THE DISTRIBUTED DEPLOYMENT OF MOBILE SENSORS I.E. THE VORONOI DIAGRAM CONSTRUCTION PROBLEM

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THE PROBLEM

2

THE DISTRIBUTED DEPLOYMENT PROBLEM (1)

We have already spoken about *mobile sensor networks…*

… and of the *deployment problem.*

A centralized solution is not always desirable because:

- Connection with a server is required
- Long delays are expected
- The solution is not fault-tolerant

The ability of moving around facilitates sensors to self-deploy starting from any initial configuration to a final distribution that guarantees that the AoI is completely covered.

THE DISTRIBUTED DEPLOYMENT PROBLEM (2)

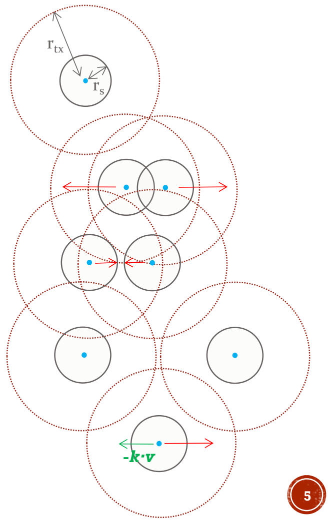
The self-deployment is necessary in "hostile" environments:

- Contaminated places
- Fires
- Battlefields…

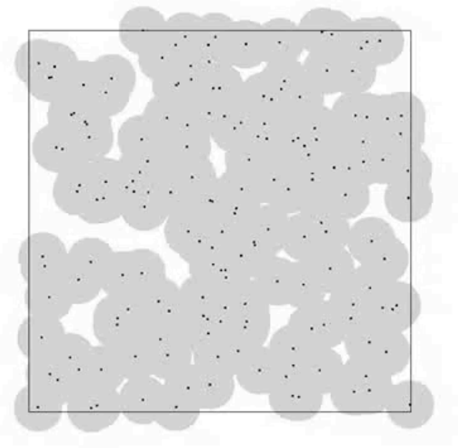
In these cases, sensors should position themselves and transmit the collected information.

A POSSIBLE APPROACH: VIRTUAL FORCES (1)

- Idea: sensors are similar to charged particles (magnetic force) having a mass (gravitational force)
- Two sensors repel each other if they are too close
- Two sensors attract each other if they are far but can anyway communicate
- Two sensors ignore each other if they cannot communicate (too far)
- Friction to attenuate oscillations



A POSSIBLE APPROACH: VIRTUAL FORCES (2)



A POSSIBLE APPROACH: VIRTUAL FORCES (3)

Weaknesses:
- It is necessary a manual tuning of parameters
- Sensor oscillation
  - Friction forces
  - Stopping conditions
- In some versions, attracting effect of the border and of the obstacles (e.g. when only repulsive forces are considered)
- …

A POSSIBLE APPROACH: VIRTUAL FORCES (4)

Weaknesses (cntd):
- Sensors tend not to pass through doors and narrows

Possible lesson

(a)

(b)

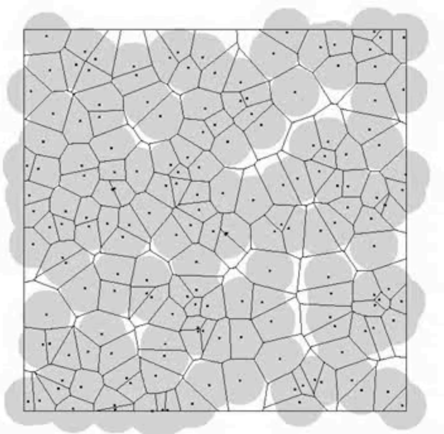
A PROTOCOL BASED ON VORONOI DIAGRAMS (1)

Idea:

- Each sensor is assigned an AoI portion and it has to take charge of it, trying to cover it as best as it can
- The sensor is "satisfied" if:
  - It completely cover its portion

  or
  - All its sensing radius is used to cover its portion
- If a sensor is not "satisfied" it has to move in order to improve its coverage
- AoI portions can be assigned according to the Voronoi diagram.

