SECOND PART: WIRELESS NETWORKS 2.B. (FIXED) SENSOR NETWORKS

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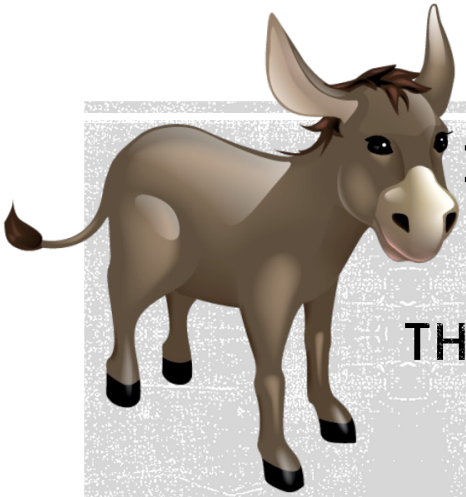

DATA MULE SCHEDULING IN SENSOR NETWORKS I.E. THE TRAVELING SALESMAN PROBLEM

2

Prof. Tiziana Calamoneri
Network Algorithms
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SENSORS

- A sensor is a device that detects and responds to some type of input from the physical environment.
- The specific input could be light, heat, motion, moisture, pressure, or any one of a great number of other environmental phenomena.
- The output is generally a signal that is converted to human-readable display at the sensor location or transmitted electronically over a network for reading or further processing.

(FIXED) SENSOR NETWORKS

- Sensor networks are wireless networks of small, typically but not necessarily low-cost sensors, which collect te environmental data.
- Sensor networks are rapidly growing for their large applicability to various purposes.

THE PROBLEM

5'

THE PROBLEM (1)

From engineering perspectives, one of the most critical issues in (fixed) sensor networks is energy, because:

- sensor networks can be deployed in remote areas where line powers are hard to obtain. This fact forces the sensor nodes to rely on batteries, but replacing the batteries can also hard.
- sensor network applications often require long-term measurements over months.

THE PROBLEM (2)

- So, improving the energy efficiency is mandatory for sensor networks.
- Since sensors do not move in this context, wireless communication is one of the most energy-consuming operations on a sensor node

try to reduce communication

THE PROBLEM (3)

A possible approach:

Multi-hop communication to the base station:

not always a good solution.

Indeed:

1. the communication infrastructure between nodes could be instable.
2. It can be inefficient depending on how densely the sensor nodes are deployed:

…

THE PROBLEM (4)

- When the node deployment is very sparse, the distance to the nearest node or base station (i.e., each hop) becomes large, so the sensor should set their transmission range to a too high value, requiring a large amount of energy.
- When the node deployment is very dense, the nodes close to the base station need to forward the data from too many remote nodes and thus tend to run out of energy soon.

THE PROBLEM (5)

Another approach:

Exploit the mobility:

A data mule is a mobile node that has:

- wireless communication capabilities
- a sufficient amount of storage to store the data from the (fixed) sensor nodes in the field.

Data mule travels across the sensing field and collects data from each sensor node when arrives at short distance from it; later it deposits all the data to the base station.

THE PROBLEM (6)

- Each sensor node can save a significant amount of energy, since it only needs to send the data over a shorter distance and has no need to forward other sensors' data to the base station.
- As data mules return to the base station after the travel, energy issue is usually not critical for them.
- This approach reduces the responsibility for message routing from the nodes thereby minimizing their processing power.

THE PROBLEM (7)

Data mule scheduling problem:

How to control a data mule such that it collects data from **all** the nodes in the **minimal amount of time**?

- We formulate it as a scheduling problem, since we view communication from each node as a job.
- We can control the movement of the data mule (path, speed) as well as its communication (i.e., which node it collects data from at certain time duration), where the latter corresponds to job allocation in classical scheduling problems.

