University of Utah

Explicit Algorithms for Probabilistic Model Checking

Igor Melatti

Thesis Contents

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  - Formal description
  - Proof of correctness
  - Implementation (FHP-Murφ)

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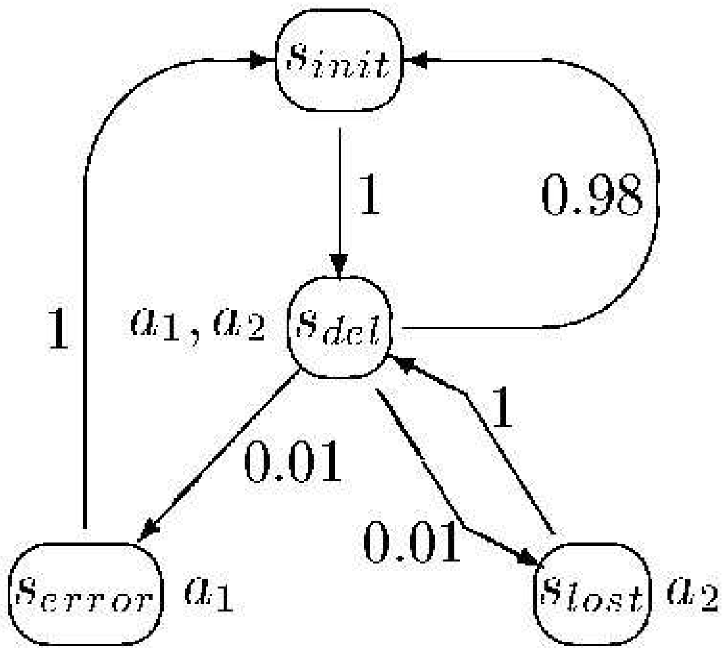
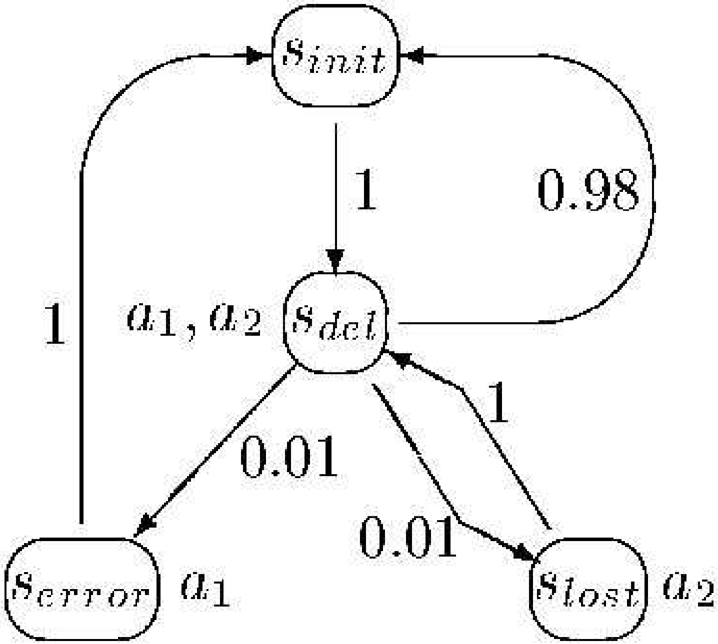
    - Verification of a "real-world" system
- Formal analysis of the proposed Markov Chain description language



Baier *et al.*, "Symbolic model checking for probabilistic processes", *ICALP'97*, LNCS 1256



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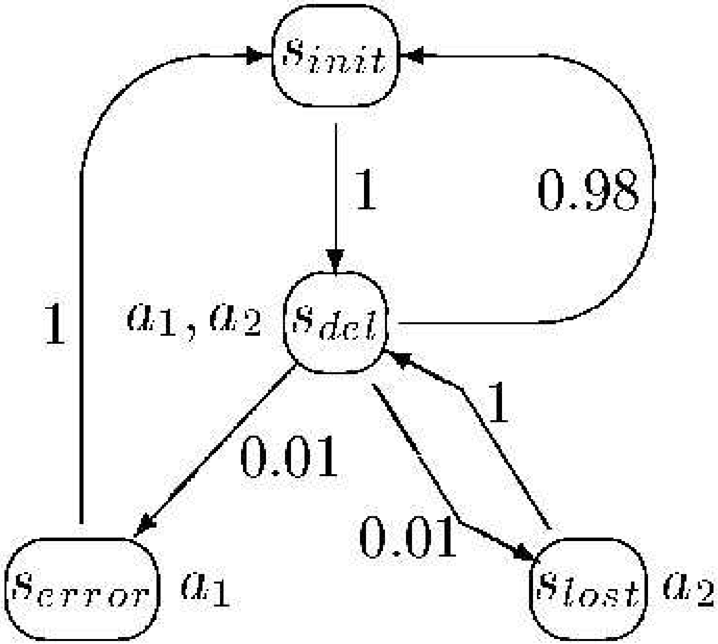
s_{init} the state in which the sender passes the message to the medium



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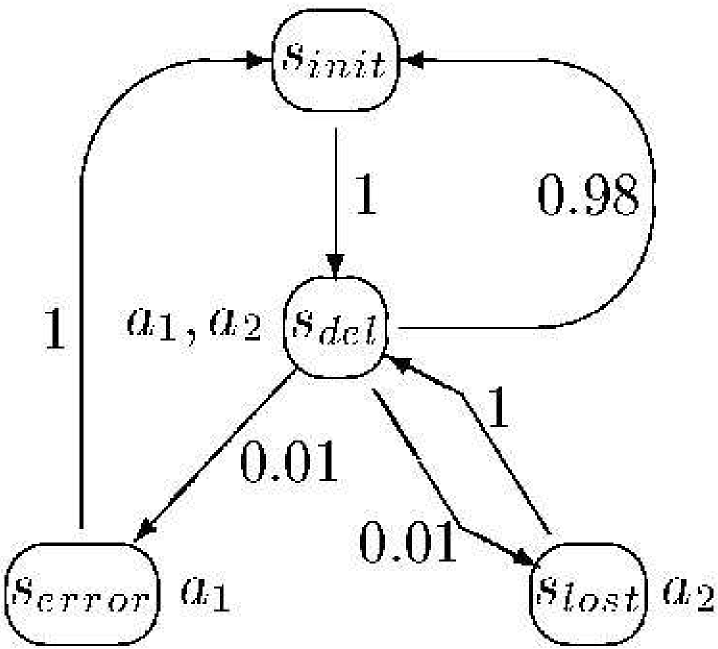
s_{del} the state in which the medium tries to deliver the message



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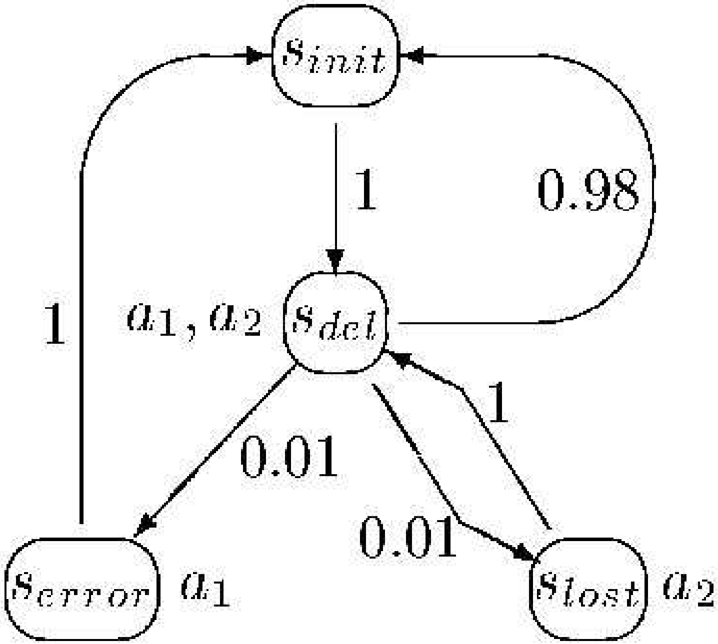

