THE DISTRIBUTED DEPLOYMENT OF MOBILE SENSORS I.E. THE VORONOI DIAGRAM CONSTRUCTION PROBLEM

1'

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THE PROBLEM

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THE DISTRIBUTED DEPLOYMENT PROBLEM (1)

About the deployment problem:

A centralized solution is not always desirable because:

- Connection with a server is required
- Long delays are expected
- The solution is not fault-tolerant

The ability of moving around facilitates sensors to self-deploy starting from any initial configuration to a final distribution that guarantees that the AoI is completely covered.

THE DISTRIBUTED DEPLOYMENT PROBLEM (2)

The self-deployment is necessary in "hostile" environments:

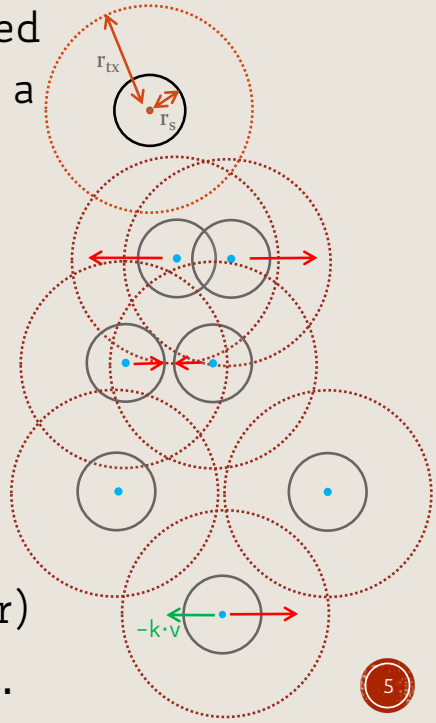
Contaminated places, Fires, Battlefields…

In these cases, sensors should position themselves and transmit the collected information.

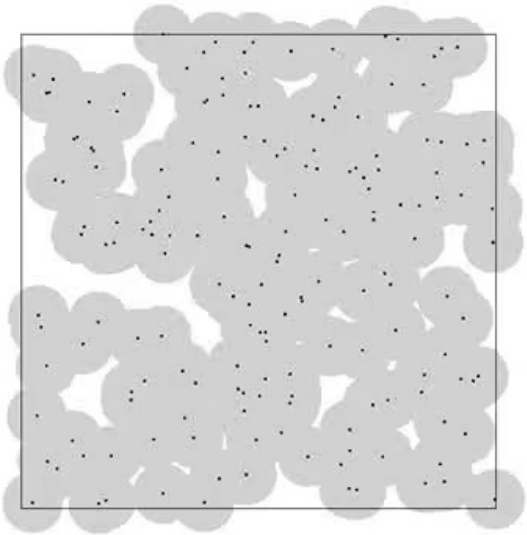
In general, each sensor repeatedly executes a **Look-Compute-Move cycle**: based on what it sees in its vicinity, it makes a decision on where to move, and moves to its next position.

A POSSIBLE APPROACH: VIRTUAL FORCES (1)

- Idea: sensors are similar to charged particles (magnetic force) having a mass (gravitational force).
- Two sensors repel each other if they are too close
- Two sensors attract each other if they are far but can anyway communicate
- Two sensors ignore each other if they cannot communicate (too far)
- Friction to attenuate oscillations.



A POSSIBLE APPROACH: VIRTUAL FORCES (2)



(video HomVF)

A POSSIBLE APPROACH: VIRTUAL FORCES (3)

Weaknesses:

- It is necessary a manual tuning of parameters
- Sensor oscillation – possible solutions:
  - Friction forces
  - Stopping conditions
- In some versions, attracting effect of the border and of the obstacles (e.g. when only repulsive forces are considered)
- …

A POSSIBLE APPROACH: VIRTUAL FORCES (4)

Weaknesses (cntd):

- Sensors tend not to pass through doors and narrows

☞ Possible students' lesson

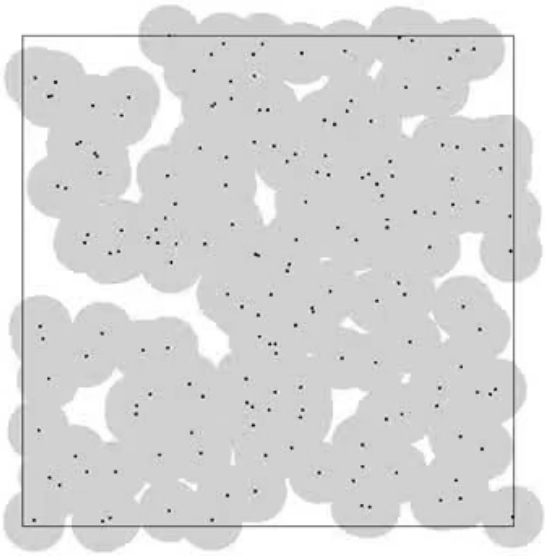
A PROTOCOL BASED ON VORONOI DIAGRAMS (1)

Idea:

- Each sensor is assigned an AoI portion and it has to take charge of it, trying to cover it as best as it can
- The sensor is "satisfied" if:
  - It completely cover its portion

  or
  - All its sensing radius is used to cover its portion
- If a sensor is not "satisfied" it has to move in order to improve its coverage
- AoI portions can be assigned according to the Voronoi diagram.

A PROTOCOL BASED ON VORONOI DIAGRAMS (2)



(video HomVor)






VORONOI DIAGRAMS

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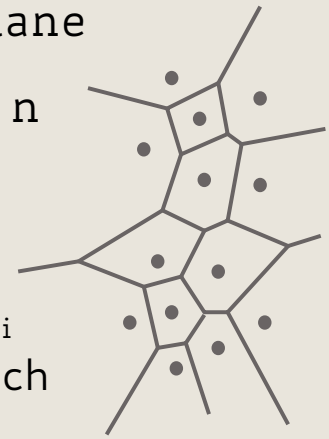
VORONOI DIAGRAM (1)

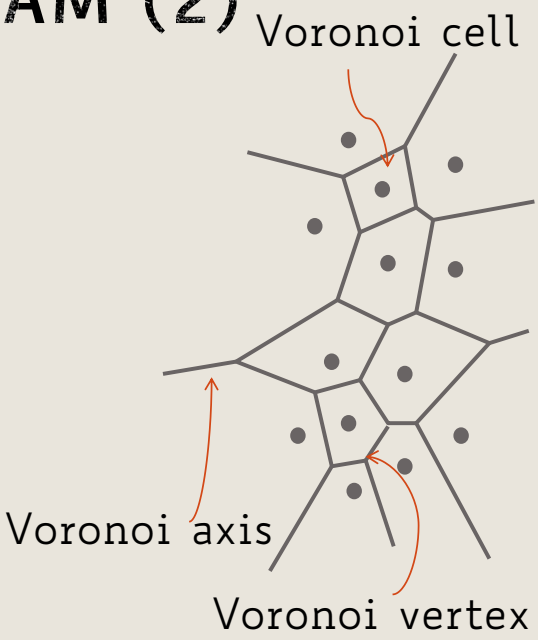
Def. of Voronoi Diagram:

- $\mathscr{P}$: set of n distinct sites on the plane
- VD($\mathscr{P}$): partition of the plane into n cells V_i such that:
  - each V_i contains exactly one site
  - if a point Q on the plane is in V_i then dist(Q, P_i) < dist(Q, P_j) for each $P_i \in \mathscr{P}$, $j \neq i$.



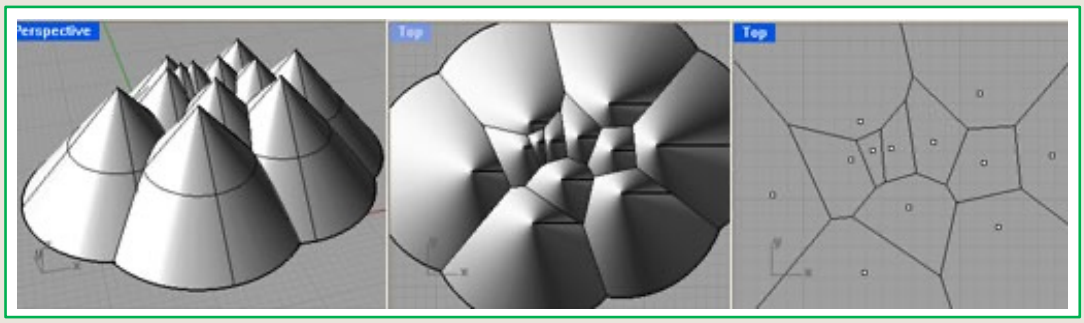
VORONOI DIAGRAM (2)

- In other words: VD($\mathscr{P}$) is a partition of the plane into convex regions $\{V_1, \ldots, V_n\}$, such that V_i contains exactly one site $P_i \in \mathscr{P}$ and for each other point in V_i the closest site in $\mathscr{P}$ is P_i.



VORONOI DIAGRAM (3)

- Alternative def. of Voronoi Diagram:
  - 2D projection of lower envelope of distance cones centered at sites



VORONOI DIAGRAM (4)

Maps like this appear frequently in various applications and under many names. To mathematicians, they are known as Voronoi diagrams.

Voronoi diagrams are rather natural constructions, and it seems that they, or something like them, have been in use for a long time.

