Intensive Computation

Prof. A. Massini

1 June 2023

End-of-term test

- Student's Name -

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- *Matricola* number -

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Exercise 1 (4 points)	
Exercise 2 (4 points)	
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Exercise 3 (4 points)	
Exercise 4 (4 points)	
Exercise 5 (4 points)	
Exercise 6 (4 points)	
Question 2 (4 points)	
Total (32 points)	

Exercise 1 (4 points) – Interconnection Networks

a) Design a Clos network of size 280 x 280, using modules having **18 inputs in the first and middle stages** (whereas the third stage is symmetrical to the first), specifying the size and the number of switches for each stage. Consider both cases, **strictly non-blocking** and **rearrangeable** network.

b) Compare the cost of the crossbar 280 x 280 and the Clos network, strictly non-blocking and rearrangeable, designed in the previous point.

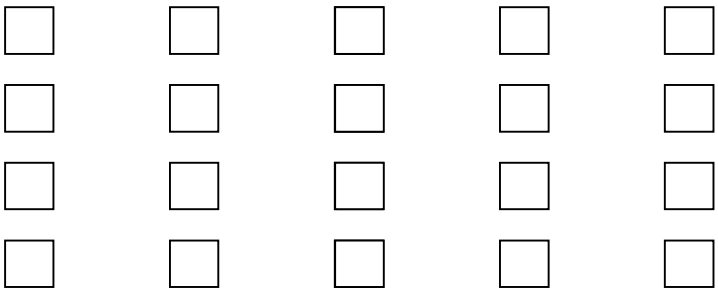
Exercise 2 (4 points) – Interconnection Networks

a) Write the permutation for eight elements obtained by rotating left the 3-bit binary representation of inputs:

$$P = \begin{pmatrix} 01\,23\,45\,67 \\ \end{pmatrix}$$

Complete the scheme of the Butterfly and Shuffle networks of size N=8 and show if they can realize permutation *P*, showing the switch setting obtained using the self-routing algorithm and explaining how to do it.

b) Complete the scheme of the Benes network of size N=8 and show how it realizes the permutation *P*, using the Looping algorithm and explaining how to do it.



Question 1 (4 points) – Interconnection networks

Explain what a fat tree is and illustrate what is the difference between a GFT and a XGFT.

Use as reference GFT(2, 2, 2) and XGFT(2; 4, 4; 1, 2).

Exercise 3 (4 points) – Amdhal law

Assume that we are considering enhancing a machine by adding vector hardware to it. When a computation is run in vector mode on the vector hardware, it is 10 times faster than the normal mode of execution. We call the percentage of time that could be spent using vector mode the *percentage of vectorization.*

a) What *percentage of vectorization* is needed to achieve a speedup of 2?

b) What percentage of the computation run time is spent in vector mode if a speedup of 4 is achieved?

Exercise 4 (4 points) – Performance

A common measure of performance for a processor is the rate at which instructions are executed, expressed as millions of instructions per second (MIPS), referred to as the **MIPS rate**.
We can express the MIPS rate in terms of the clock rate and CPI as follows:

$$\text{MIPS rate} = \frac{IC}{T \times 10^6} = \frac{f}{CPI \times 10^6}$$

Consider two different machines, with two different instruction sets, both of which have a **clock rate of 200 MHz**. The following measurements are recorded on the two machines running a given set of benchmark programs:

Instruction Type Machine A	**Instruction Count (millions)**	**Cycles per Instruction**
Arithmetic and logic	8	1
Load and store	4	3
Branch	2	4
Others	4	3

Instruction Type Machine B	**Instruction Count (millions)**	**Cycles per Instruction**
Arithmetic and logic	10	1
Load and store	8	2
Branch	2	4
Others	4	3

a) Determine the effective CPI and MIPS rate for each machine.

b) Assume that a design alternative for Machine A is to decrease the CPI of *Load and store* instructions and of *Branch* instructions to 2. Determine the new CPI of Machine A and compute the speedup.

Exercise 5 (4 points) – Quantum circuits

a) Verify which of the following qubit is valid:

$$\psi_1 = \frac{\sqrt{6}}{3}\,|0\rangle + \frac{1}{\sqrt{3}}i|1\rangle$$

$$\psi_2 = \frac{1}{\sqrt{3}}\,|0\rangle + \frac{3}{\sqrt{15}}i|1\rangle$$

$$\psi_3 = \frac{1}{2}\,|0\rangle - \frac{\sqrt{3}}{2}i|1\rangle$$

b) Compute the state vector of a two-qubit system using two of the previous qubits verified as valid.

c) Verify the normalization condition of the two-qubit system.

d) Compute the probability of measuring $|00\rangle$ and the probability of measuring $|10\rangle$.

Exercise 6 (4 points) – Quantum circuits

a) Show that the matrices $V = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$ and $W = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\frac{\pi}{4}} & e^{-i\frac{\pi}{4}} \\ e^{-i\frac{\pi}{4}} & e^{i\frac{\pi}{4}} \end{bmatrix}$ correspond to the same operator and that it is unitary.

b) Show that $VV = WW = X$

Question 2 (4 points) – Quantum circuits

Explain what entanglement between qubits is and show how to obtain the four Bell states.