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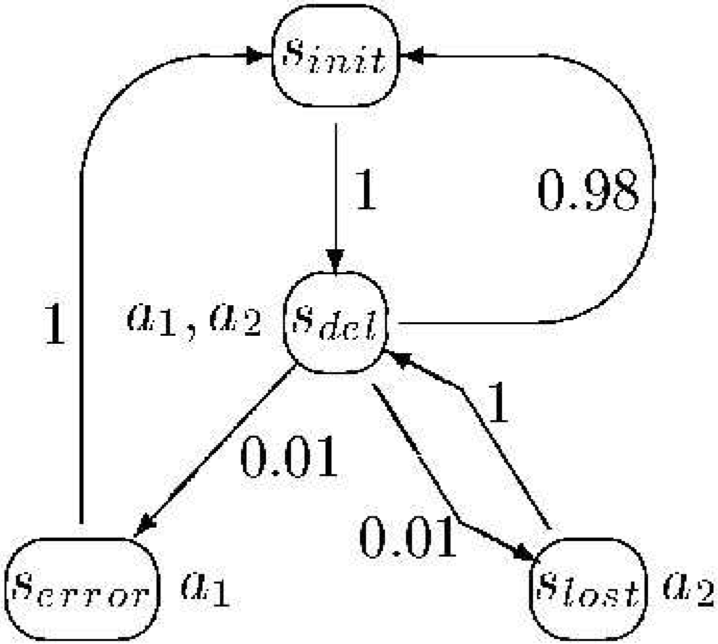
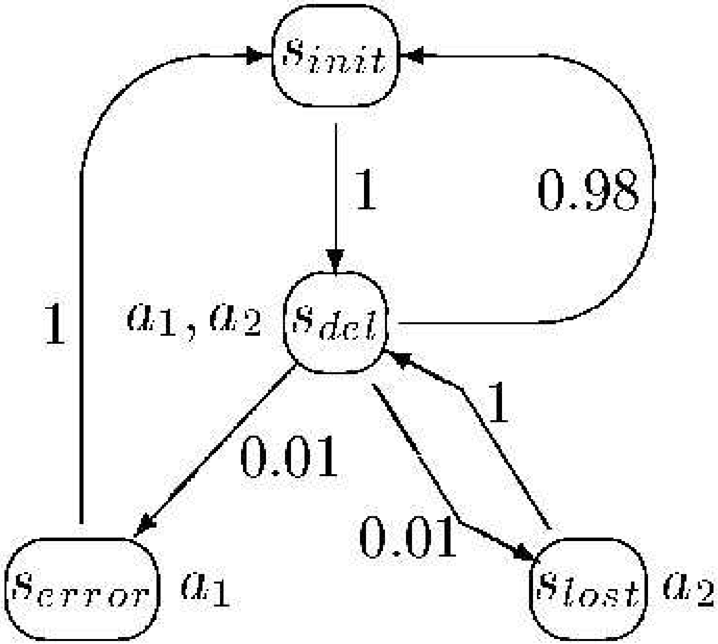
- Probability of a finite path ρ: $\mathbf{P}(\rho) = \prod_{i=0}^{|\rho|-1} \mathbf{P}(\rho(i), \rho(i+1))$.


s_{init}
1
0.98
1
a_1, a_2
s_{del}
1
0.01
0.01
s_{error}
a_1
s_{lost}
a_2



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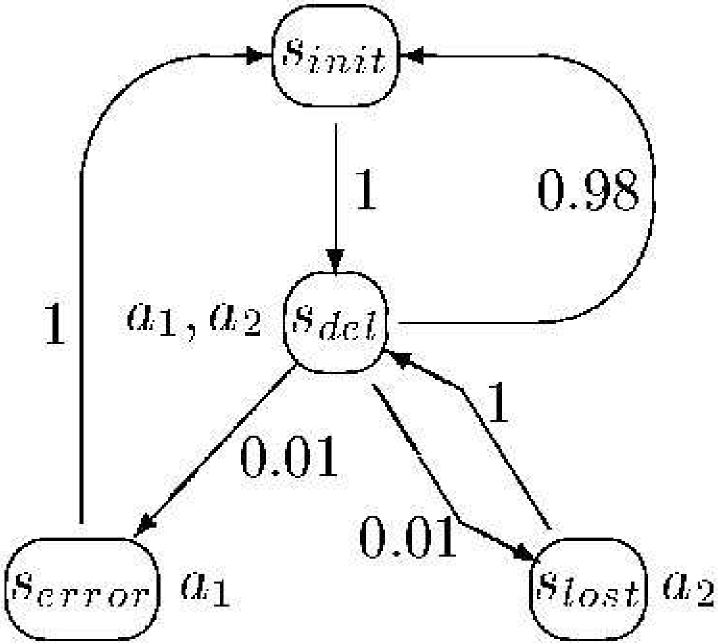
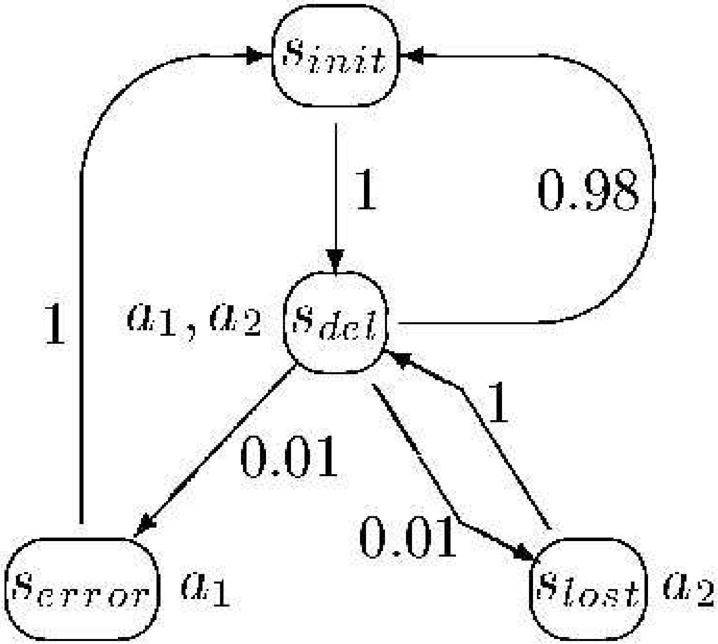


- $S = \{s_{init}, s_{del}, s_{lost}, s_{error}\}$
- $$\mathbf{P} = \begin{array}{cccc} s_{init} & s_{del} & s_{lost} & s_{error} \end{array} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0.98 & 0 & 0.01 & 0.01 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$



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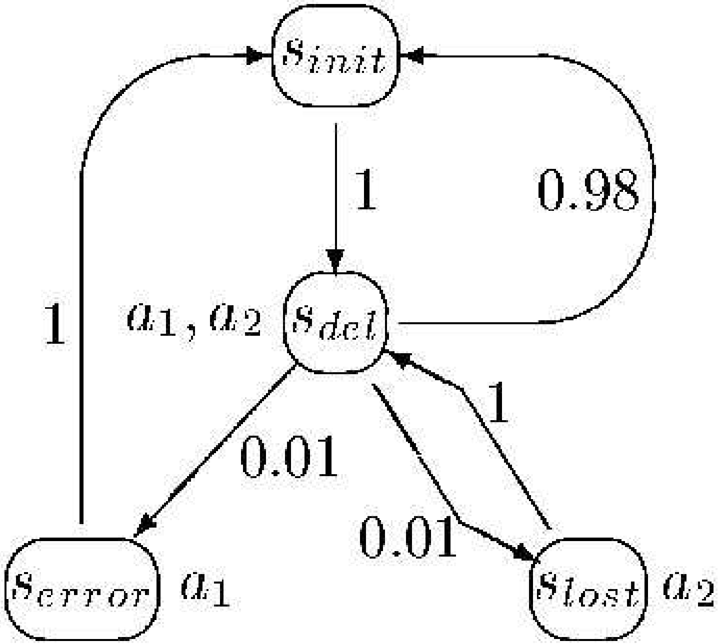
- 2 possible paths and their probabilities:
  - $\mathbf{P}(s_{init}s_{del}s_{error}s_{init}) = 1 \cdot \frac{1}{100} \cdot 1 = \frac{1}{100}$
  - $\mathbf{P}(s_{init}(s_{del}s_{lost})^k s_{del}s_{init}) = 1 \cdot (\frac{1}{100} \cdot 1)^k \cdot \frac{98}{100} = \frac{98}{10^{2(k+1)}}$



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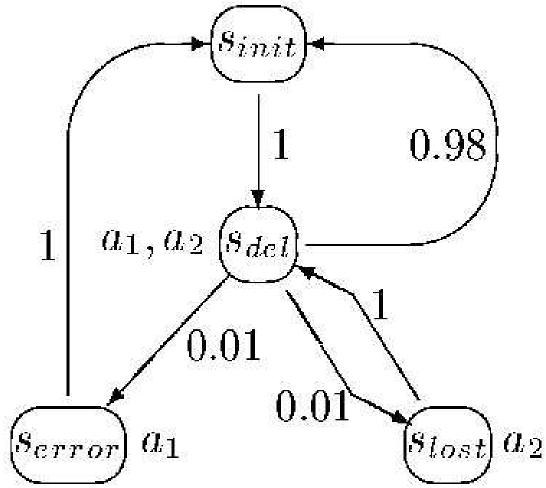
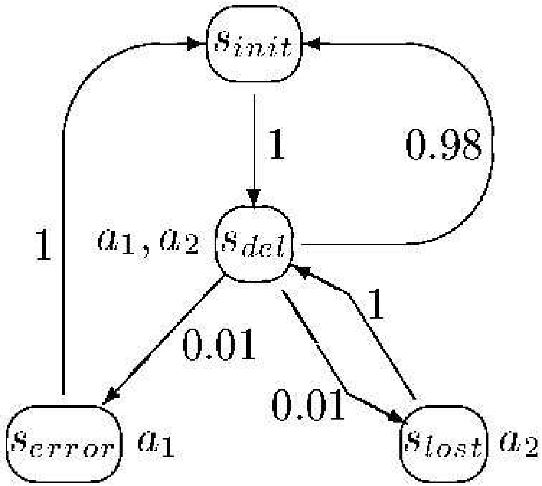
- BPCTL: Bounded PCTL
  - Proper subset of PCTL
  - All Untils (**U**) must be bounded
  - $[tt\ \mathbf{U}^{\leq k_1}\ (\neg\phi \wedge \neg[tt\ \mathbf{U}^{\leq k_2}\phi]_{\geq 1})]_{\leq 0}$
  - $[tt\ \mathbf{U}^{\leq k_1}\ (\phi_{und} \wedge \neg[tt\ \mathbf{U}^{\leq k_2}\neg\phi_{err}]_{\geq 1})]_{\leq 0}$