VORONOI DIAGRAM (5)

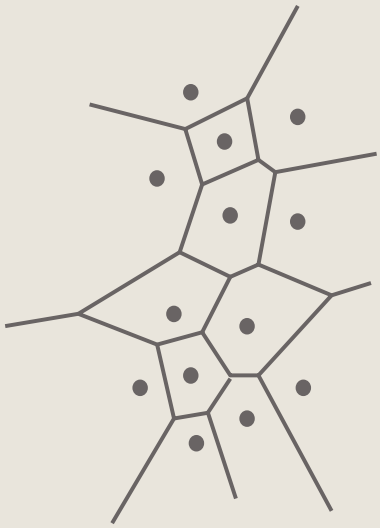
Voronoi diagrams have been used by:

- anthropologists to describe regions of influence of different cultures;
- crystallographers to explain the structure of certain crystals and metals;
- ecologists to study competition between plants;
- economists to model markets in a certain economy;
- …

VORONOI DIAGRAM (6)

Example of application 1:

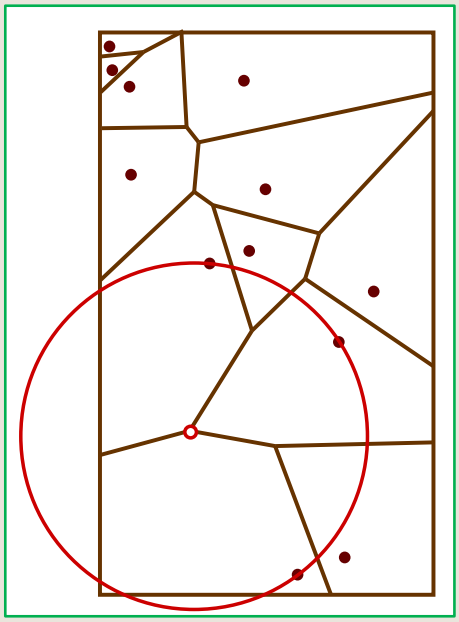
in a desert where the only water sources are a few springs, for each of them, you would like to determine the locations nearest that spring. The result could be a map in which the terrain is divided into regions of locations nearest the various springs.



VORONOI DIAGRAM (7)

Example of application 2:

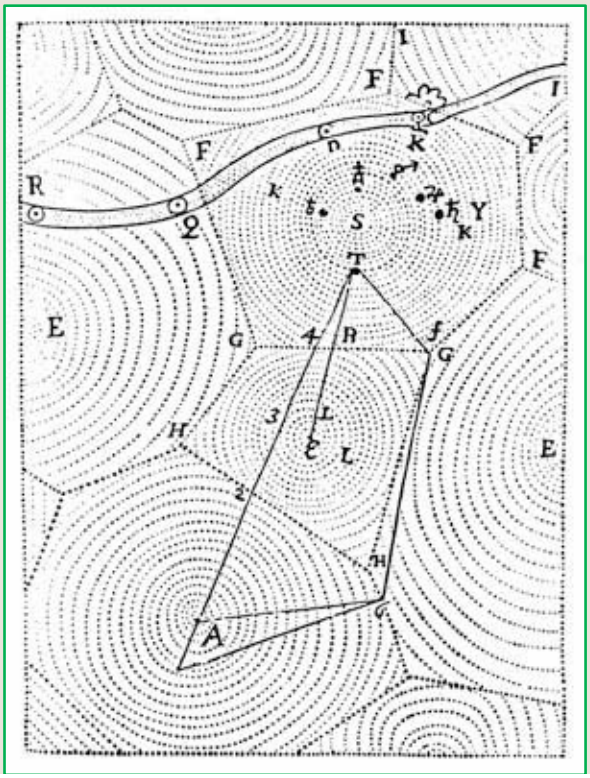
if one wants to determine a location for their shop to maximize the distance to its competitors, it is necessary to find the center of the largest empty circle, that must be a vertex of the Voronoi diagram.



VORONOI DIAGRAM (8)

History:

- An informal study of Voronoi diagrams dates back to Descartes (1644): he includes the following figure with his demonstration of how matter is distributed throughout the solar system.
- …



VORONOI DIAGRAM (9)

History (contd)

The English physicist Snow uses them for his analysis of the London cholera outbreak of 1854:

Snow considers the pumps of drinking water distributed throughout the city, and draws a map, which essentially is the street pump's Voronoi cell.

This map supports Snow's hypothesis that the cholera deaths are associated with contaminated water.

Snow recommends to the authorities to close the contaminated pumps, and the cholera outbreak ends.

- …

VORONOI DIAGRAM (10)

History (contd)

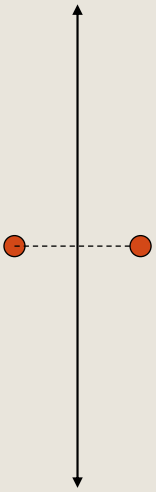
- Dirichlet uses Voronoi diagrams in his studies on quadratic equations in 1850.
- Voronoi diagrams are so called in honor of the Russian mathematician Georgy F. Voronoi, who defined and studied them in the n-dimensional space in 1908.
- They are also called Thiessen polygons in meteorology in honor of the US meteorologist Alfred H. Thiessen; Wigner-Seitz cells in physics, fundamental domains in group theory and fundamental polygons in topology.

VORONOI DIAGRAM (11)

Voronoi diagram of a single site

VORONOI DIAGRAM (12)

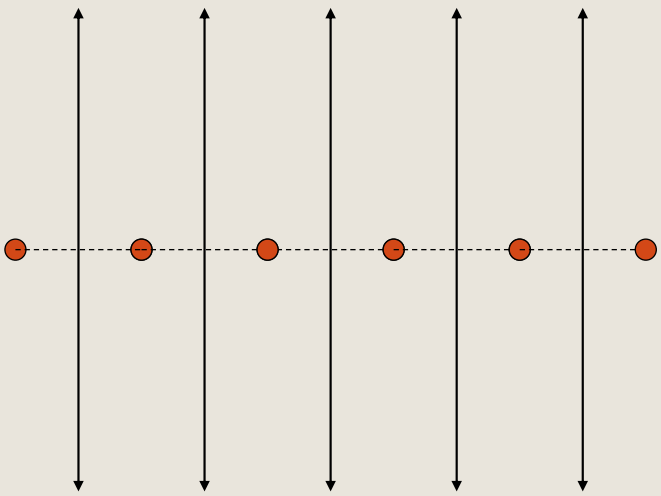
Voronoi diagram of two sites



The axis extends to infinity in both directions, generating two halfplanes

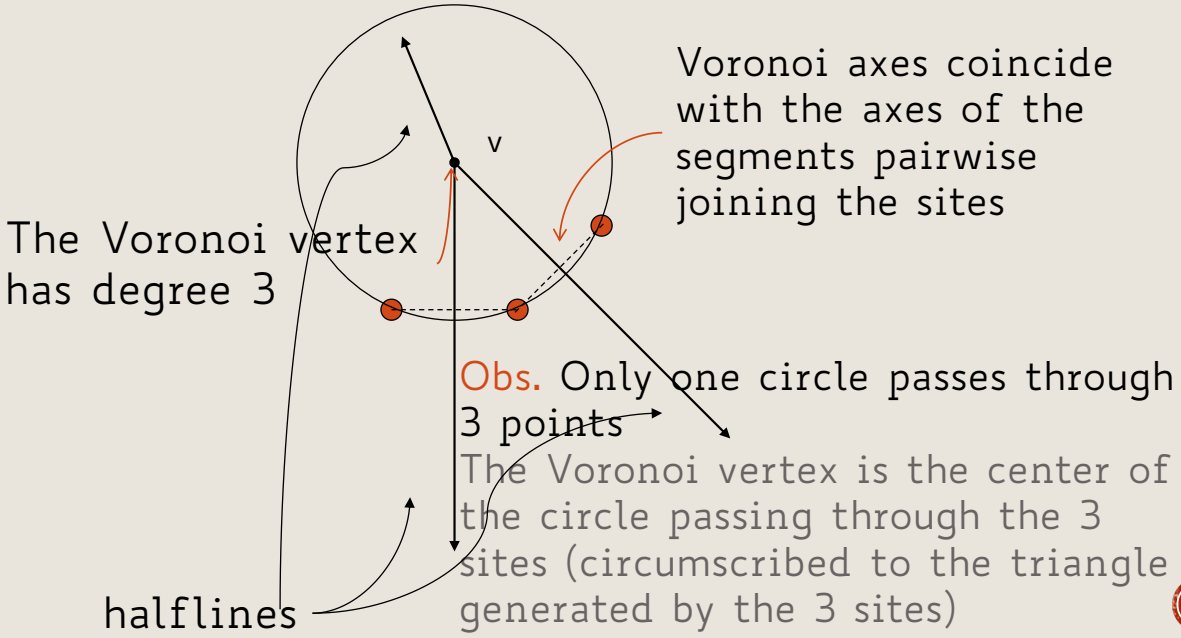
VORONOI DIAGRAM (13)

Voronoi diagram of some colinear sites



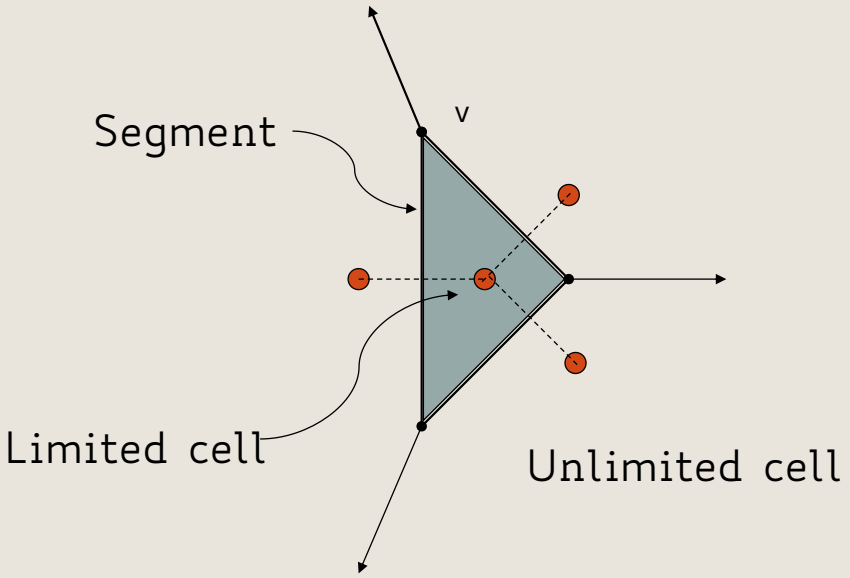
VORONOI DIAGRAM (14)