A PROTOCOL BASED ON VORONOI DIAGRAMS (2)

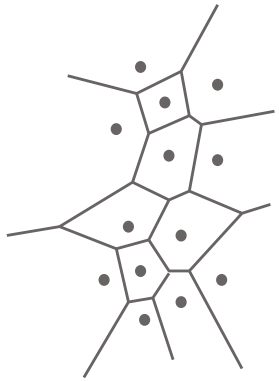





VORONOI DIAGRAMS


VORONOI DIAGRAM (1)

Suppose that you live in a desert where the only sources of water are a few springs scattered here and there. For each spring, you would like to determine the locations nearest that spring. The result could be a map, like the one shown here, in which the terrain is divided into regions of locations nearest the various springs.



VORONOI DIAGRAM (2)

Maps like this appear frequently in various applications and under many names. To mathematicians, they are known as *Voronoi diagrams*.

Voronoi diagrams are rather natural constructions, and it seems that they, or something like them, have been in use for a long time.

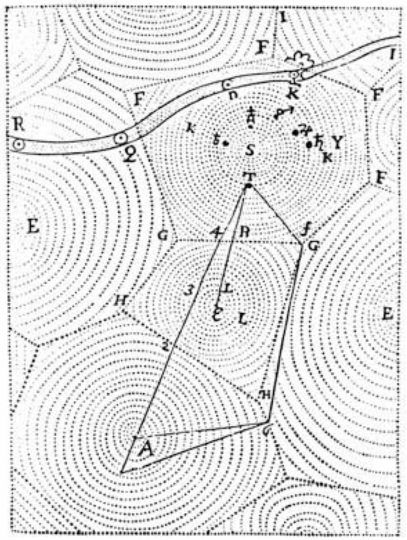
VORONOI DIAGRAM (3)

Voronoi diagrams have been used by:

- anthropologists to describe regions of influence of different cultures;
- crystallographers to explain the structure of certain crystals and metals;
- ecologists to study competition between plants;
- economists to model markets in a certain economy;
- …

VORONOI DIAGRAM (4)

- An informal study of Voronoi diagrams dates back to Descartes (1644): he includes the following figure with his demonstration of how matter is distributed throughout the solar system.
- …



VORONOI DIAGRAM (5)

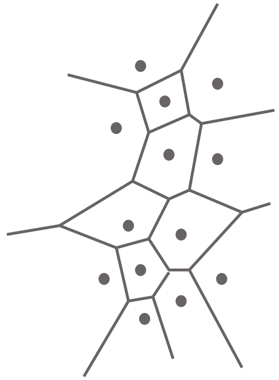
- …
- The English physicist Snow uses them for his analysis of the London cholera outbreak of 1854:
  - Snow considers the sources of drinking water, pumps distributed throughout the city, and draws a line labeled "Boundary of equal distance between Broad Street Pump and other Pumps," which essentially indicates the Broad Street Pump's Voronoi cell.
  - This map supports Snow's hypothesis that the cholera deaths are associated with contaminated water, in this case, from the Broad Street Pump. Snow recommends to the authorities that the pump handle be removed, after which action the cholera outbreak quickly ends.
- …

VORONOI DIAGRAM (6)

- Dirichlet uses Voronoi diagrams in his studies on quadratic equations in 1850.
- Voronoi diagrams are so called in honor of the Russian mathematician Georgy F. Voronoi, who defined and studied them in the n-dimensional space in 1908.
- They are also called *Thiessen polygons* in meteorology in honor of the US meteorologist Alfred H. Thiessen; *Wigner-Seitz cells* in physics, *fundamental domains* in group theory and *fundamental polygons* in topology.

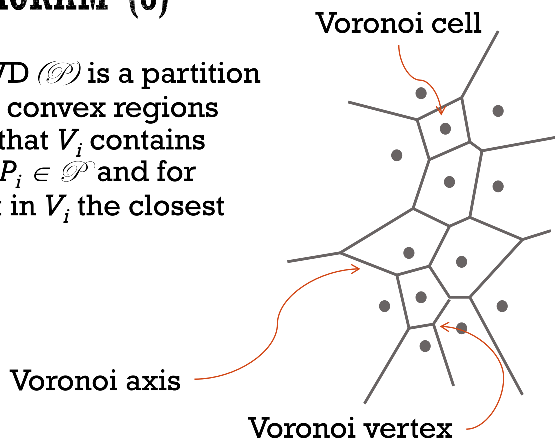
VORONOI DIAGRAM (7)

- Def. of Voronoi Diagram:
  - $\mathscr{P}$: set of n distinct sites on the plane
  - VD $(\mathscr{P})$: partition of the plane into n cells V_i such that:
    - each V_i contains exactly one site
    - if a point Q on the plane is in V_i then $dist(Q, P_i) < dist(Q, P_j)$ for each $P_j \in \mathscr{P}, j \neq i$.