Property: $[\text{Try\_to\_deliver}\ U^{\leq 100}\ \text{Correctly\_delivered}]_{\geq 0.9}$



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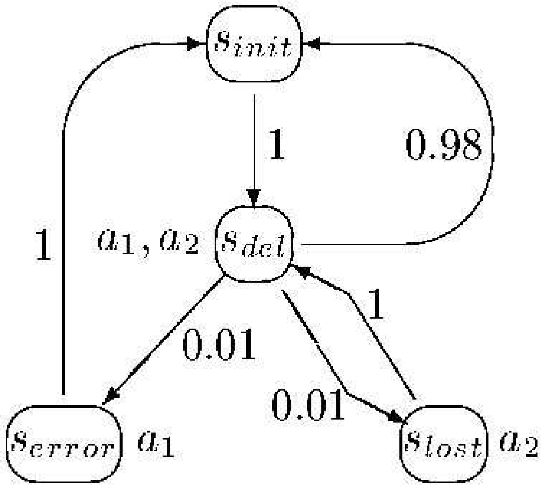
- Try_to_deliver (T in the following) is true if we are in state s_{del} or s_{lost}
- Correctly_delivered (C in the following) is true if we are in state s_{init}
- Initial state is s_{del}



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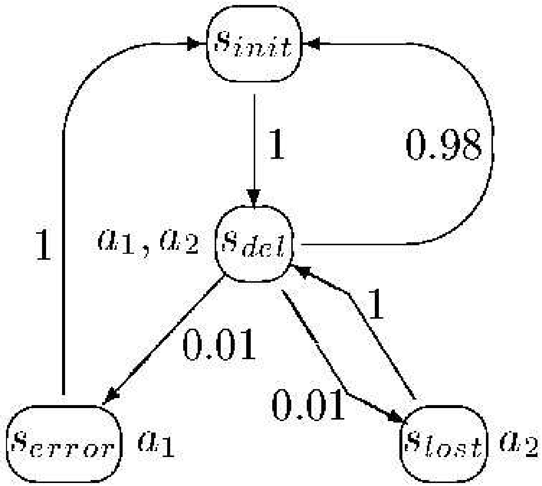

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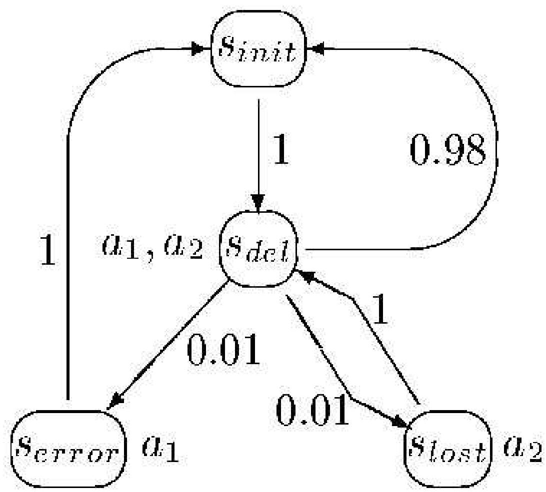

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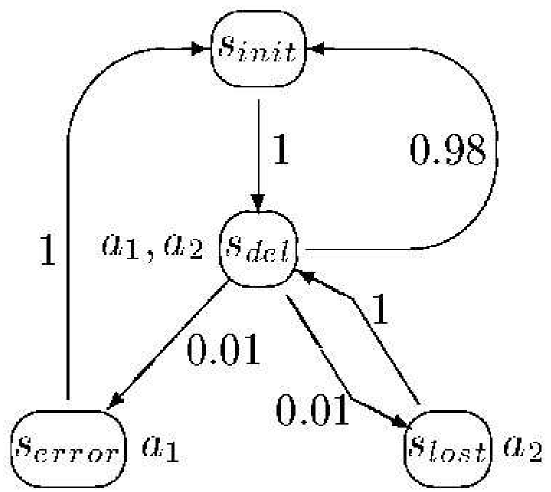
- Is the probability of the paths of the form $T^k C$ ($0 \leq k \leq 100$) at least 0.9?
  - A path of the form $T^k C$ corresponds to an execution of the system in which, after a bounded trials, the message is finally transmitted
  - Thus, we are requiring the probability of a "correct behavior" to be high enough (i.e. ≥ 0.9)



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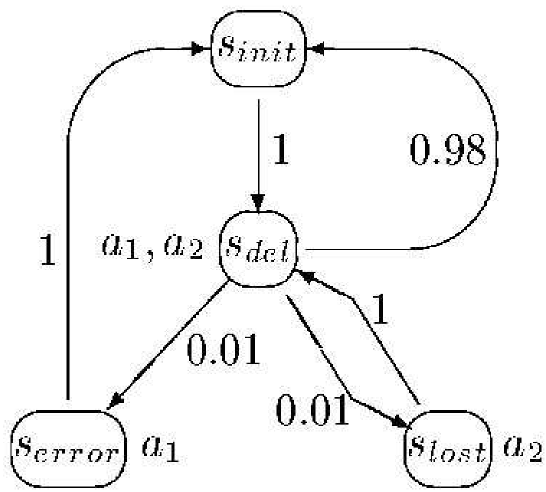

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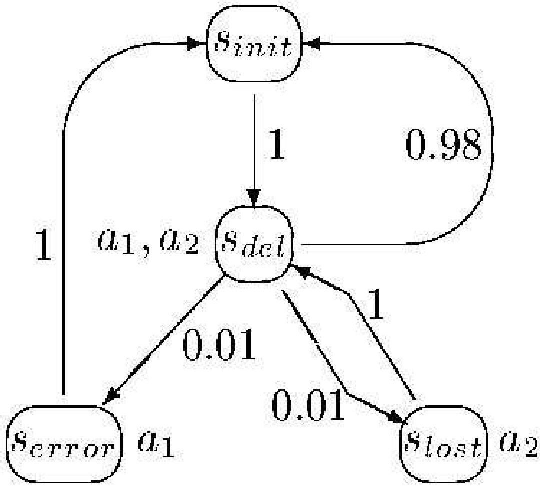

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- Given a BPCTL formula $[\Phi]_{\geq 0.9}$, our algorithms computes $P[\Phi] = \sum_{\pi | \pi \models \Phi} \mathbf{P}(\pi)$

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  - If it is not, an exponential amount of RAM memory is needed
  - Our approach tries to avoid this, at least for some classes of Markov Chains

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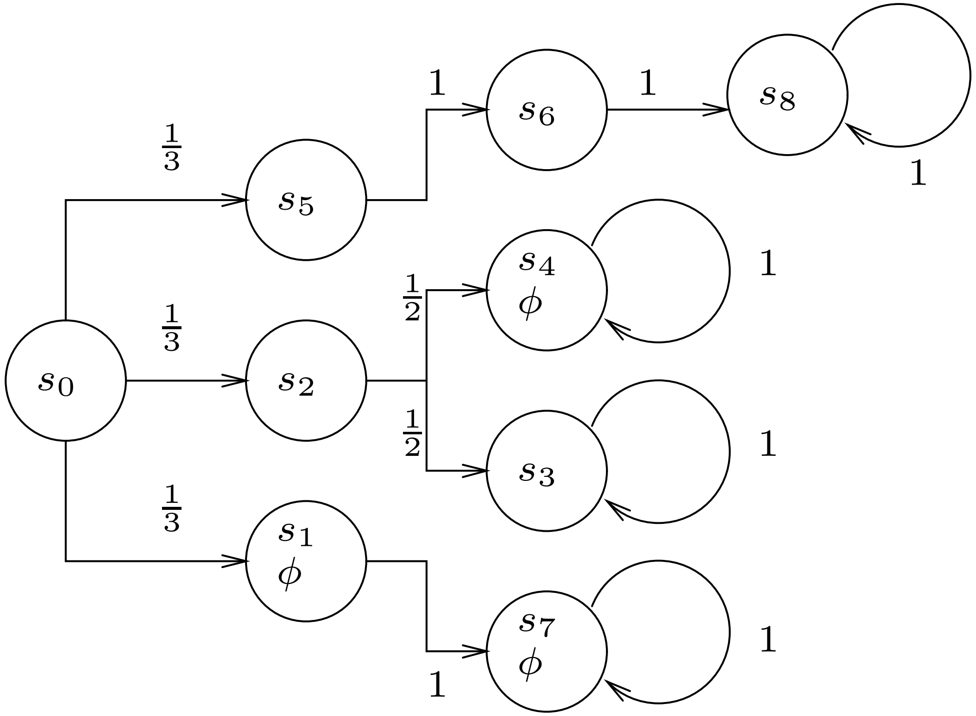
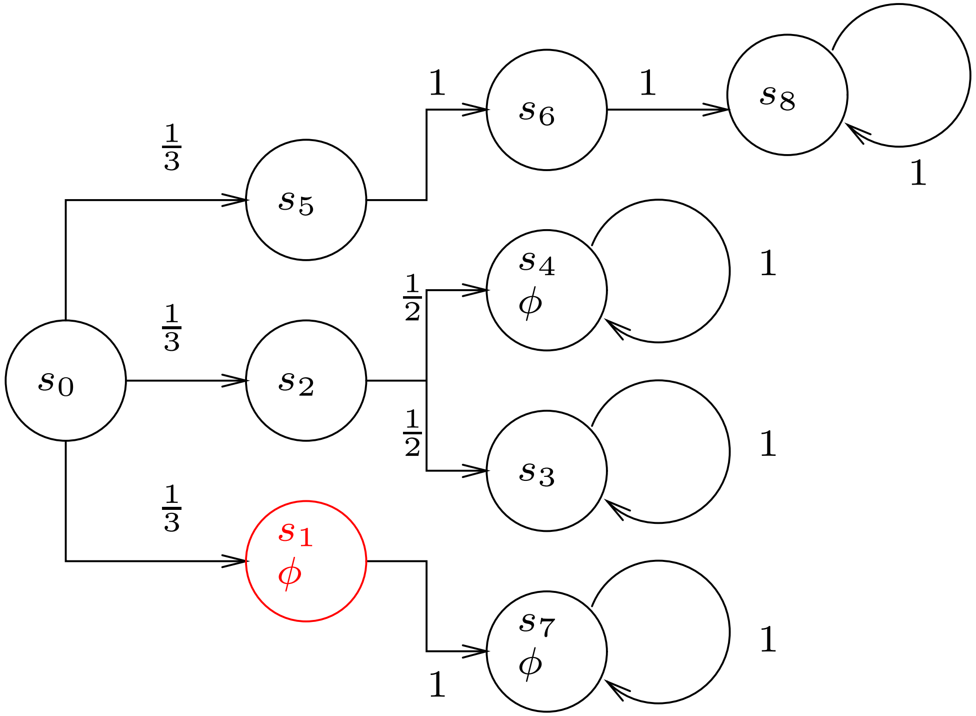
- Murφ modified in the input language and in the verification algorithm
- Two explicit algorithms developed
  - BF visit: only for finite horizon safety properties
    - $*$ Able to compute error probabilities
  - DF visit: all BPCTL formulas