THE PROBLEM (8)

Data mule scheduling problem (contd)

- Despite the similarities with a scheduling problem, data mule scheduling problem is more complex because it has both location and time constraints.
- location: Availability of each job is determined by the range of wireless communication, which primarily depends on the distance from a node and thus serves as a location constraint.
- …

THE PROBLEM (9)

Data mule scheduling problem (contd)

- time: since the bandwidth of wireless communication is constant and the data mule never stops, we also have a time constraint for each node necessary for transmitting the data to a data mule.

Namely, the problem can be decomposed into the following three subproblems:

THE PROBLEM (10)

Data mule scheduling problem (contd)

1. Path selection is to determine the trajectory of the data mule in the sensor field. To collect data from each sensor, the data mule needs to go within the sensor's communication range at least once.
2. …

THE PROBLEM (11)

Data mule scheduling problem (contd)

2. Speed control is to determine how the data mule changes its speed along the chosen path, so that it stays within each node's communication range long enough to collect all the data from it without stopping.
   More technical than algorithmic.
3. …

THE PROBLEM (12)

Data mule scheduling problem (contd)

3. Job scheduling: at each instant, the data mule can be close to more than one sensor, and it has to decide from which one to collect data. Data collection from each sensor is a job, having certain time-intervals in which it can be executed.
   It can be reduced to a classical job scheduling problem = to determine the allocation of time-slots to jobs so that all jobs can be completed.

   ☞ possible students' lesson

THE PROBLEM (13)

Data mule scheduling problem (contd)

We focus on the first sub-problem (path selection):

- Consider sensor nodes operating at different sampling rates (e.g., in case of pollution sensors).
- Each sensor has a finite buffer for storing the sensed values and a data mule (acting as a base station) does the job of the data gathering.
- Once the mobile element visits a sensor node, it transfers the data to its own memory and the sensor's memory is freed.

THE PROBLEM (14)

Data mule scheduling problem (contd)

- Problem: scheduling of the visits of the data mule so that none of the sensor nodes' buffer overflows: **Mobile Element Scheduling (MES)** problem.

THE PROBLEM (15)

Observe the similitude with the **Traveling Salesman Problem (TSP)**:

Given a set of cities, a salesman has to visit each one of the cities starting from a certain one (e.g., the hometown) and returning to the same city. The challenge of the problem is that the traveling salesman wants to minimize the total length of the trip.

THE PROBLEM (16)

In fact, MES and TSP are different:

- In TSP, the goal is to find a (minimum cost) tour that visits each node exactly once.
- In MES, a node may need to be visited multiple times before all the other nodes are visited depending on the strictness of its deadline i.e., frequency of sampling.
- …

THE PROBLEM (17)

In fact, MES problem and TSP are different (contd):

- In TSP, the optimization function minimizes the cost of the tour and costs are fixed in the time.
- In MES, as soon as a node is visited, its deadline (i.e., time before which it should be revisited to avoid buffer overflow), is updated. Thus, deadlines are dynamically updated as the mobile element performs the job of data gathering.

Nevertheless, TSP seems very useful to solve MES problem.

THE TRAVELING SALESMAN PROBLEM

THE TRAVELING SALESMAN PROBLEM (1)

HC (decisional problem):

- Let G=(V,E) be a graph.
- A Hamiltonian cycle (HC) is a cycle passing through all nodes exactly once.
- Question: Does G contains a Hamiltonian cycle?
- The problem of finding a Hamiltonian cycle (HC) is NP-complete.

THE TRAVELING SALESMAN PROBLEM (2)

TSP (decisional version):

- Let K_n=(V,E) be a complete graph, w a non negative edge-weight function, and t a non negative real value.
- Question: Does K_n contains a Hamiltonian cycle with cost not exceeding t?
- Note: K_n always contains a HC because it is complete; here the problem is to minimize the cost…

THE TRAVELING SALESMAN PROBLEM (3)

- The origins of TSP are unclear:
- A handbook for travelling salesmen from 1832 mentions the problem and includes example tours through Germany and Switzerland but contains no mathematical treatment.
- It was mathematically formulated in the 1800s by W.R. Hamilton and T. Kirkman.
- The general form has been first studied during the 1930s considering the obvious brute-force algorithm and observing the non-optimality of the nearest neighbor heuristic.