Voronoi diagram of 3 not colinear sites



VORONOI DIAGRAM (15)

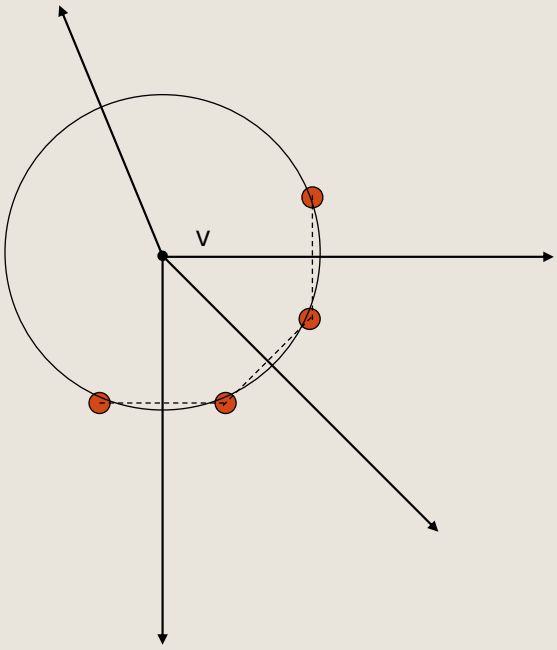
Voronoi diagram of 4 not colinear sites



VORONOI DIAGRAM (16)

Not always 4 not colinear sites create a limited cell:

General position assumption: each 3 sites are not colinear and each 4 sites are not cocircular.
Thanks to this assumption, all vertices have degree 3!

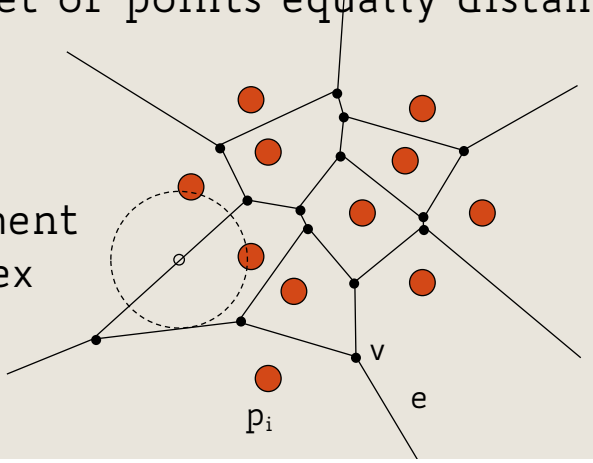


VORONOI DIAGRAM PROPERTIES (1)

A point q on the plane lies on the Voronoi segment between p_i and p_j iff the largest empty circle centered in q touches only p_i and p_j.

- A Voronoi segment is a subset of a Voronoi axis, i.e., the set of points equally distant from p_i and p_j

p_i : sites of $\mathscr{P}$
e : Voronoi segment
v : Voronoi vertex

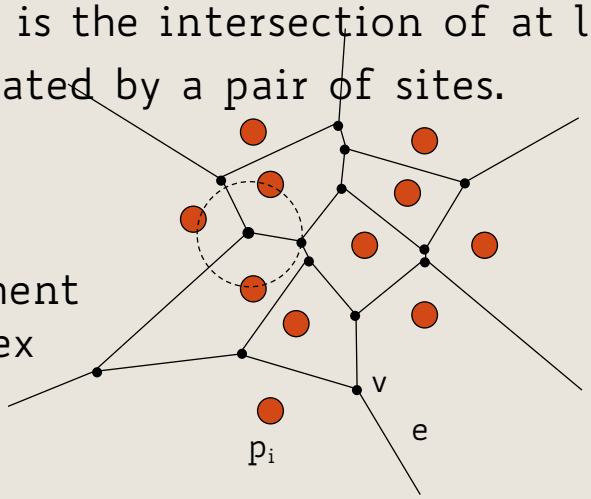


VORONOI DIAGRAM PROPERTIES (2)

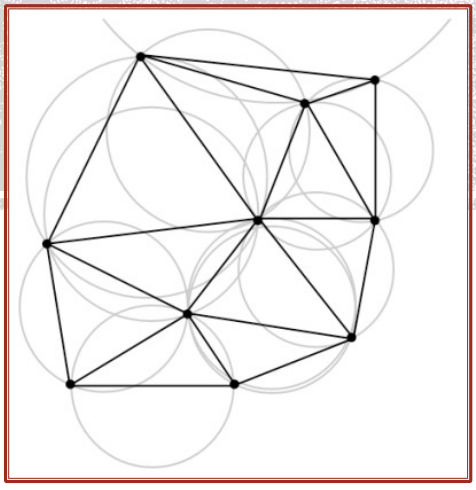
A point q in the plane is a Vornoi vertex iff the largest empty circle centered in q touches at least 3 sites of P.

A Voronoi vertex is the intersection of at least 3 axes, each generated by a pair of sites.

p_i : sites of $\mathscr{P}$
e : Voronoi segment
v : Voronoi vertex

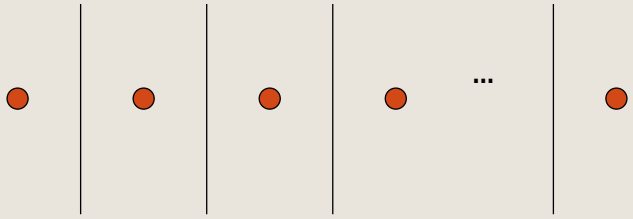


VORONOI DIAGRAM COMPLEXITY



VORONOI DIAGRAM COMPLEXITY (1)

- Th.: $|v| \leq 2n - 5$ and $|e| \leq 3n - 6$ for each $n \geq 3$.
- Proof: (Easy case)



Colinear sites → $|v| = 0$, $|e| = n - 1$

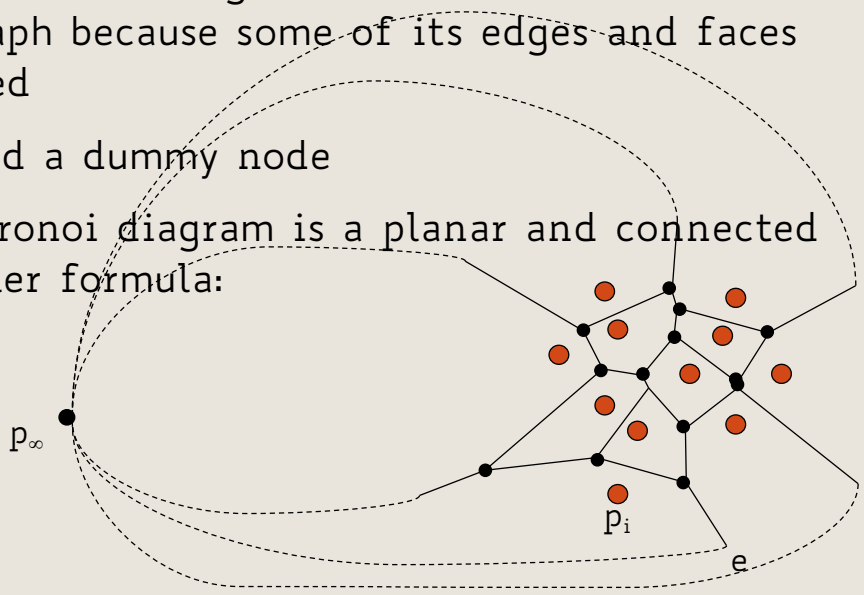
VORONOI DIAGRAM COMPLEXITY (2)

Proof of Th.: $|v| \leq 2n - 5$ and $|e| \leq 3n - 6$ for each $n \geq 3$ – contd.