- We want to verify if $s_0 \models [tt\ \mathbf{U}^{\leq 2}\ \phi]_{\geq 0.5}$
- ϕ holds in s_1, s_4, s_7
- The searched probability is: 0

The Basic DF Visit



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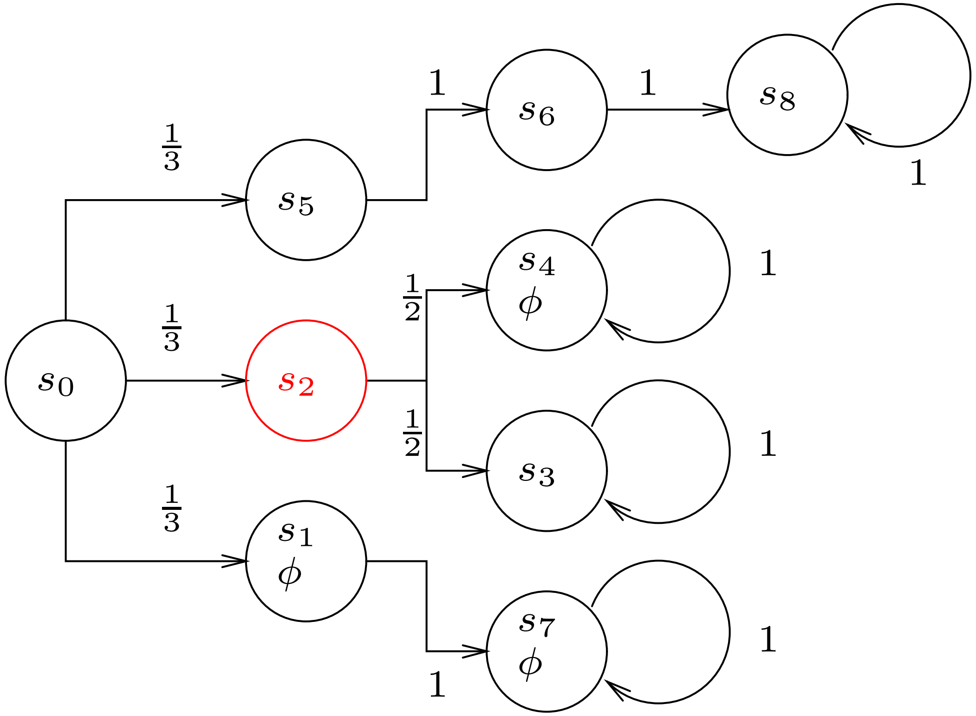

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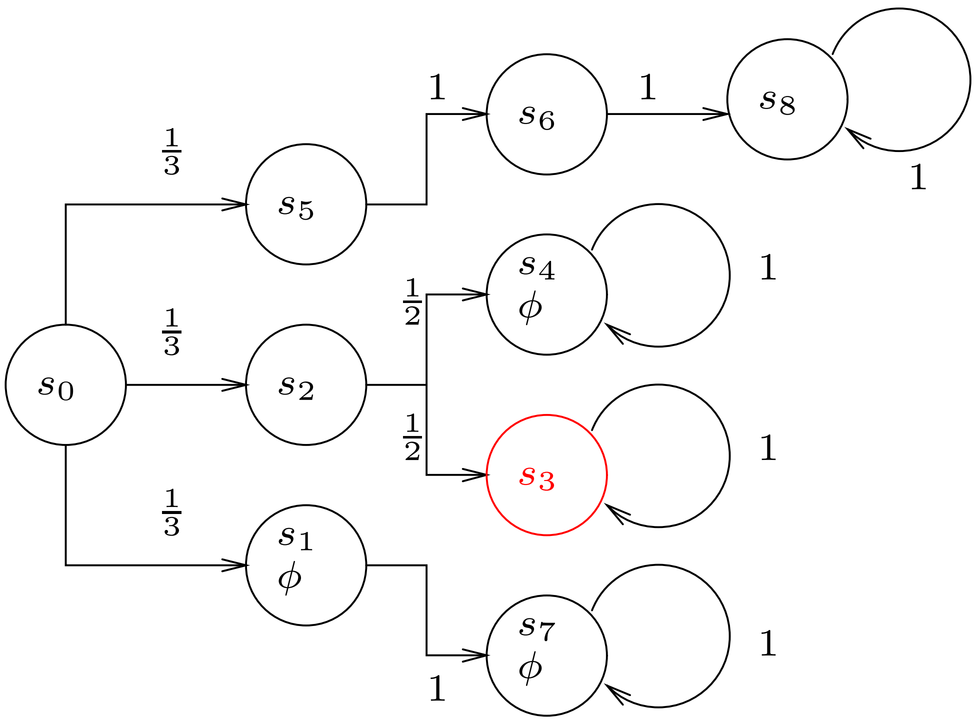

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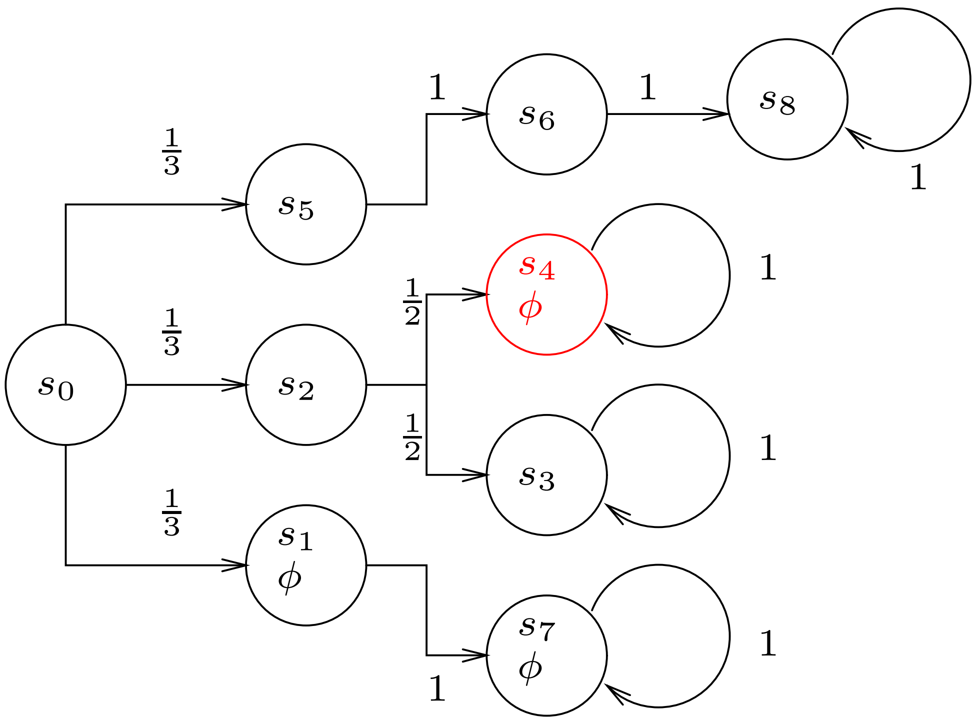