VORONOI DIAGRAM (8)

- In other words: VD $(\mathscr{P})$ is a partition of the plane into convex regions $\{V_1, \ldots, V_n\}$, such that V_i contains exactly one site $P_i \in \mathscr{P}$ and for each other point in V_i the closest site in $\mathscr{P}$ is P_i.

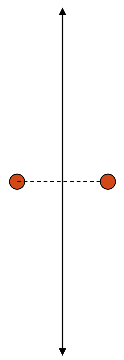


VORONOI DIAGRAM (9)

Voronoi diagram of a single site

VORONOI DIAGRAM (10)

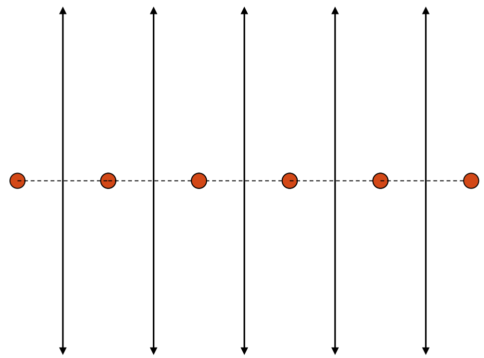
Voronoi diagram of two sites



The axis extends to infinity in both directions, generating two halfplanes

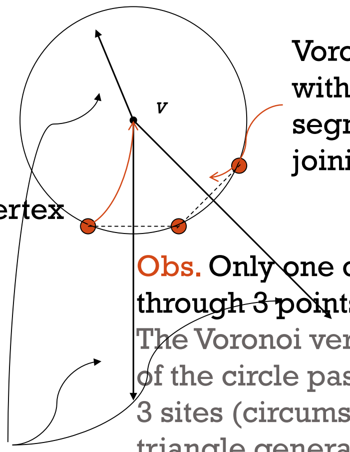
VORONOI DIAGRAM (11)

Voronoi diagram of some colinear sites



VORONOI DIAGRAM (12)

Voronoi diagram of 3 not colinear sites



Voronoi axes coincide with the axes of the segments pairwise joining the sites

The Voronoi vertex Has degree 3

Obs. Only one circle passes through 3 points

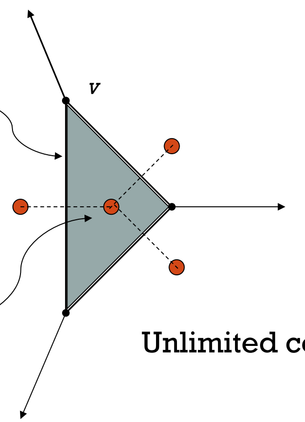
The Voronoi vertex is the center of the circle passing through the 3 sites (circumscribed to the triangle generated by the 3 sites)

halflines

VORONOI DIAGRAM (13)

Voronoi diagram of 4 not colinear sites

Segment



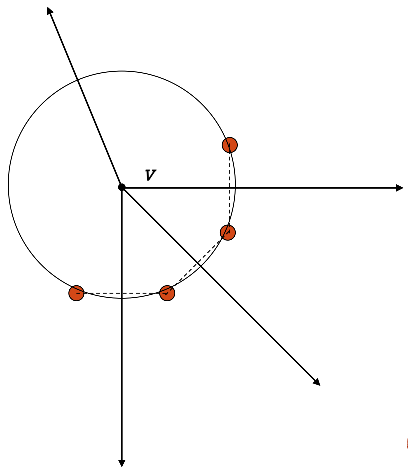
Limited cell

Unlimited cell

VORONOI DIAGRAM (14)

Not always 4 not colinear sites create a limited cell:

General position assumption: each 3 sites are not colinear and each 4 sites are not cocircular.
Thanks to this assumption, all vertices have degree 3!