Proof: (General case)

- Problem: A Voronoi diagram cannot be considered as a planar graph because some of its edges and faces are unlimited
- Solution: add a dummy node
- Now the Voronoi diagram is a planar and connected graph → Euler formula:

  $|v| - |e| + f = 2$



VORONOI DIAGRAM COMPLEXITY (3)

Proof of Th.: $|v| \leq 2n - 5$ and $|e| \leq 3n - 6$ for each $n \geq 3$ – contd.

f=n+1. Euler formula becomes:

$|v|-|e|+n+1=2$ (1)

Moreover: $\sum_{v \in VD} \deg(v) = 2|e|$

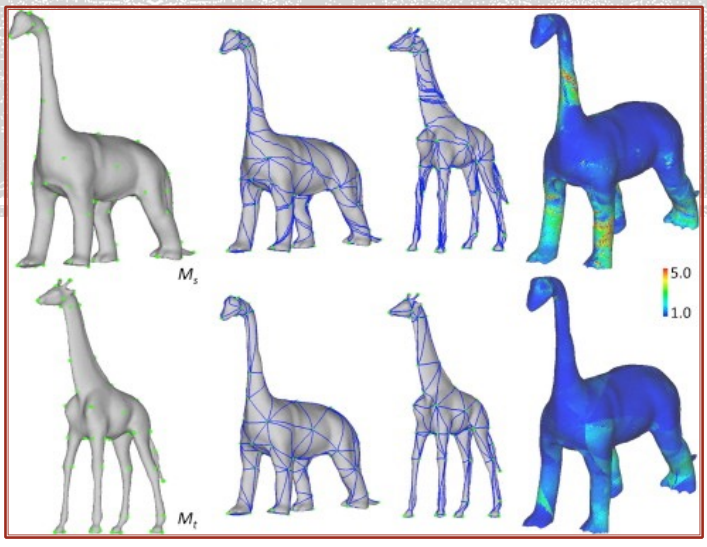
since $\deg(v) \geq 3 \rightarrow 2|e| \geq 3|v|$ (2)

Joining (1) e (2):

$|v| \leq 2n-5$

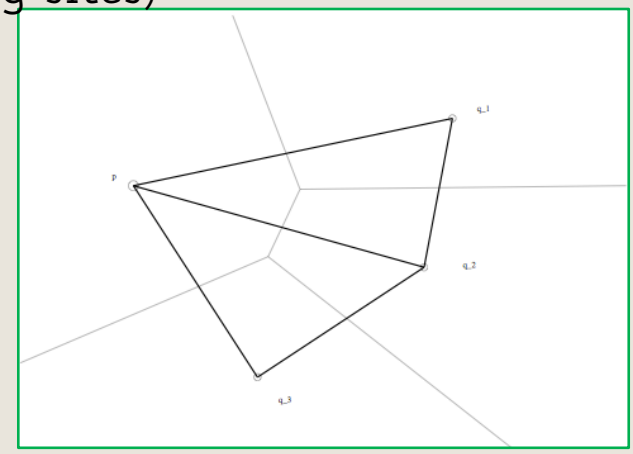
$|e| \leq 3n-6$

THE DUAL PROBLEM OF THE VORONOI DIAGRAM



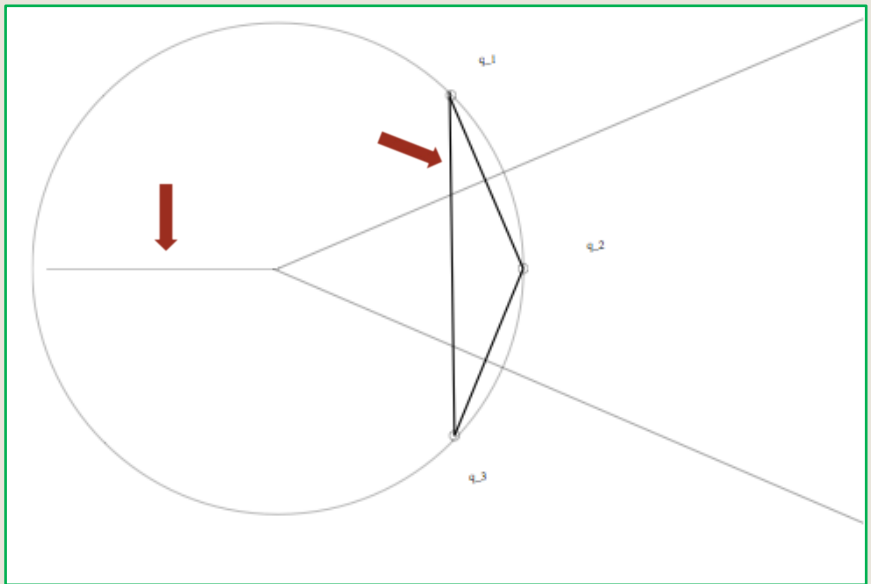
THE DUAL PROBLEM OF THE VORONOI DIAGRAM

- The dual problem w.r.t. the decomposition of the plane into Voronoi cell is the Delaunay triangulation (obtained interescting each Voronoi axis with a segment joining the generating sites)



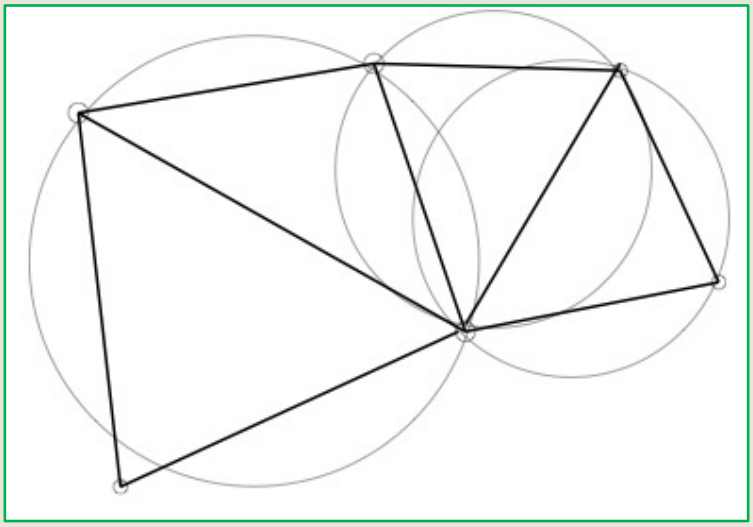
DELAUNAY TRIANGULATION (1)

- Obs. Dual segments not necessarily intersect!



DELAUNAY TRIANGULATION (2)

- Property: the circle circumscribed to a Delaunay triangle does not contain any site inside it

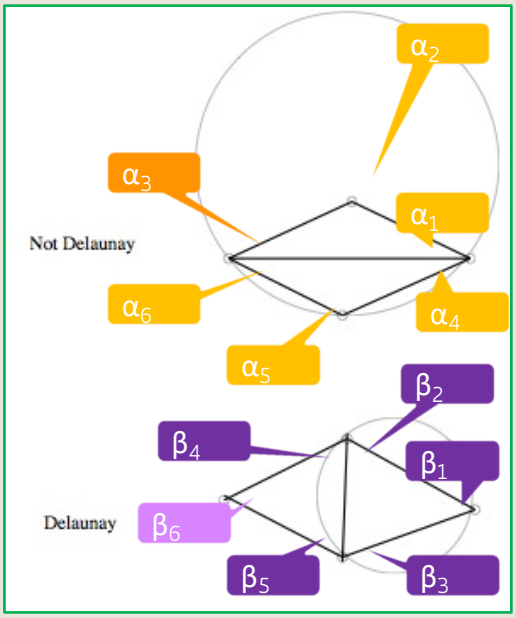


DELAUNAY TRIANGULATION (3)

- A segment is illegal if:

  min α_i<min β_i

- Property: no segment can be illegal.
- If e is an illegal edge, then it is possible to swap the triangles to get a Delaunay triangulation.



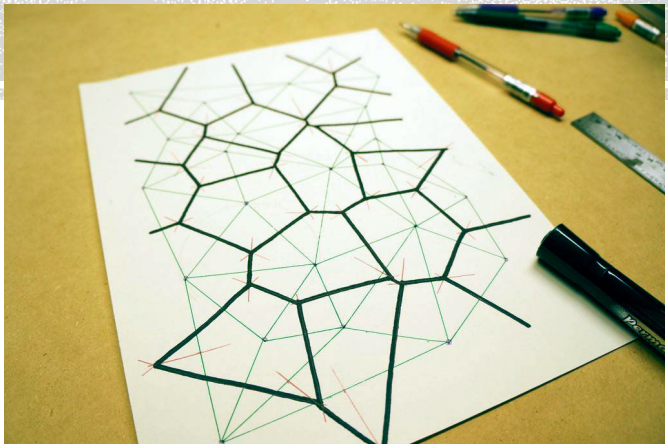
DELAUNAY TRIANGULATION (4)

- Some papers exploit a Delaunay triangulation to route sensors towards a position contributing to a complete coverage.
- There are several algorithms to compute a Delaunay triangulation

☞ Possible students' lesson

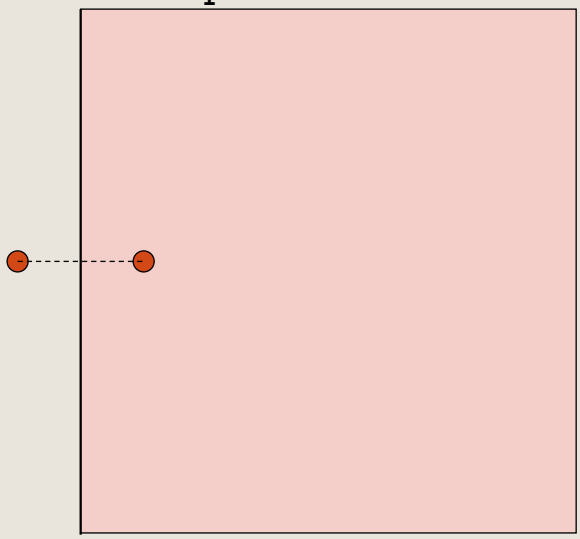
- The Voronoi Diagram can be computed as dual construction of the Delaunay triangulation.
- Otherwise…

ALGORITHMS TO COMPUTE THE VORONOI DIAGRAM

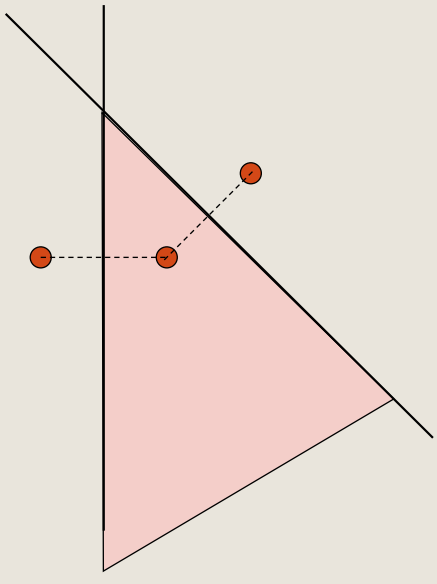


ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (1)

A Voronoi cell can be obtained repeatedly intersecting opportune half-planes:

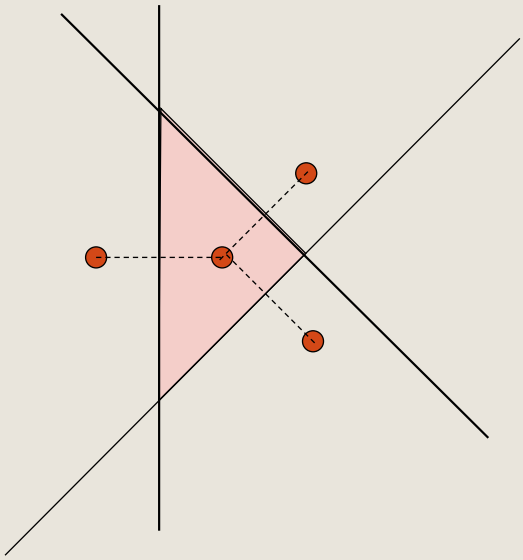


ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (2)



ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (3)

This operation needs to be iterated for each site.



ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (4)

- Each of the n Voronoi cells is the result of the intersection of a number k of halfplanes with $k=\Theta(n)$
- Cost to determine the intersection of a certain number k of halfplanes?
  From computational geometry, a possible algorithm exploits the divide-et-impera technique…

ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (5)

DIVIDE: The set of k halfplanes is recursively split until k single halfplanes are obtained (bin. tree structure).

IMPERA: The halfplane on each leaf is intersected with a rectangle R (the search space). In this way, each leaf contains now a polygon.

COMBINE: Recursively, bottom-up, compute the intersection of two sibling polygons and put the result on the father node.

ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (6)

Time Complexity of the Combine step:
FACT 1: Two polygons with p and p' vertices each, they can be intersected in O(p+p') time.

FACT 2: It can be proved that the computational time of the whole algorithm is O(k log k)

FACT 3: This is optimum because the sorting problem (using comparisons) can be reduced to the intersection of halfplanes.

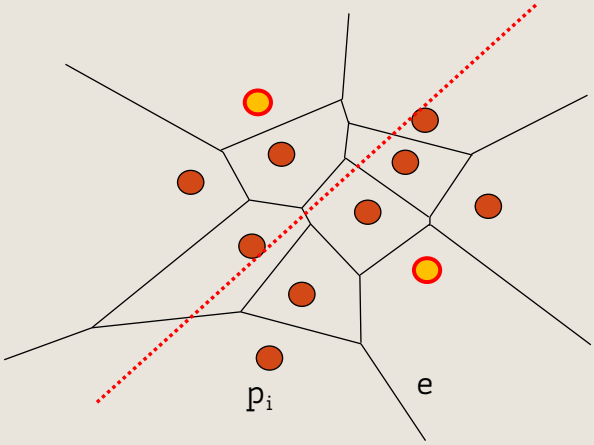
ALGORITHM BASED ON THE INTERSECTION OF HALF-PLANES (7)

Computational Time of the whole algorithm to compute the Voronoi diagram through the intersection of halfplanes:
in order to find a single cell, O(n) halfplanes need to be intersected, so O(n log n) per cell and $O(n^2 \log n)$ for the whole algorithm.

Can we do better?

INTUITION (1)

Not all the site pairs give raise to an axis!



INTUITION (2)

- Idea: use a well known technique in computational geometry.
- The sweep line is used to solve geometrical bidimensional problems through a sequence of almost onedimensional subproblems.

INTUITION (3)

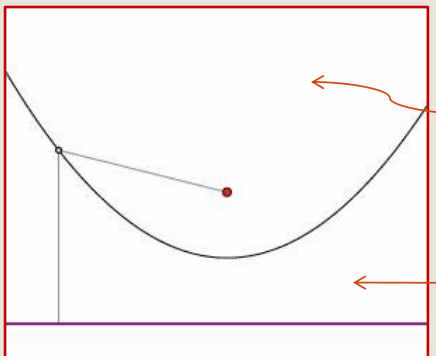
- Example: [Bentley, Ottmann'79] Compute the intersection points of n segments sweeping the plane with a horizontal line.
- When the sweep line moves, it encounters objects, and the algorithm solves the single problem related to each single object.

INTUITION (4)

- This method cannot work as it is for Voronoi diagrams, because it would be necessary to "predict" the site position before the sweep line encounters them.
- Fortune [1986] designed an algorithm based on a different line, called beach line.

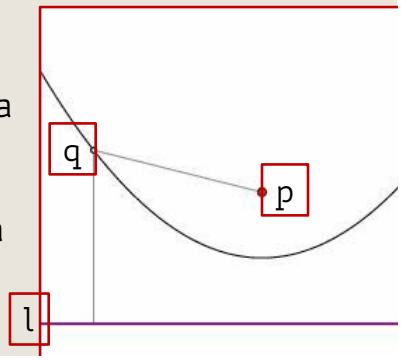
FORTUNE ALGORITHM (1)

- Idea: we introduce a line l sweeping the plane (sweep line) helping us to compare distances.
- Somehow, l "discovers" the Voronoi diagram on the just sweeped plane portion.
- Note. Given any point p, the set of points equally distant from p and l is a parabola $P_{p,l}$.



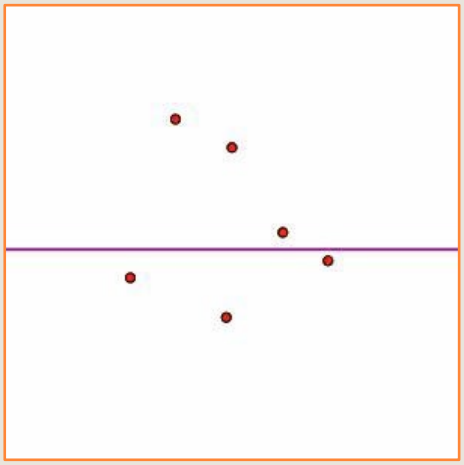
FORTUNE ALGORITHM (2)

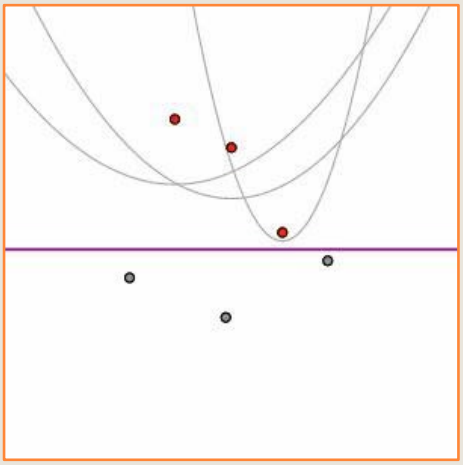
- Consider any point $q=(q_x, q_y)$.
- The sweep line l is horizontal and its y-coordinate is l_y. Hence $dist(q,l)=l_y-q_y$.
- Given another point p, q lies on the parabola generated by p and l iff $dist(q,p)=l_y-q_y$.
- More in general:
  - $dist(q,p)< l_y-q_y$ if q lies above the parabola
  - $dist(q,p)=l_y-q_y$ if q lies on the parabola
  - $dist(q,p)>l_y-q_y$ if q lies under the parabola



FORTUNE ALGORITHM (3)

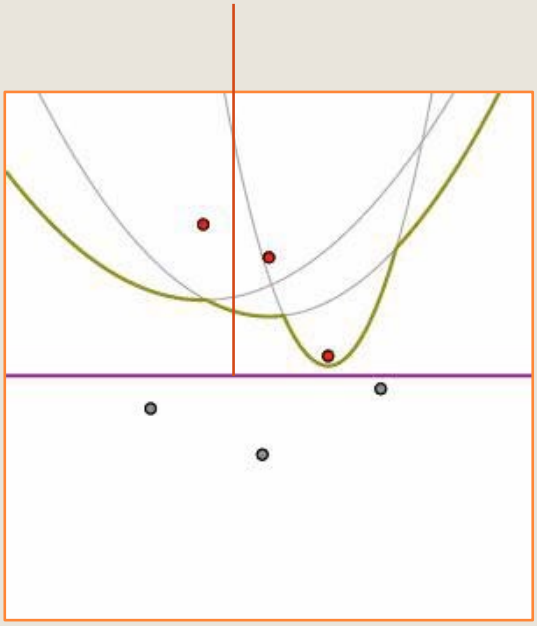
- The sweep line l goes down.
- At each instant, consider the sites above l and the parabolas they define with l.
- Example:





FORTUNE ALGORITHM (4)

- Define the beach line *b* as the line formed by the union of the lower parabola arches.
- In other words: each vertical line crosses many parabolas; the lower intersection point belongs to *b*.
- Note. Each arch of *b* is associated with a site above *l*.



FORTUNE ALGORITHM (5)

- If a point is above *b*, it is closer to one of the sites above *l* than to *l* itself.
- In other words, this point lies inside the Voronoi cell of a site that *l* has already encountered.
- Hence, the Voronoi diagram above *b* is completely determined by the sites above *l*.

To see an animation of the beach line:
https://www.youtube.com/watch?v=k2P9yWSMaXE

FORTUNE ALGORITHM (6)

Let us determine the condition such that *b* passes through any point q.