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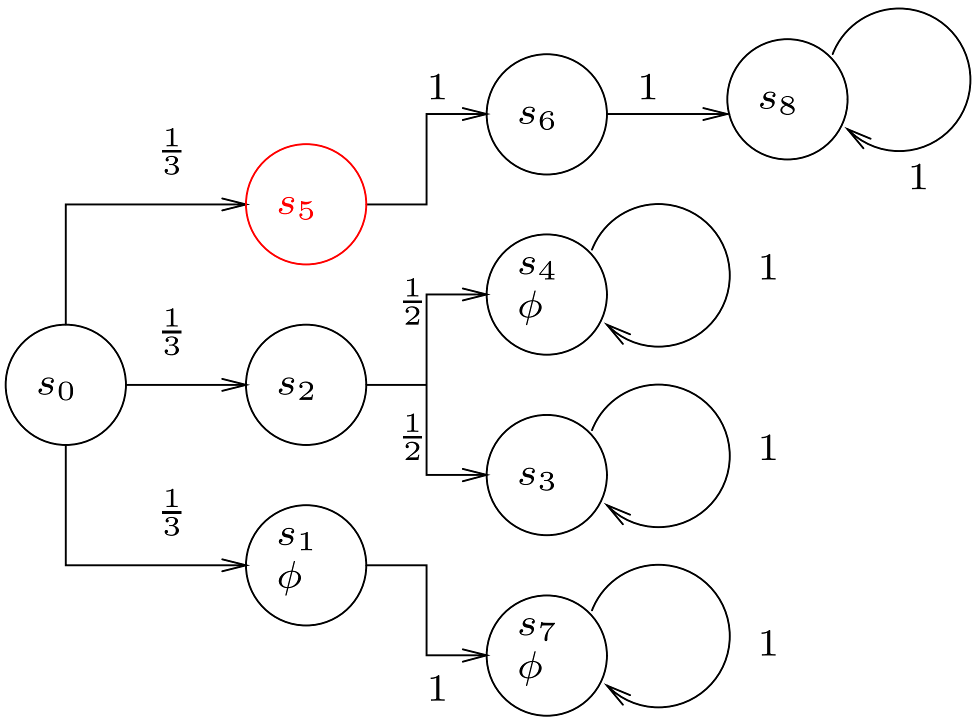

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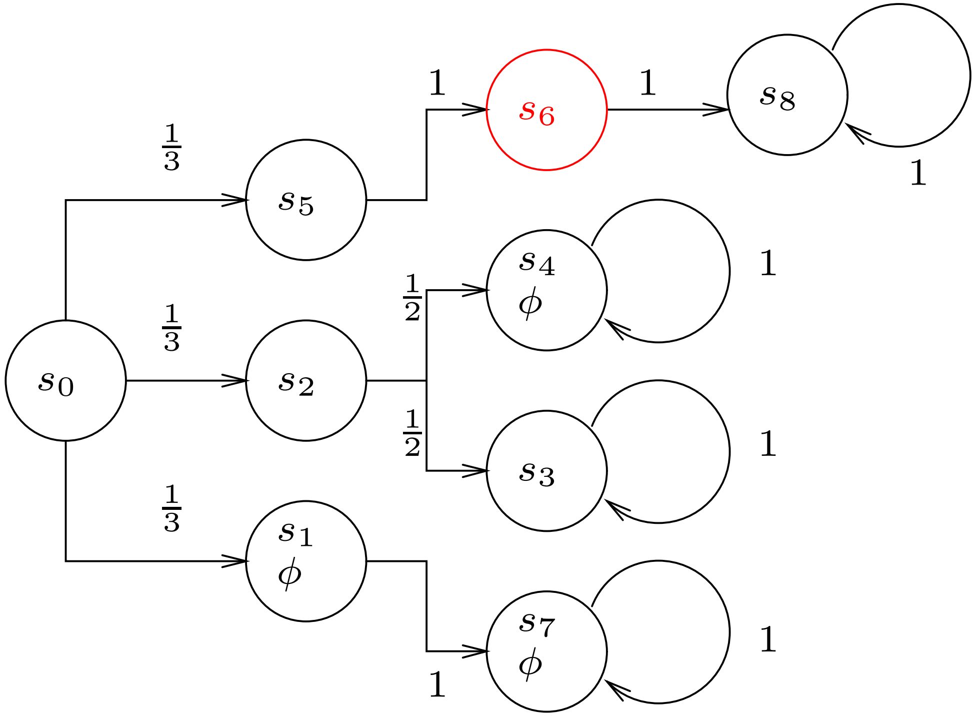

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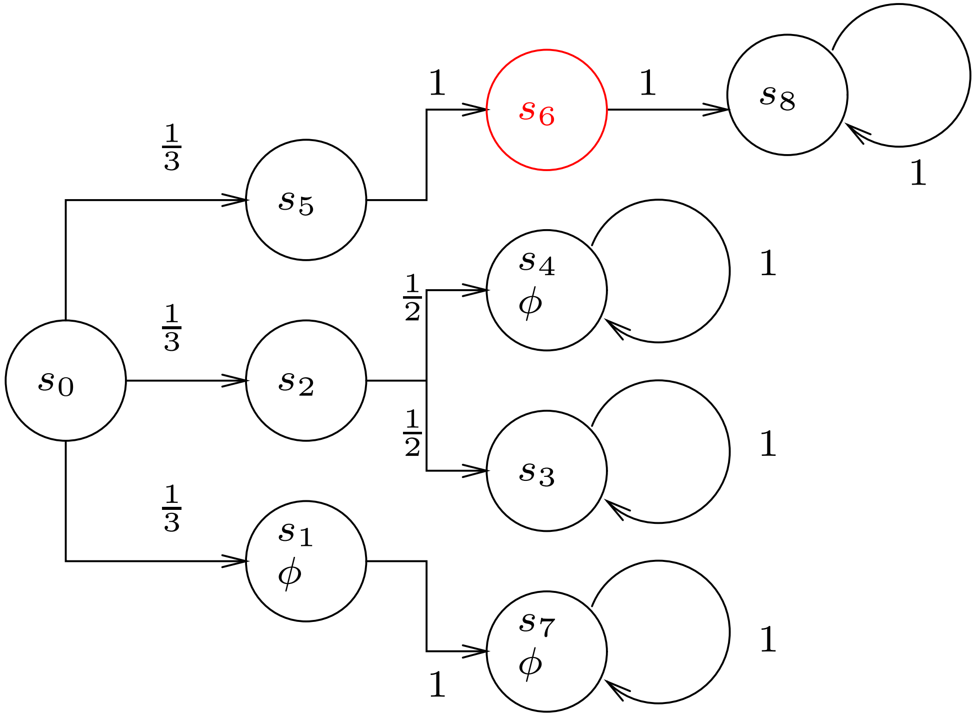

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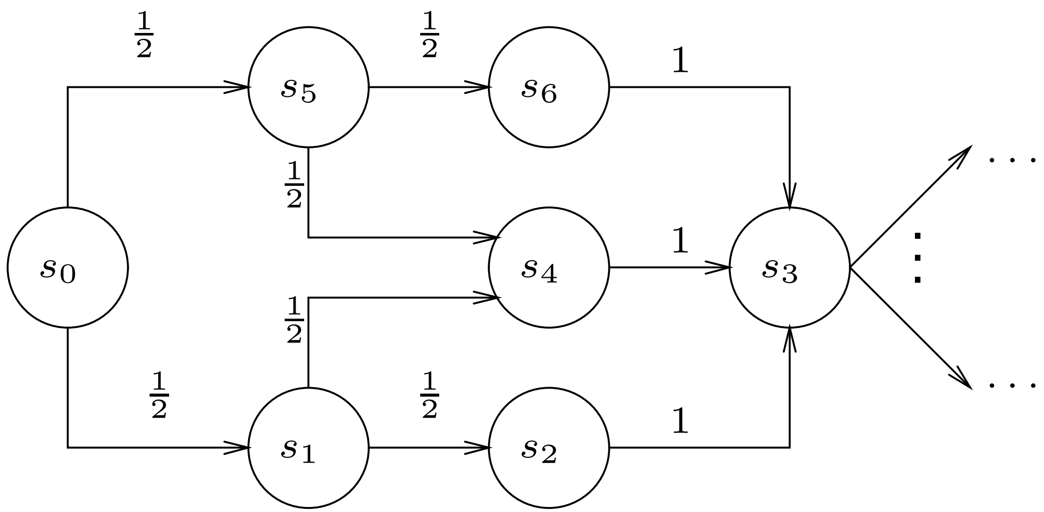
- ϕ holds in s_1, s_4, s_7
- The searched probability is: $\frac{1}{3} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 0$
- Finally, we have $\frac{1}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{1}{2} \geq 0.5$, so the property is verified

Cache Usage (1)



- We want to verify if $s_0 \models F$, being $F \equiv [\Phi \ \mathbf{U}^{\leq k} \ \Psi]_{\leq 0.5}$
- The cache stores 4-tuples $\{s, F, h, p\}$
  - p is the probability of $\Phi \ \mathbf{U}^{\leq h} \ \Psi$

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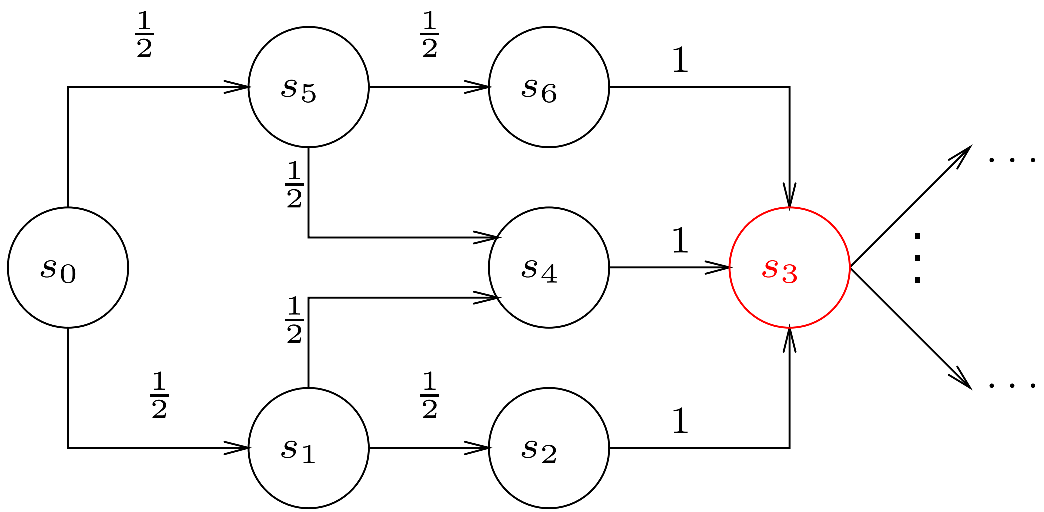

