Formal Methods in Software Development

Written Test – Rome, January 22, 2020

Name
Surname

Q1 (Modelling): In a Kripke structure there is always at least an infinite path. Why?

Q2 (Temporal Logic): Let g be an atomic proposition. Exhibit two Kripke structures $\mathcal{M}_1$ and $\mathcal{M}_2$ such that: *a*) $\mathcal{M}_1 \models \mathbf{GF}\, g$ but $\mathcal{M}_1 \not\models \mathbf{FG}\, g$, and *b*) $\mathcal{M}_2 \models \mathbf{FG}\, g$. [Hint: $\mathcal{M}_1$ and $\mathcal{M}_2$ can have just two states.]

Q3 (Safety Properties): Let AP be a set of atomic propositions and let $P_1, P_2 \subseteq (2^{AP})^\omega$ be two safety properties. Is $P_1 \cap P_2$ a safety property? Motivate your answer shortly.

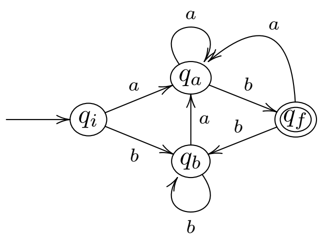
Q4 (Fixed Points): Let S be a *finite* set. Define an operator $T : 2^S \mapsto 2^S$ that does not have any fixed point.

Q5 (Equivalences): Among the following temporal logics, which ones are always invariant on equivalence classes induced by a bisimulation?

☐ LTL
☐ CTL*
☐ CTL
☐ None of them

Q6 (Algorithms): Let $T : 2^S \mapsto 2^S$ be monotone. Why does the sequence $T^n(\varnothing)$ converge to the minimum fixpoint in at most $|S|$ steps?

Q7 (Automata): Which is the language recognised by the following Büchi automaton?



☐ $\{w \in \{a,b\}^\omega \mid w$ contains infinitely many $a\}$
☐ $\{w \in \{a,b\}^\omega \mid w$ contains infinitely many $b\}$
☐ $\{w \in \{a,b\}^\omega \mid w$ contains infinitely many sequences $ab\}$
☐ $\{w \in \{a,b\}^\omega \mid w$ contains finitely many a or finitely many $b\}$

Q8 (Probabilistic Model Checking and Fairness): Provide an example of a Markov chain such that, for some state s we have: a) in the corresponding Kripke structure, $s \not\models \mathbf{F}\, g$, b) $Pr(s \models \mathbf{F}\, g) = 1$, and c) $s \models_{\text{Fair}} \mathbf{F}\, g$, for suitable fairness constraints.