If q is touched by the portion of *b* generated by p_i, it will enter in the Voronoi cell of p_i. So:

$$dist(q,p_i) \leq dist(q, p_j) \text{ for any other } j \neq i.$$

Point q lies on the parabola generated by p_i and *l* iff

$$dist(q, p_i) = l_y - q_y.$$

FORTUNE ALGORITHM (7)

…

Joining the inequality and the condition:

$$dist(q, p_j) \geq dist(q,p_i) = l_y - q_y = dist(q,l).$$

Remind that $dist(q,p) > dist(q,l)$ if q lies under $P_{p,l}$

So, q is on $P_{pi,l}$ and under any other parabola $P_{pj,l}$, i.e. q is on *b*. In other words:

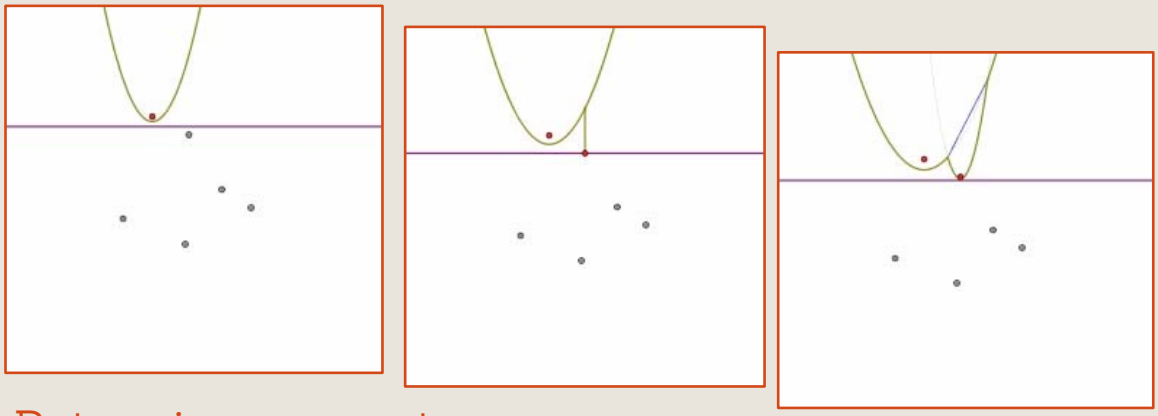
When a point appears on *b*, it is on the parabola associated to its closest site

FORTUNE ALGORITHM (8)

- Points on *b* lying at the intersection of two parabola archs are called breakpoints.
- Breakpoints are at the same time closest to two sites. In other words,

  Breakpoints lie on the segments of the Voronoi diagram
- In order to construct the Voronoi diagram, it is enough to keep trace of the breakpoints.

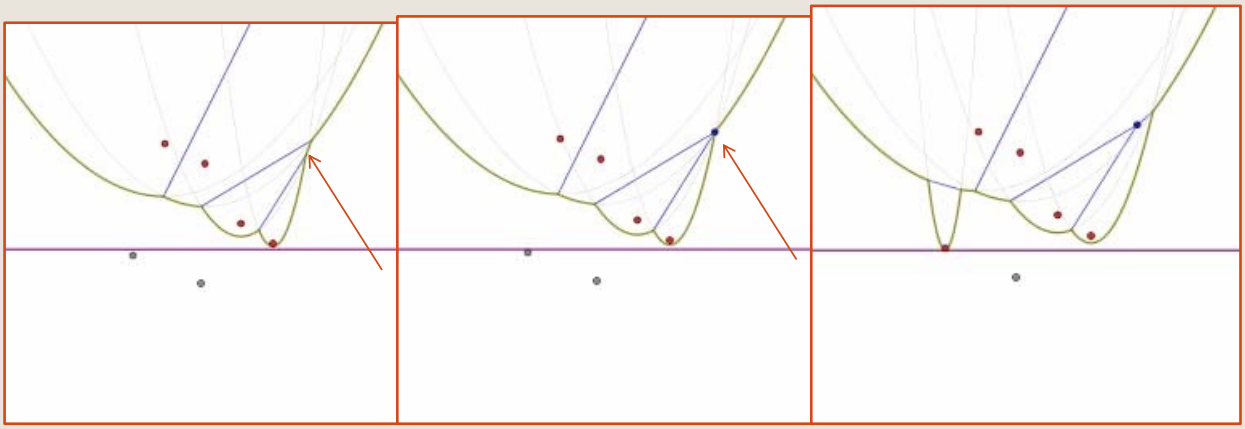
FORTUNE ALGORITHM (9)



Determine segments:

- A pair of breakpoints, corresponding to a segment in the Voronoi diagram, appear on *b* exactly when the sweep line encounters a new site ⇒ site event.

FORTUNE ALGORITHM (10)



Determine vertices:

- While l moves, breakpoints move too, and they follow a segment; they reach a vertex when a parabola arch disappear.

FORTUNE ALGORITHM (11)

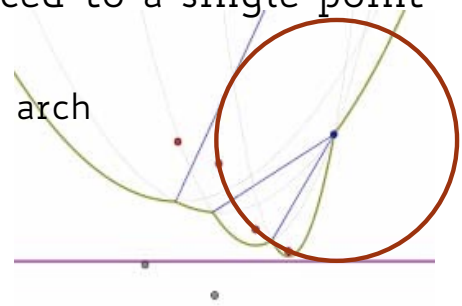
a new parabola arch appearing on the beach line:

Easy to detect: it appears when l encounters a site.

a parabola arch disappearing from the beach line:

Easy to detect: when this arch is reduced to a single point x, it lies on 3 parabolas:

- The one containing the disappearing arch
- The one to its right
- The one to its left



So x is equally distant from 3 sites

⇒ a circle centered at x passes through these 3 sites.

We determine a Voronoi vertex when l has finished to sweep this circle ⇒ circle event.

FORTUNE ALGORITHM (12)

Fortune Algorithm

- Resume: In order to determine segments and vertices of the Voronoi diagram, keep trace of the parabola arches appearing and disappearing on *b*.
- Imagine to walk on *b* left to right and sort the order of the sites producing the parabola arches on it.
- This order cannot change until either a site event or a circle event happens.

FORTUNE ALGORITHM (13)

Fortune algorithm (contd.)

- Breakpoints are implicitely stored as intersections of parabola arches on *b*.
- If the next event encountered by *b* is:
  - A site event, insert the new site in the list of sites in the order indicated by its parabola arch and store a new segment in the Voronoi diagram.
  - a circle event, store both the new Voronoi vertex and the information that it is an extreme of the segments corresponding to two breakpoints joining in a single point.
- …

FORTUNE ALGORITHM (14)

Fortune algorithm (contd.)

- …
- In both cases, verify whether a new triple of sites producing a next circle event has been discovered.
- The Voronoi diagram is computed considering the (finite) sequence of these events.

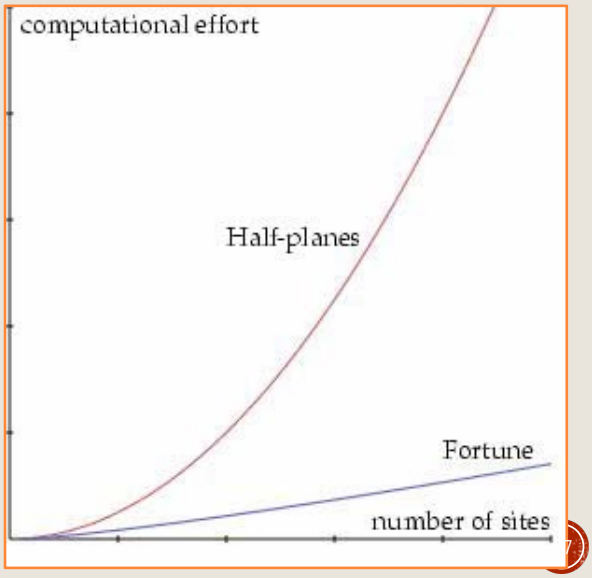
COMPUTATIONAL TIME ANALYSIS

(typical scheme of all the algorithms based on the sweep line)

- Each event takes O(1) time to be detected + a constant number of accesses to the data structures to be stored.
- Each data structure contains O(n) information
- Each one of these accesses needs O(logn) time
- The whole computational time is O(n log n), and the occupied space is O(n).
- This time is optimum because the sorting problem (based on comparisons) can be reduced to the computation of the Voronoi diagram.

CONCLUSIONS

- The time complexity for computing a Voronoi diagrams clearly grows up with the growth of the number of sites.
- The algorithm based on the intersection of halfplanes runs in $O(n^2 \log n)$ time if there are n sites.
- Fortune algorithm runs in $O(n \log n)$ time.