- When the DF visit of s_3 is completed, $\{s_3, F, k-3, p_3\}$ is inserted in the cache
  - p_3 is the probability value computed by the DF on s_3
  - k is decremented of 3 because s_3 is reached in 3 steps from s_0

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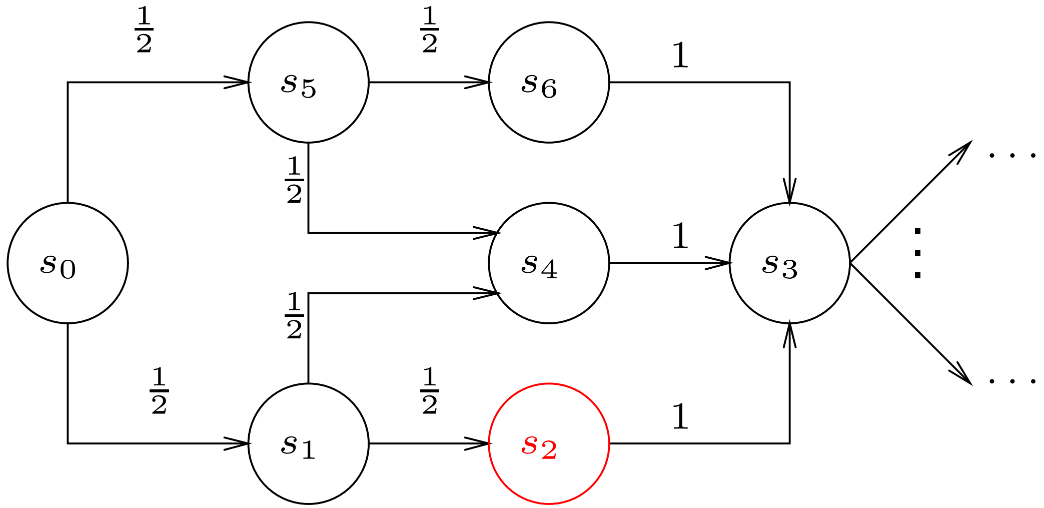

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  - p_3 is the probability value computed by the DF on s_3
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- Analogously, $\{s_2, F, k-2, p_2\}$ is inserted in the cache

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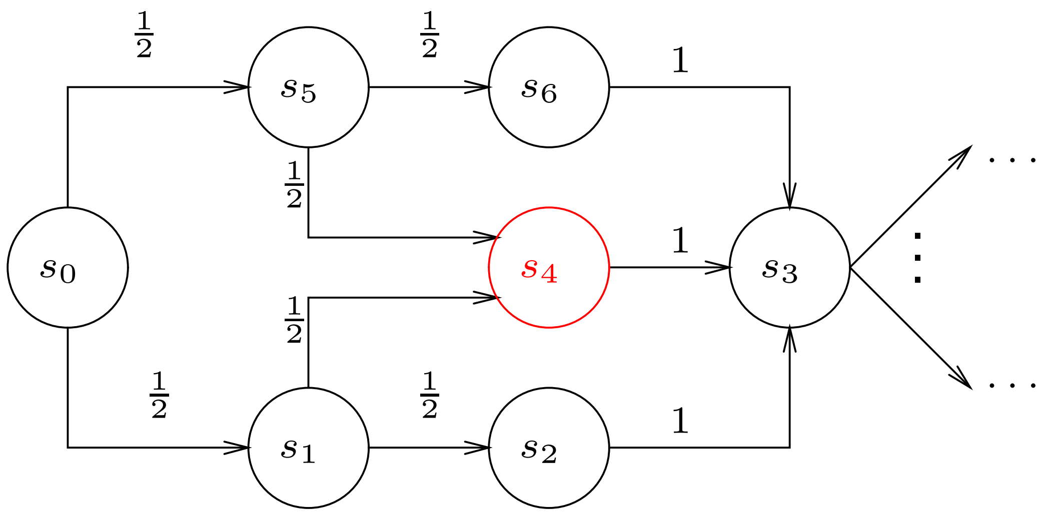
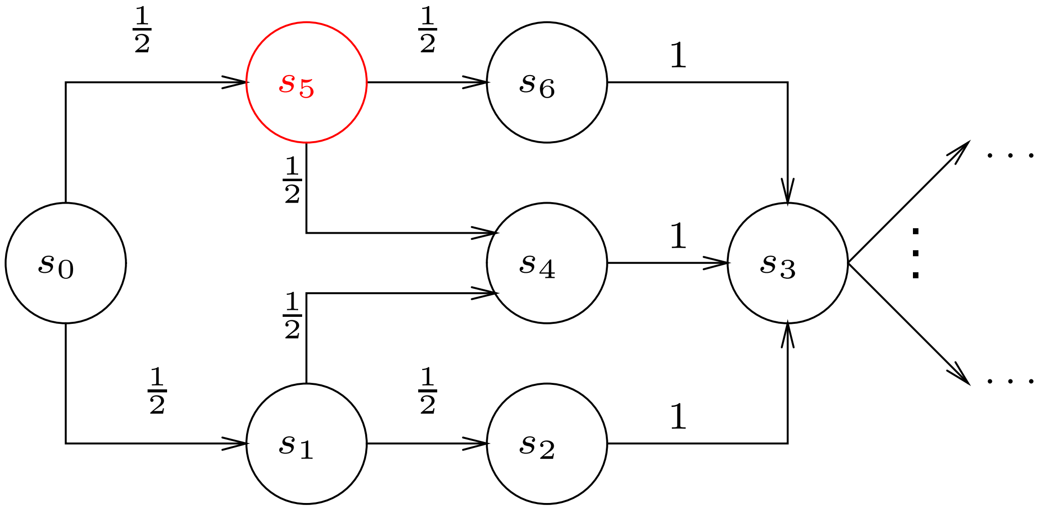


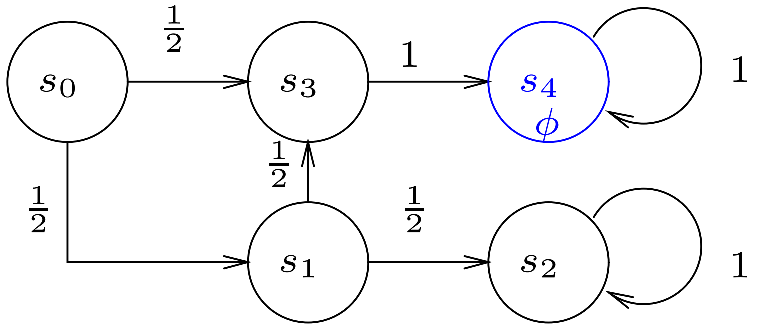
- In this way, the DF visit of s_4 can directly compute $p_4 = p_3 \times 1$
  - p_3 is not computed, but it is found on the cache
- Then, $\{s_4, F, k-2, p_4\}$ is inserted in the cache

Cache Usage (1)



- Analogously, when the DF visit of s_5 starts, the nested DF visit of s_4 is skipped
  - p_4 is not computed, but it is found on the cache
- The result of the DF visit of s_6 will be multiplied by $\frac{1}{2}$ and then added to $\frac{1}{2} \times p_4$

Cache Usage (2)



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  - $\phi(s_4) = 1, \forall s \neq s_4.\ \phi(s) = 0.$

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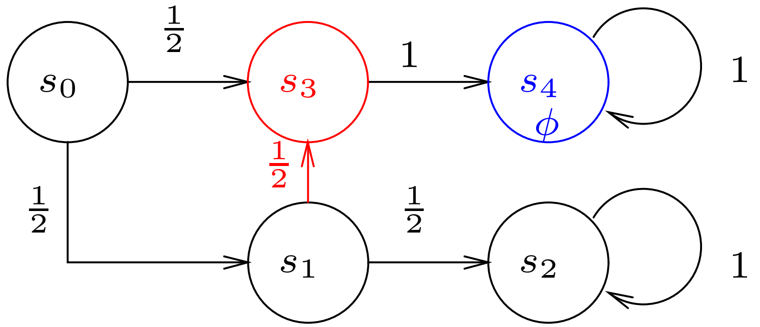

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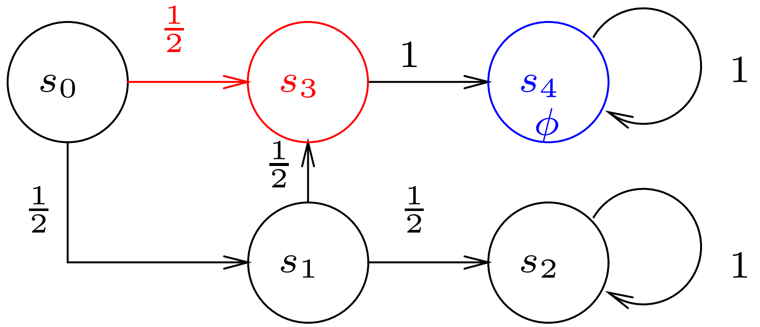