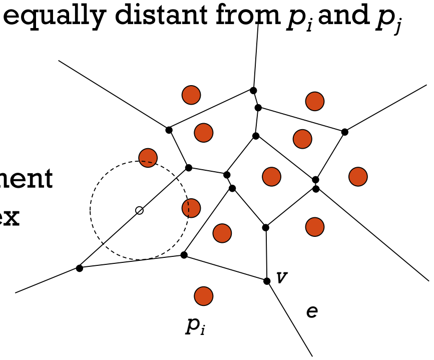


VORONOI DIAGRAM PROPERTIES (1)

A point q on the plane lies on the Voronoi segment between p_i and p_j *iff* the largest empty circle centered in q touches only p_i and p_j.

– A Voronoi segment is a subset of a Voronoi axis, i.e. the set of point equally distant from p_i and p_j

p_i : sites of $\mathscr{P}$
e : Voronoi segment
v : Voronoi vertex

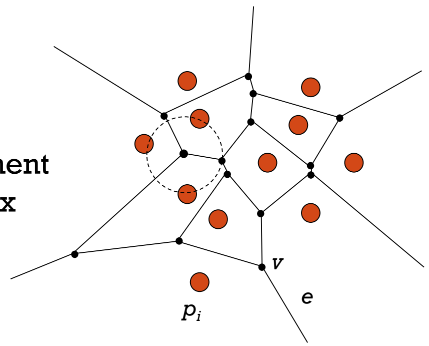


VORONOI DIAGRAM PROPERTIES (2)

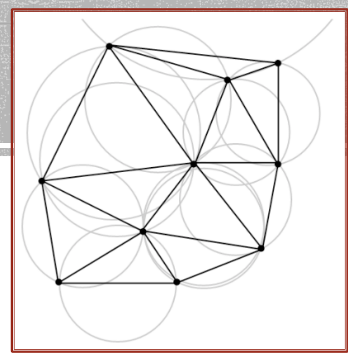
A point q in the plane is a Vornoi vertex *iff* the largest empty circle centered in q touches at least 3 sites of $\mathscr{P}$.

A Voronoi vertex is the intersection of at least 3 axes, each generated by a pair of sites.

p_i : sites of $\mathscr{P}$
e : Voronoi segment
v : Voronoi vertex

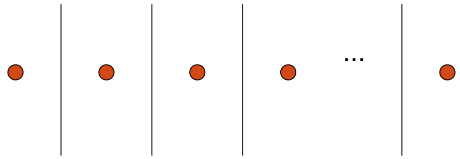


VORONOI DIAGRAM COMPLEXITY



VORONOI DIAGRAM COMPLEXITY (1)

- Th.: $|v| \le 2n - 5$ and $|e| \le 3n - 6$ for each $n \ge 3$.
- Proof: (Easy case)



Colinear sites → $|v| = 0$, $|e| = n - 1$

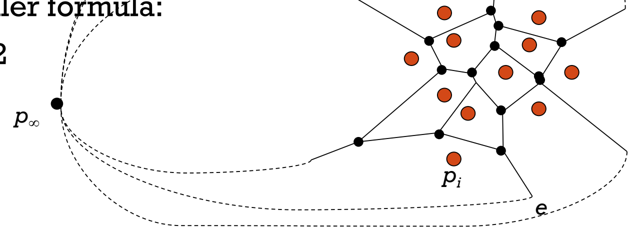
VORONOI DIAGRAM COMPLEXITY (2)

Proof of Th.: $|v| \le 2n - 5$ and $|e| \le 3n - 6$ for each $n \ge 3$ – cntd.

Proof: (General case)

- Problem: A Voronoi diagram cannot be considered as a planar graph because some of its edges and faces are unlimited
- Solution: add a dummy node
- Now the Voronoi diagram is a planar and connected graph → Euler formula:

$|v|-|e|+f=2$



VORONOI DIAGRAM COMPLEXITY (3)

Proof of Th.: $|v| \le 2n - 5$ and $|e| \le 3n - 6$ for each $n \ge 3$ – cntd.