HETEROGENEOUS SENSORS

HETEROGENEOUS SENSORS (1)

Sensors are not necessarily all equal. In a heterogeneous sensor network:

- the devices are different or
- the sensing and communicating ability depend on their position (not smooth terrain, obstacles, …)

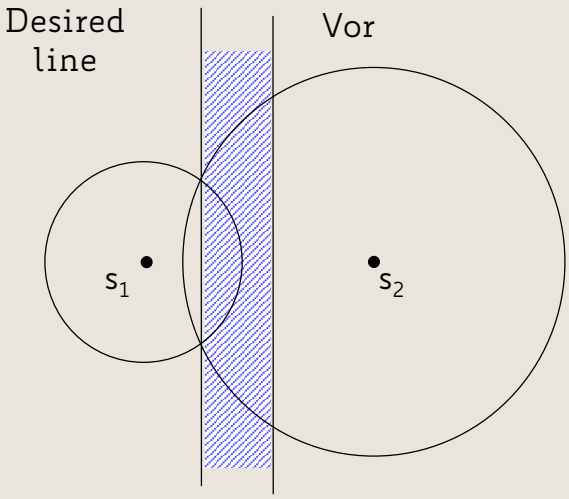
HETEROGENEOUS SENSORS (2)

The previously described approaches (based on virtual forces and on Voronoi cells) do not work well with heterogeneous sensors:

- Virtual forces: forces depend on the distance
- Voronoi: cells do not take into account the coverage capability

LIMITATIONS OF THE PROTOCOLS BASED ON VORONOI CELLS (1)

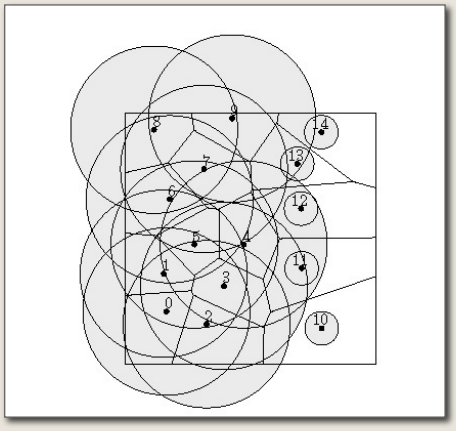
- A protocol based on the construction of Voronoi cells would assign:
  - The left halfplane (included the blue zone) to s_1
  - The right halfplane (included the blue zone) to s_2



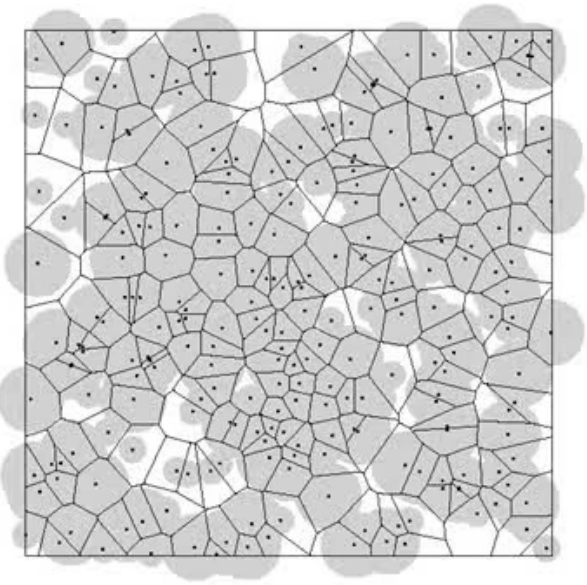
LIMITATIONS OF THE PROTOCOLS BASED ON VORONOI CELLS (2)

Stale situation:

- the sensors on the left (big circles) do not move since they completely cover their cells
- the sensors on the right (small circles) do not move since their circles are completely used to cover a portion of their cell (in other words, their coverage capacity is maximized).



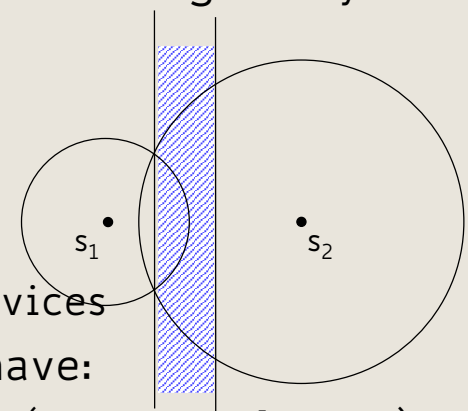
LIMITATIONS OF THE PROTOCOLS BASED ON VORONOI CELLS (3)



(video HetVor)

A NEW NOTION OF DISTANCE

- In the known algorithms, the heterogeneity is usually ignored
- New notion of distance keeping into account:
  - The Euclidean distance
  - The heterogeneity of the devices
- Many possibilities, wish to have:
  - Diagrams with straigh edges (convex polygons)
  - the set of points equally distant from two sensors contains the intersection of their sensing circles



LAGUERRE DISTANCE (1)

[W. Blaschke. Vorlesungen uber Differentialgeometrie III. Springer Berlin. 1929]

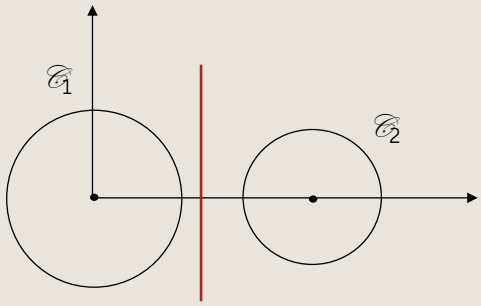
- Defined in $\mathbb{R}^3$
- Given two points P=(x,y,z) and Q=(x',y',z'), their Laguerre distance is:
  - $d_L^2(P,Q)=(x-x')^2+(y-y')^2-(z-z')^2$
- P can be seen as the (oriented) circle centered at (x,y) and having radius |z |

LAGUERRE DISTANCE (2)

- Given two circles $\mathscr{C}_1$ and $\mathscr{C}_2$, centered at C_1 and C_2 respectively, and with radii r_1 and r_2, their Laguerre distance is:
  - $d_L^2(\mathscr{C}_1, \mathscr{C}_2)= d_E^2(C_1, C_2) - (r_1 - r_2)^2$
- The Laguerre distance between a point P=(x,y) and a circle $\mathscr{C}$=(x',y',r) is:
  - $d_L^2(P, \mathscr{C})=(x-x')^2+(y-y')^2-r^2$

LAGUERRE DISTANCE (3)

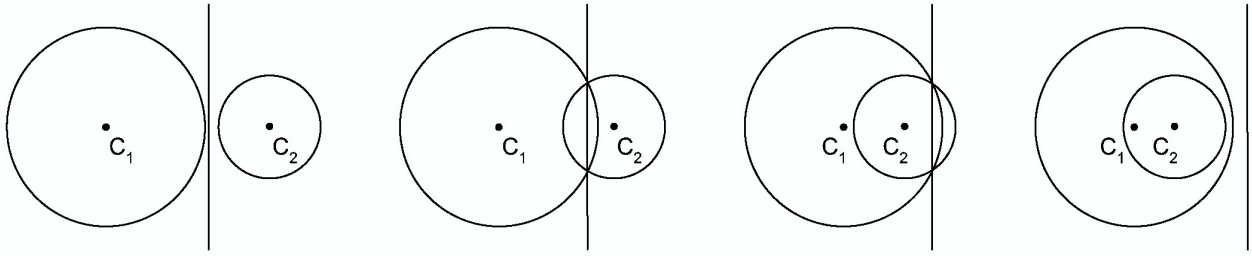
Lemma. Given two circles $\mathscr{C}_1$ and $\mathscr{C}_2$ centered at C_1 and C_2 ($C_1 \neq C_2$) and radii r_1 and r_2, the sets of point equally distant from $\mathscr{C}_1$ and $\mathscr{C}_2$ (called radical axis) is a straight line orthogonal to the segment joining C_1 and C_2.



LAGUERRE DISTANCE (4)

Lemma. Given two circles $\mathscr{C}_1$ and $\mathscr{C}_2$ centered at C_1 and C_2 ($C_1 \neq C_2$) and having radii r_1 and r_2, their centers may lie on the same side w.r.t. the radical axis (if and only if $d_E^2(C_1,C_2) < |r_1^2 - r_2^2|$).

Possible positions of the radical axis of two cricles $\mathscr{C}_1$ and $\mathscr{C}_2$:

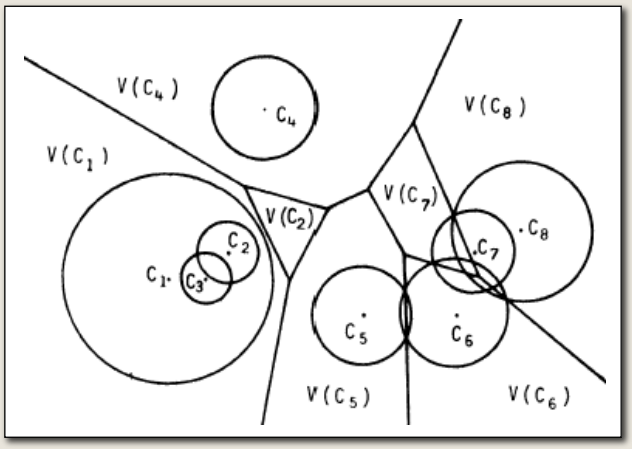


VORONOI-LAGUERRE DIAGRAM (1)

Voronoi-Laguerre diagram of $\mathscr{C}_1$, …, $\mathscr{C}_n$:

- $V_i = \cap \{p \in \mathscr{R}^2 \mid d_L^2(\mathscr{C}_i,P) \leq d_L^2(\mathscr{C}_j,P)\}$

[H. Imai, M. Iri, K. Murota. "Voronoi Diagram in the Laguerre Geometry and its Applications". SIAM J. Comput. 14(1), 93-105. 1985]



They have similarities and differences w.r.t. the classical Voronoi diagrams…

VORONOI-LAGUERRE DIAGRAM (2)

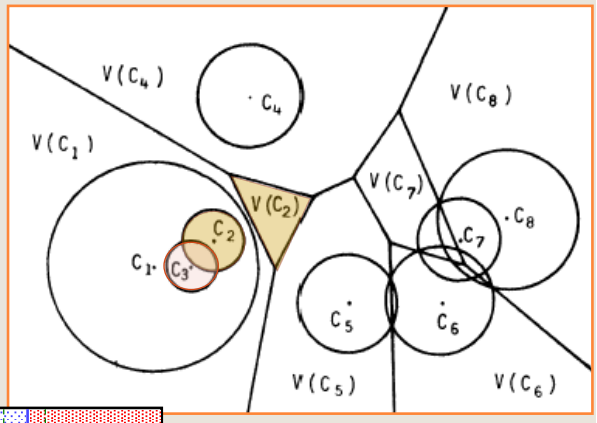
Similarities:

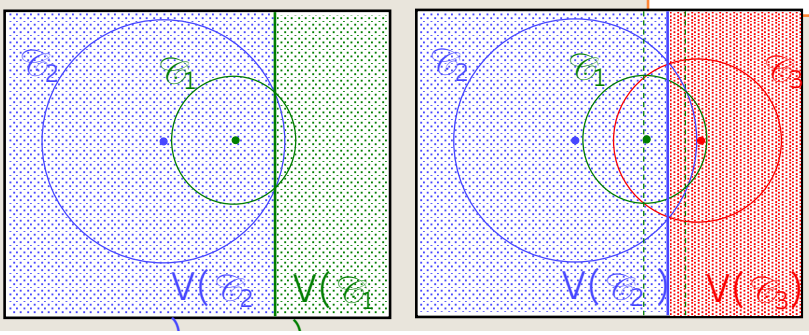
- Voronoi-Laguerre polygons partition the plane
- V_i is always convex because it is the intersection of some halfplanes
- if r_i=0 for each i=1, …, n, the Voronoi-Laguerre diagram is in fact the classical Voronoi diagram.