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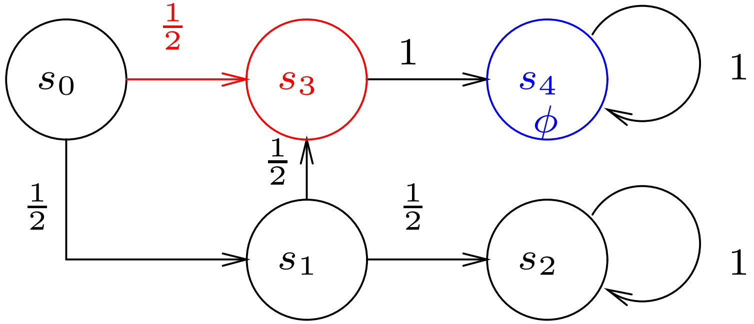

- s_3 is visited for the second time as a successor of s_0
  - The required 4-tuple $< s_3, \Phi, 2, p >$ is not on the cache

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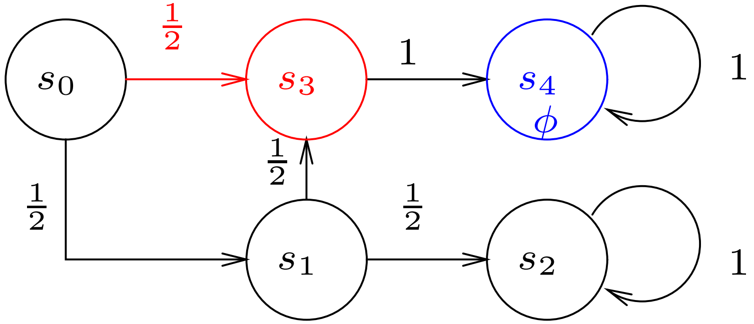

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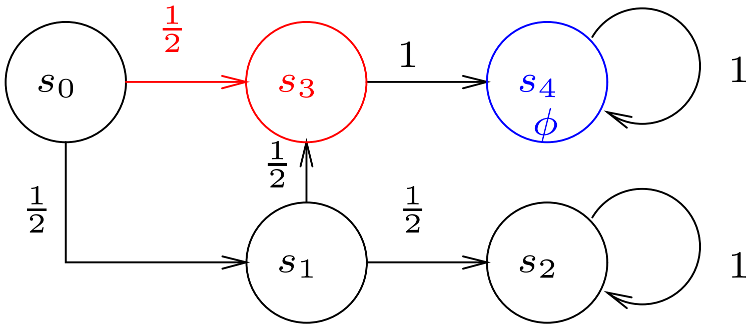

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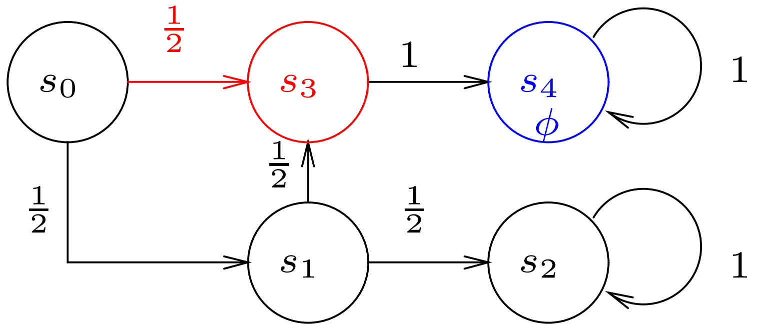

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  - So, the second DF visit on s_3 is avoided

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Hybrid systems Verification of a turbogas control system, assuming a probability distribution on the user demand

- Probabilistic Safety Verification
- Probabilistic Robustness Verification

NPHIL	MAX_WAIT	Result	Murφ Mem (MB)	PRISM Mem (MB)	Murφ Time (s)	PRISM Time (s)
Modified Pnueli-Zuck						
5	3	false	5.0e+2	9.168246e+02	1.28381900e+04	1.196793e+03
5	4	false	5.0e+2	N/A	1.27377300e+04	N/A
Modified Lehmann-Rabin						
3	4	true	5.0e+2	7.014830e+01	5.00634000e+03	5.359870e+02
4	3	true	5.0e+2	N/A	1.11480680e+05	N/A

Property verified:

- If a philosopher risks to die, then it will eat soon
- $[tt \ \mathbf{U}^{\leq k_1} \ (\phi_{und} \wedge \neg[tt \ \mathbf{U}^{\leq k_2} \neg\phi_{err}]_{\geq 1})]_{\leq 0}$.

`NPHIL`, `MAX_WAIT`: protocol parameters

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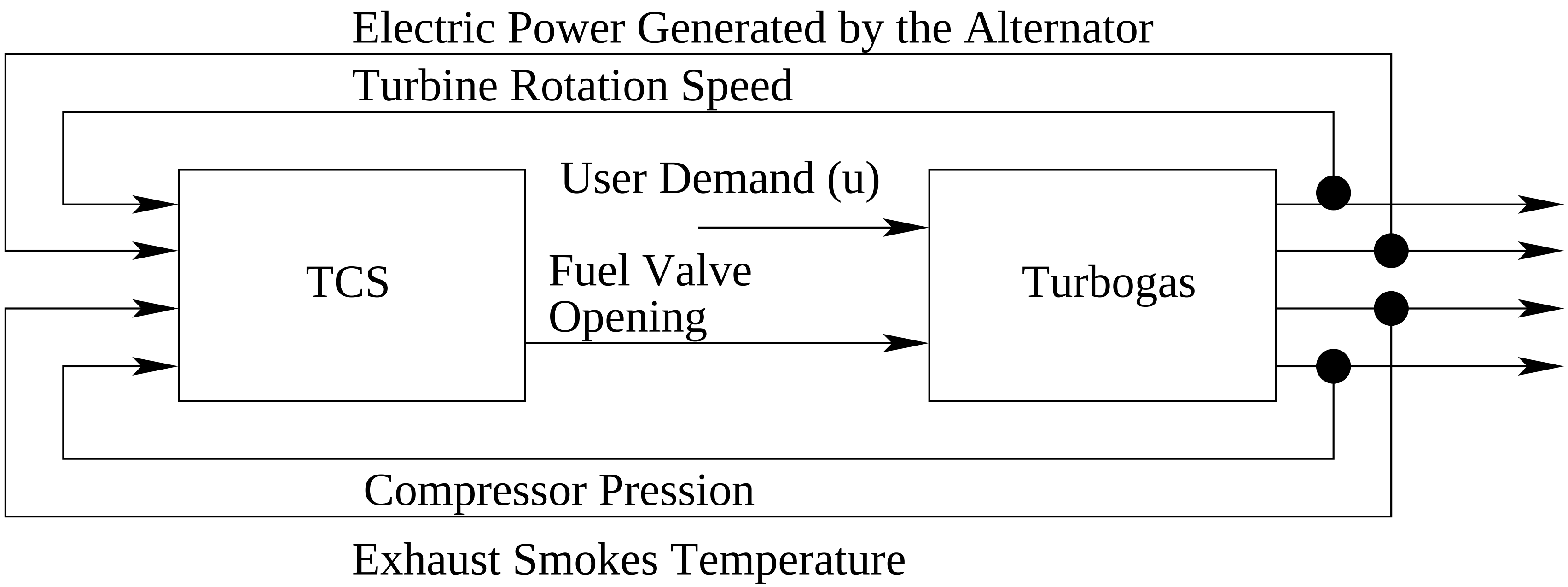
  - It is also the most complex one
- It is an hybrid system: it has both continuous (e.g., power and user demand) and discrete variables (execution modality)
  - This kind of systems are hard to analyze with OBDD-based model checkers
  - Thus, there is no hope to verify TCS with PRISM