$f=n+1$. Euler formula becomes:

$|v|-|e|+n+1=2$ (1)

Moreover: $\sum_{v \in VD} \deg(v) = 2|e|$

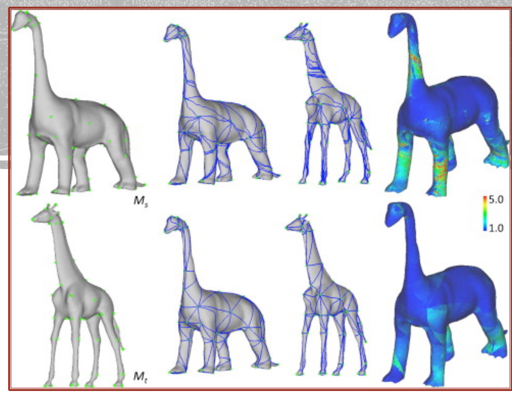
since $\deg(v) \ge 3$ → $2|e| \ge 3|v|$ (2)

Joining (1) e (2):

$|v| \le 2n-5$

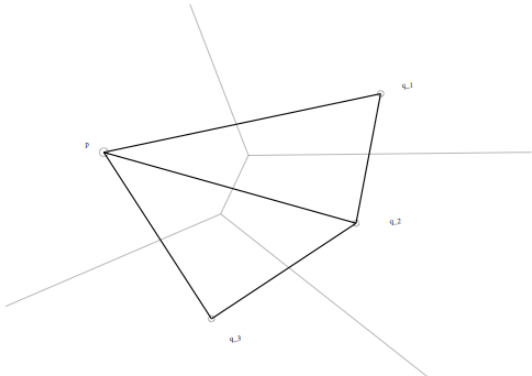
$|e| \le 3n-6$

THE DUAL PROBLEM OF THE VORONOI DIAGRAM



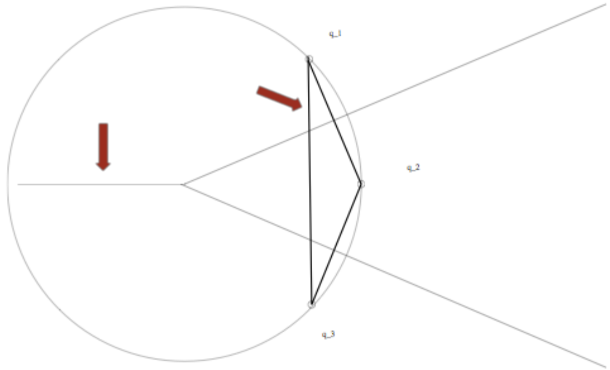
THE DUAL PROBLEM OF THE VORONOI DIAGR.

- The dual problem w.r.t. the decomposition of the plane into Voronoi cell is the Delaunay triangulation (obtained interescting each Voronoi axis with a segment joining the generating sites)



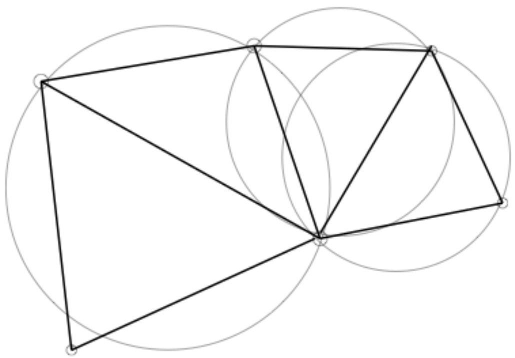
DELAUNAY TRIANGULATION (1)

- Obs. Dual segments not necessarily intersect!



DELAUNAY TRIANGULATION (2)

- Property: the circle circumscribed to a Delaunay triangle does not contain any site inside it

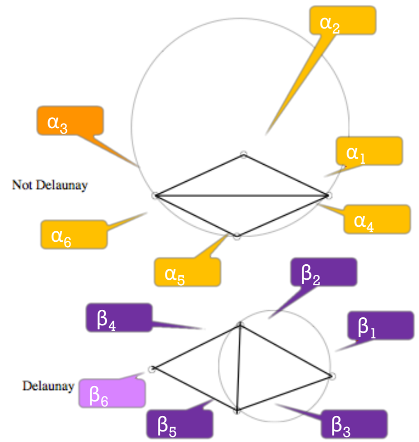


DELAUNAY TRIANGULATION (3)

- Property: no segment can be illegal.
- A segment is *illegal* if:

  min α_i<min β_i

- If *e* is an illegal edge, then it is possible to swap the triangles to get a Delaunay triangulation.s



DELAUNAY TRIANGULATION (4)

- Some papers exploit a Delaunay triangulation to route sensors towards a position contributing to a complete coverage.
- There are several algorithms to compute a Delaunay triangulation -> Possible lesson
- The Voronoi Diagram can be computed as dual construction of the Delaunay triangulation.
- Otherwise…