VORONOI-LAGUERRE DIAGRAM (3)

Differences:

- $\mathscr{C}_i$ can be external to V_i (see $\mathscr{C}_2$)
- V_i can be empty (e.g. if $\mathscr{C}_i$ is inside the union of other circles - see $\mathscr{C}_3$)





VORONOI-LAGUERRE DIAGRAM (4)

- Theorem. Given n circles $\mathscr{C}_i$ centered at $C_i=(x_i,y_i)$ and having radii r_i, i=1, …, n, let V_i be their Voronoi-Laguerre polygons.

  For each i and j, $V_i \cap \mathscr{C}_j \subseteq \mathscr{C}_i$.

  In other words, the intersection of V_i with a circle $\mathscr{C}_j$ is included in $\mathscr{C}_i$.

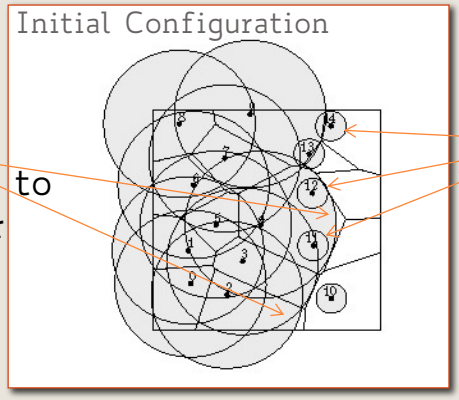
ALGORITHM BASED ON VORONOI-LAGUERRE DIAGRAM (1)

Algorithm executed by each sensor s_i:

- Compute V_i
- If s_i is inside V_i, move toward the minimax (at most by $d_i^{max}=r_{tx}/2-r_i$ where$r_{tx}=\min_i r_i^{tx}$) if the coverage of V_i is increased
- If s_i is outside V_i, move toward the minimax (by at most $d_i^{max}=r_{tx}/2-r_i$)
- if V_i is empty, do nothing.

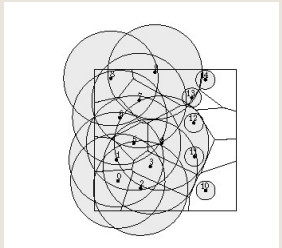
ALGORITHM BASED ON VORONOI-LAGUERRE DIAGRAM (2)

Big Circles
Some of them move to better cover their polygons

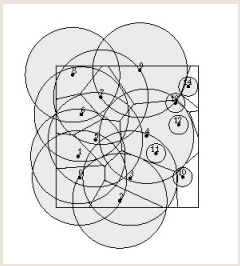


Small circles
Many of them move because they are external w.r.t. their polygon

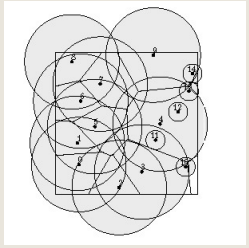
Initial Configuration



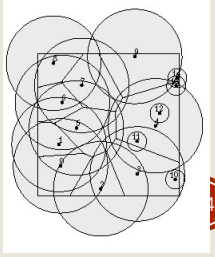
Round 6



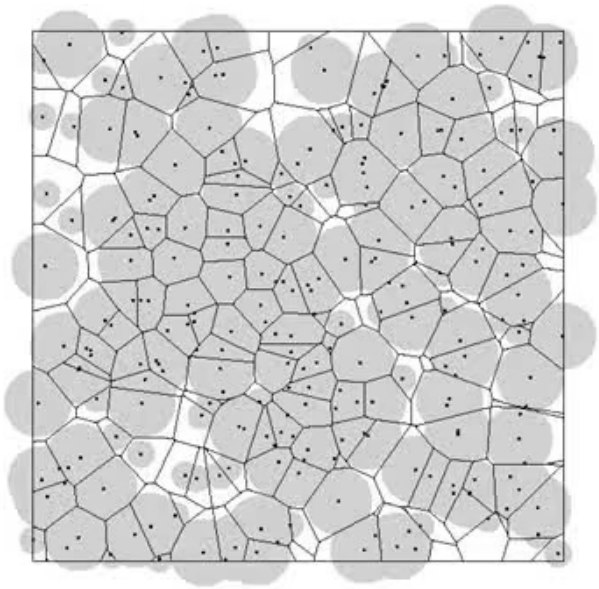
Round 9



Round 12:
The Stale is solved!



ALGORITHM BASED ON VORONOI-LAGUERRE DIAGRAM (3)

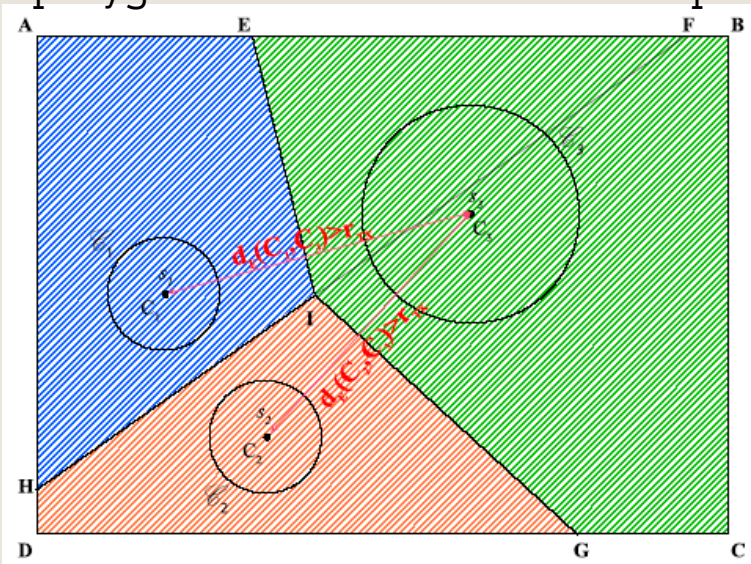


(video HetVorLag)

PROPERTIES OF THE ALGORITHM (1)

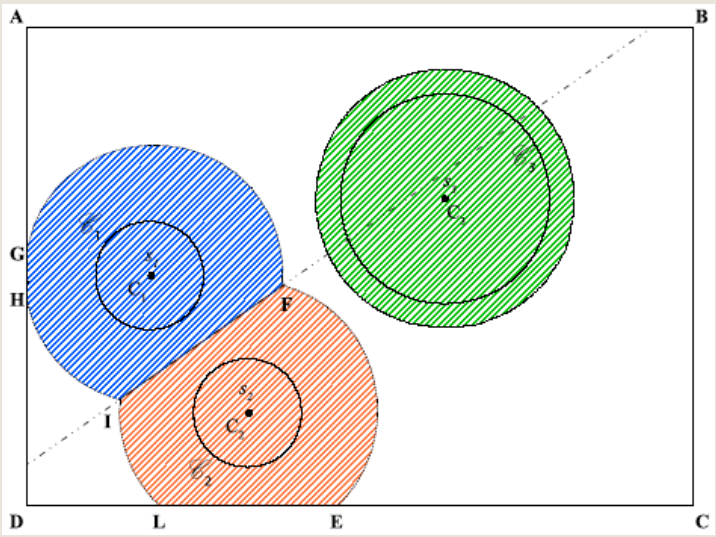
Obs.:

- "local" polygon≠"global" polygon and the set of local polygons do not constitute a partition!



PROPERTIES OF THE ALGORITHM (2)

- Exploiting the minimax, we define a curve polygon V'_i generated intersecting the "local" polygon with the circle of radius $d_i^{max}+r_i=r_{tx}/2$.



PROPERTIES OF THE ALGORITHM (3)

Lemma. $V'_i \cap V'_j = \emptyset \ \forall \ i \neq j$

Lemma. $\forall \ i \neq j$, $V'_i \cap \mathcal{G}_j \subseteq \mathcal{G}_i$.

In other words, each curve polygon can be covered by the sensor generating it better than by any other sensor.

PROPERTIES OF THE ALGORITHM (6)

- Th. The algorithm converges.
- Convergence does not imply termination.
- In order to guarantee termination, we introduce a minimum movement threshold ε, so that sensors do not move if they are suppose to do by less than ε.
- Corollary. The algorithm, with the addition of the minimum movement threshold, terminates.

MORE COMPLEX PROBLEMS

- Obstacles and terrain asperities
  - Anisotropy
  - Movement obstacles
- AOI with complex shape
  - concave regions and corridors
- …