Intensive Computation

Prof. A. Massini

6 June 2024

End-of-term test

- **Student's Name** -

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- ***Matricola* number** -

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Exercise 1 (6 points)	
Exercise 2 (6 points)	
Exercise 3 (4 points)	
Exercise 4 (6 points)	
Exercise 5 (5 points)	
Exercise 6 (5 points)	
Total (32 points)	

Exercise 1 (6 points) – Interconnection Networks

a) Design a Clos network of size 250 x 250, using modules having **12 inputs in the first and middle stages** (the third stage is symmetrical to the first for the number of inputs and outputs). Specify the size and the number of switches for each stage. Consider both cases, **strictly non-blocking** and **rearrangeable** network.

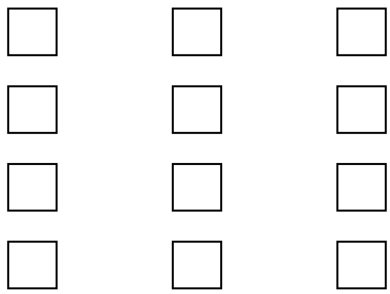
b) Compare the cost of the crossbar 250 x 250 and the Clos networks strictly non-blocking and rearrangeable designed in the previous point.

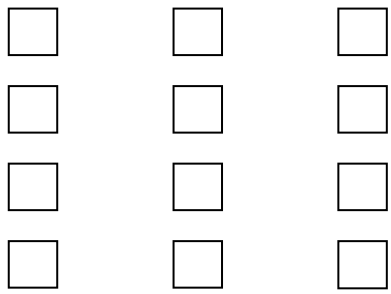
c) Compare the cost of the Benes with 256 inputs and the Clos networks strictly non-blocking and rearrangeable designed in point a).

Exercise 2 (6 points) – Interconnection Networks

a) Briefly explain how the self-routing algorithm works.

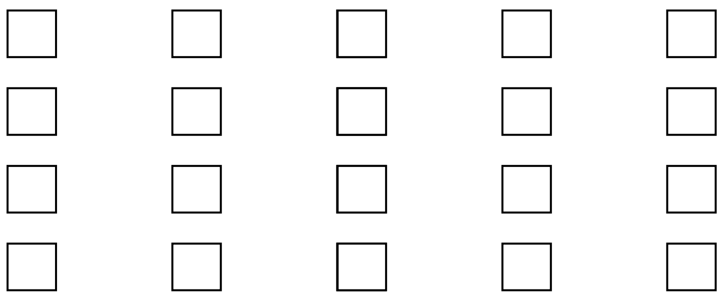
b) Complete the scheme of the Butterfly and Shuffle networks of size N=8 and show if they can realize permutation $P = \begin{pmatrix} 01234567 \\ 50274361 \end{pmatrix}$, showing the switch setting obtained using the self-routing algorithm.





c) Briefly explain how the Looping algorithm works.

d) Complete the scheme of the Benes network of size N=8 below and show how it can realize the permutation *P* using the Looping algorithm. Show how the algorithm proceeds in the diagram below.



Exercise 3 (4 points) – Interconnection networks

Explain how an Extended Generalized Fat Tree is made and show the representation of the XGFT(3; 2, 4, 2; 2, 4, 2).

Exercise 4 (6 points) – Sparse matrices

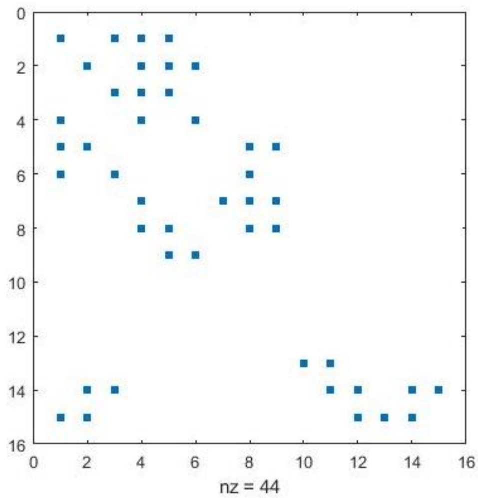
Consider the sparse matrix 15x15 and its pattern shown here below

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	**1,571**	0	**-94,252**	**0,785**	**-283**	0	0	0	0	0	0	0	0	0	0
2	0	**256**	0	**-6,28**	**314,17**	**-942,52**	0	0	0	0	0	0	0	0	0
3	0	0	**0,609**	**94,252**	**0,785**	0	0	0	0	0	0	0	0	0	0
4	**-94,252**	0	0	**508,4**	0	**-754,02**	0	0	0	0	0	0	0	0	0
5	**0,785**	**3,142**	0	0	0	0	0	**-42,52**	**0,785**	0	0	0	0	0	0
6	**-83**	0	**256**	0	0	0	0	**-28**	0	0	0	0	0	0	0
7	0	0	0	**-0,304**	0	0	**0,609**	**-0,304**	**94,258**	0	0	0	0	0	0
8	0	0	0	**-754,02**	**-94,25**	0	0	**154,5**	**-75,022**	0	0	0	0	0	0
9	0	0	0	0	**0,609**	**94,252**	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	**-0,304**	**0,609**	0	0	0	0
14	0	**150,45**	**94,252**	0	0	0	0	0	0	0	**942,52**	**0,785**	0	**314,17**	**0,785**
15	**-40,2**	**-94,252**	0	0	0	0	0	0	0	0	0	**942,52**	**0,785**	**157,08**	0

a) Specify which arrays you need for the following compressed representations and how many bytes they occupy in memory.

b) Explain how arrays change after the deleting the elements $m_{14,2}$, and $m_{14,3}$ and what the new memory occupation corresponds to.

c) Explain how arrays change after inserting the elements $m_{10,11}$=75,35 $m2_{11,10}$=92,81 and what the new memory occupation corresponds to.



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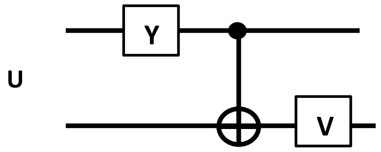
Exercise 5 (5 points) – Quantum systems

a) Consider the two-qubit system state $|\psi_1\psi_2\rangle$, where $|\psi_1\rangle = \frac{\sqrt{3}}{3}|0\rangle - \frac{\sqrt{6}}{3}i|1\rangle$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{\sqrt{3}}{\sqrt{6}}i|1\rangle$ and give the state vector representing it.

b) Compute the probability of measuring $|11\rangle$ and the probability of measuring $|10\rangle$.

Exercise 6 (5 points) – Quantum circuits

Consider the two-qubit transformations U shown below:



where $\mathrm{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ and $\mathrm{V} = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$.

a) Show what transformation U represents writing the associated 4x4 matrix.

b) Show if U is unitary.

c) Show how U acts on the state $|01\rangle$ and on the state represented by the statevector $[\ \frac{i}{2};\ 0;\ \frac{1}{2};\ \frac{\sqrt{2}}{2}\ i]$.