A Turbogas Control System



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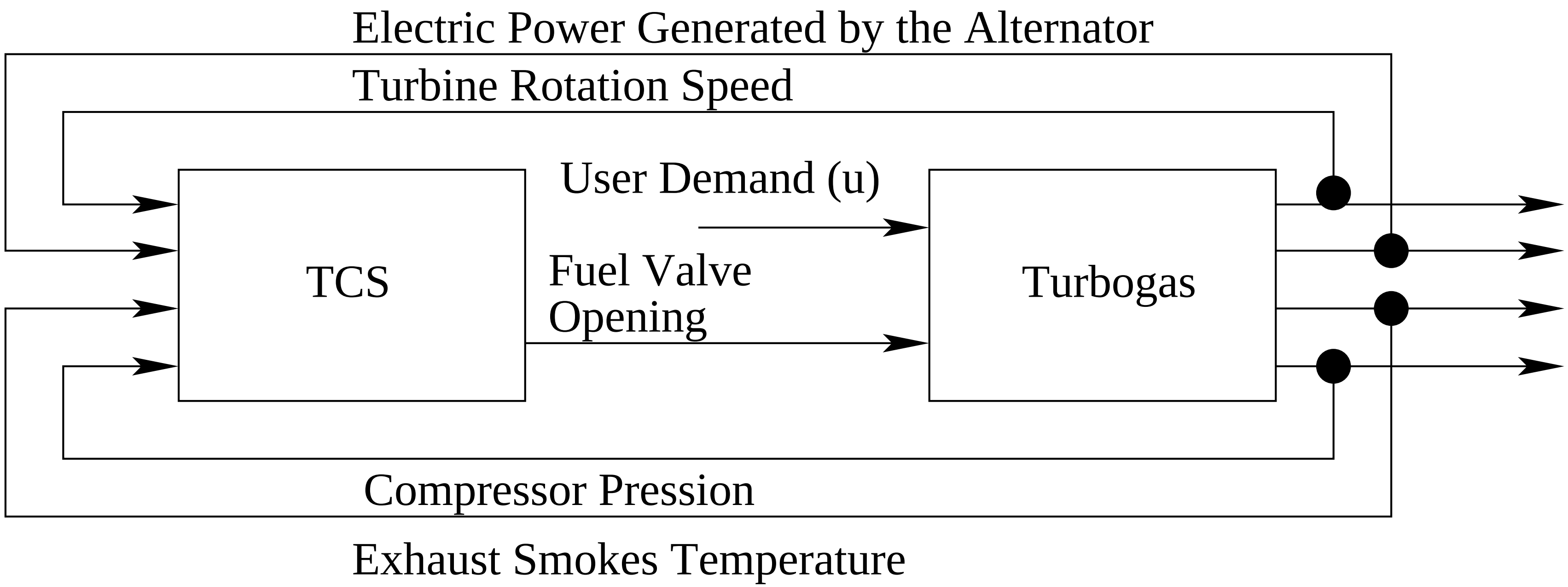

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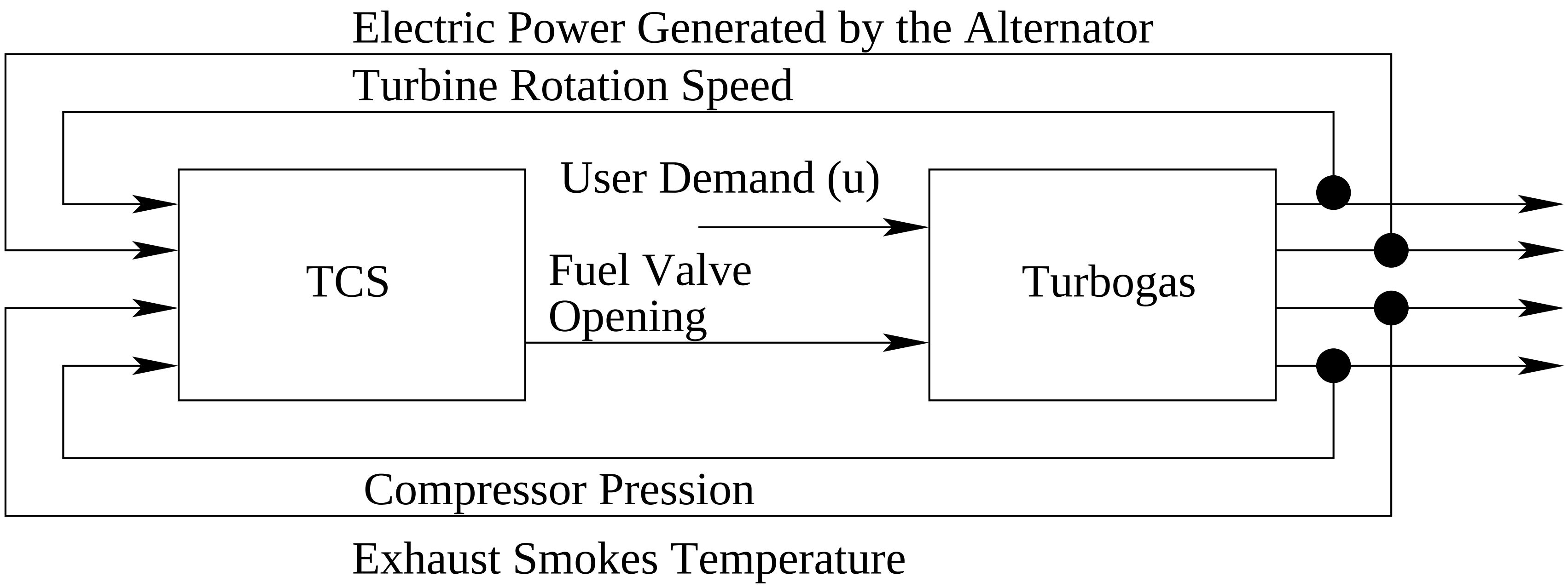

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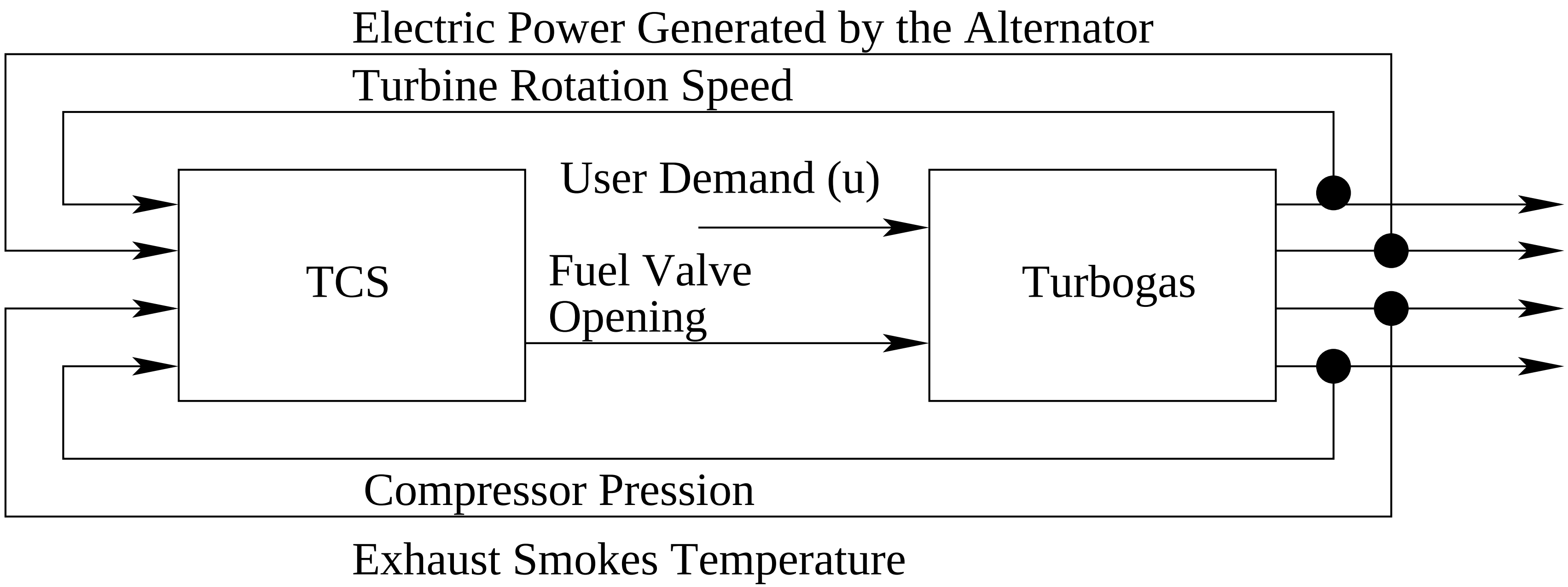

- TCS is an electronic circuit, its detail are known
- The turbogas is modeled by a set of ODEs
- The user demand is modeled as a nondeterministic disturbance
  - Its variation is bounded by a verification parameter (MAX_D_U)

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- As a result, if the user demand varies too much rapidly (i.e. MAX_D_U is too high), the controller fails

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$$p(u,i) = \begin{cases} 0.4 + \beta\frac{(u-\frac{M}{2})|u-\frac{M}{2}|}{M^2} & \text{if } i = -1 \\ 0.2 & \text{if } i = 0 \\ 0.4 + \beta\frac{(\frac{M}{2}-u)|u-\frac{M}{2}|}{M^2} & \text{if } i = +1 \end{cases}$$

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  - Then

$$u(t+1) = \begin{cases} \max(u(t) - \alpha, 0) & \text{with prob. } p(u(t), -1) \\ u(t) & \text{with prob. } p(u(t), 0) \\ \min(u(t) + \alpha, M) & \text{with prob. } p(u(t), +1) \end{cases}$$

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MAX_D_U	Reachable States	Finite Horizon	CPU Time	Probability
25	3018970	1600	68562.570	7.373291768e-05
35	2226036	1400	50263.020	1.076644427e-04
45	1834684	1300	41403.150	9.957147381e-05
50	83189	900	2212.360	3.984375e-03

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- k_1 is sufficient to reach an undesired state
- $k_2 = \frac{k_1}{100}$

`MAX_D_U`	Visited States	k_1	CPU Time (s)	Probability
35	1.159160e+05	800	3.702400e+03	4.104681e-03
45	4.098000e+04	700	1.313900e+03	1.792883e-02
50	4.067700e+04	700	1.307850e+03	3.825000e-02

Results on a machine with 2 processors (both INTEL Pentium III 500Mhz) and 2GB of RAM.

Murφ options used: `-m500` (use 500 MB of RAM)

Future Works

More features for FHP-Murφ and then comparison with PRISM

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Improving performances

- Try to apply symmetry reduction (to be investigated)