}\,a) \equiv \text{Sat}(\mathbf{E}\,\mathbf{X}\,a \wedge a_{\text{fair}})$$

$$\text{Sat}_{\textbf{fair}}(\mathbf{E}\,[a\ \mathbf{U}\ a']) \equiv \text{Sat}(\mathbf{E}\,[a\ \mathbf{U}\ a' \wedge a_{\text{fair}}])$$

And those on **the right-hand side are pure CTL formulas** that can be computed by the usual CTL algorithm (see lesson **3**).

Therefore, **we are left with the problem of computing**:

$$\text{Sat}_{\textbf{fair}}(\mathbf{E}\,\mathbf{G}\,a)$$

That, in particular, can be used to compute $a_{\text{fair}} \in L(s) \equiv s \in \text{Sat}_{\textbf{fair}}(\mathbf{E}\,\mathbf{G}\ \text{true})$.

Checking $\mathcal{M}$, $s \vDash_{fair}$ EG a (1)

Lemma. Let sfair = $\bigwedge_{1 \le i \le k}$ **G F** a_i $\rightarrow$ **G F** b_i be a fair constraint. Then $\mathcal{M}, s \vDash_{\text{sfair}}$ **EG** a if and only if there exists a finite path $s_0 s_1 \ldots s_n$ and a cycle $s'_0\, s'_1 \ldots s'_r$ such that:

i. $s = s_0$ and $s'_0 = s'_r$

ii. $s_i \vDash a$ for all $0 \le i \le n$ and $s'_j \vDash a$ for all $0 \le j \le r$

iii. For all $0 \le i \le k$, Sat(a_i) $\cap \{s'_0, s'_1, \ldots, s'_r\} = \varnothing$ or Sat(b_i) $\cap \{s_0, s_1, \ldots, s_n\} \neq \varnothing$

Proof (**if**): Clearly $s_0 s_1 \ldots s_n (s'_0 s'_1 \ldots s'_r)^\omega$ is a fair path according to sfair satisfying **EG** a.

(**Only if**) $\mathcal{M}, s \vDash_{\text{sfair}}$ **EG** a implies that there exists an infinite fair path $\pi = s_0 s_1 s_2 \ldots$ such that $\pi \vDash_{\text{sfair}}$ **G** a and $\pi \vDash$ sfair. Two cases:

1. $\pi \vDash$ **G F** a_i. This implies exists $s' \vDash b_i$ visited infinitely often in π. Let n and r be the first and second occurrence of s'. Clearly $\{s_0, s_1, \ldots, s_n\}$ and $\{s_n, s_{n+1}, \ldots, s_r\}$ satisfies *iii*.
2. $\pi \nvDash$ **G F** a_i. Then there exists m such that $s_m, s_{m+1}, \ldots \notin$ Sat(b_i). There are finitely many states, there is a cycle $s_n, s_{n+1}, \ldots, s_r$ ($n > m$) such that Sat(a_i) $\cap \{s_n, s_{n+1}, \ldots, s_r\} = \varnothing$. $\square$

Checking $\mathcal{M}, s \vDash_{fair} EG\ a$ (2)

The previous Lemma can be used as follows. Let us consider the graph G_a whose nodes $V_a = \{ s \mid \mathcal{M}, s \vDash a \}$ and edges $E_a = \{(s, s') \in R \mid s, s' \in V_a\}$.

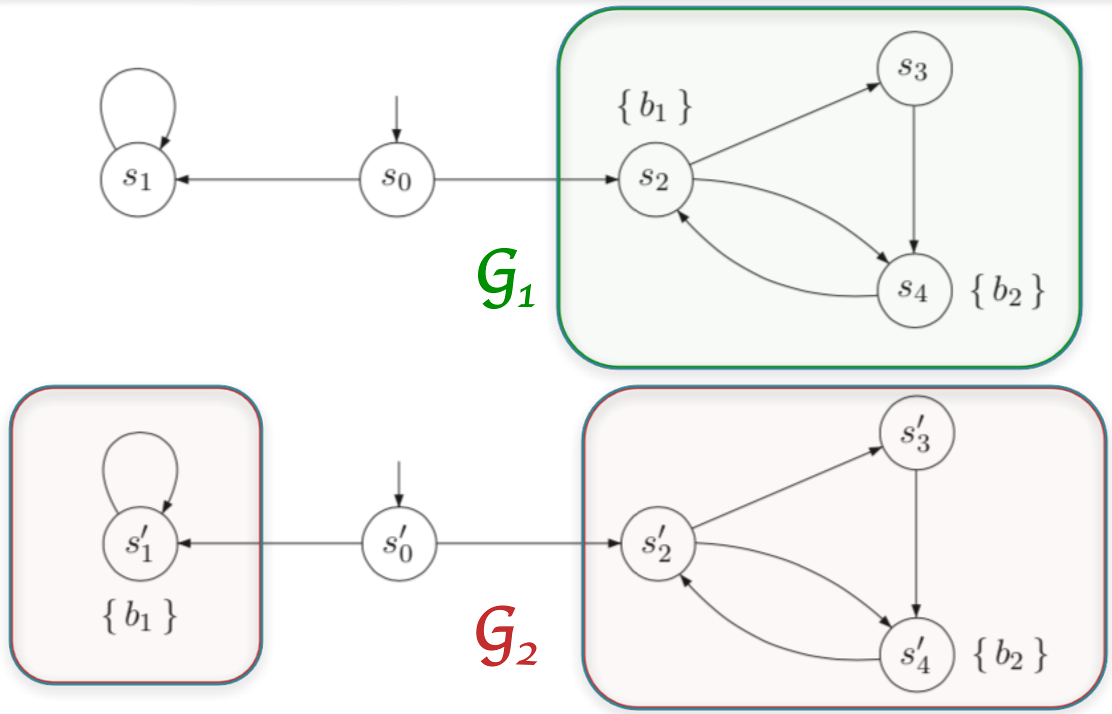
Each infinite path in G_a is a path in $\mathcal{M}$ satisfying $\mathbf{G}\ a$. Conversely, each path in $\mathcal{M}$ satisfying $\mathbf{G}\ a$ is a path in G_a.

$\mathcal{M}, s \vDash_{sfair} \mathbf{EG}\ a$ if and only if there exists a nontrivial SCC C in G_a reachable from s and a set of nodes $D \subseteq C$ such that for all $0 \leq i \leq k$, $D \cap \mathrm{Sat}(a_i) = \varnothing$ or $D \cap \mathrm{Sat}(b_i) \neq \varnothing$.

$$\mathrm{Sat}_{fair}(\mathbf{EG}\ a) = \{s \mid \text{exists } C \text{ reachable from } s \text{ in } G_a\}$$

Unconditional Fairness: in this case, a_i is true for all i. Observe that in this case, fair paths **correspond to accepting runs of a Generalised Büchi automaton**.

Example: unconditional fairness



$\mathcal{G}_1$ satisfy unconditional fairness constraint $\mathbf{G}\,\mathbf{F}\, b_1 \wedge \mathbf{G}\,\mathbf{F}\, b_2$ **because there is the SCC $\{s_2, s_3, s_4\}$.**

By contrast, $\mathcal{G}_2$ does not satisfy $\mathbf{G}\,\mathbf{F}\, b_1 \wedge \mathbf{G}\,\mathbf{F}\, b_2$ because there is the SCC $\{s_2, s_3, s_4\}$ that contains b_2 and the SCC $\{s_1\}$ that contains only b_1, but **no one of them contains both** b_1 and b_2.

That's all Folks!

Thanks for your attention…

…Questions?