THE TRAVELING SALESMAN PROBLEM (4)

- Th. TSP is NP-complete.
- Proof.

1. TSP belongs to NP: check that the tour contains each node exactly once + the sum of the costs of all the edges in the tour is bounded by t.
2. We reduce HC to TSP:

   Assume G=(V,E) is an instance of HC and construct the complete graph K_n=(V,E'). Let t=n and function w is defined as follows:

   - w(i,j)=1 if (i,j) is in E
   - w(i,j)=2 if (i,j) is in E'\E

THE TRAVELING SALESMAN PROBLEM (5)

NP-completeness proof (contd)

- Assume now that a HC C exists in G.
- All edges in C have weight 1 in K_n since they are all in G.
- So, if G has a HC then K_n has a TS tour of cost n.
- Conversely, if K_n has a TS tour of cost n then all the used edges must come from G and hence the tour is a HC for G. ■

THE TRAVELING SALESMAN PROBLEM (6)

TSP can be formulated as an ILP

[Dantzig, Fulkerson, Johnson '54]:

- Assume that the tour is oriented.
- We define:
  - boolean variables x_{ij}=1 iff the tour traverses oriented edge (i,j), x_{ij}=0 otherwise.
  - w_{ij} the weight of oriented edge (i,j).
- …

THE TRAVELING SALESMAN PROBLEM (7)

ILP formulation (contd)

…

The objective is: $\min \sum_{i,j=1..n} w_{ij}\ x_{ij}$

subject to: $\sum_{j=1..n} x_{ij}=1$, $\sum_{i=1..n} x_{ij}=1$ (only one arc from i and to j)

and the subtour elimination constraints (guaranteeing that a cycle cover is not a solution):

$\sum_{i,j \text{ in } S} x_{ij} < |S|$ for each S proper subset of V,

indeed if S forms a cycle the sum is =|S|.

Note: exponential number of constraints!

INAPPROXIMABILITY (1)

The following negative result holds:

Th. If there exists a polyomial time algorithm for TSP with any approximation ratio r>1 then P=NP.

Proof. We prove that if TSP is r-approximable, then there exists a polynomial time exact algorithm for HC.

Let G=(V,E) be an instance of HC and K_n a complete graph with |V|=n.

…

INAPPROXIMABILITY (2)

If there exists a polyomial algorithm for TSP with any approximation ratio r>1 then P=NP. (proof contd)

…

We define edge weights on K_n as follows:

w(i,j)=1 if (i,j) is an edge of E

w(i,j)=2+(r-1)n otherwise.

Then, a tour with cost n exists in K_n if and only if G has a HC.

In this case, if we assume there exists an r-approximation algorithm A for TSP, if the cost is n, A will find a solution H with cost(H) ≤ r n.

…

INAPPROXIMABILITY (3)

If there exists a polyomial algorithm for TSP with any approximation ratio r>1 then P=NP. (proof cntd)

…

If H contains an edge that is not in E, then:

$$\text{cost}(H) \geq (n-1)+2+(r-1)n=rn+1$$

Hence, a solution of TSP with cost(H) ≤ r n exists iff an HC is in G and hence HC can be solved in polynomial time: a contradiction. ■

APPROXIMATE ALGORITHMS (1)

Inapproximability result: Bad news!

How to manage the problem?

Some special cases…

APPROXIMATE ALGORITHMS (2)

Def. Given any three nodes a, b, c if:

$$w(a,c) \leq w(a,b)+w(b,c)$$

we say that w satisfies the triangle inequality.

- Note. The weigth of a minimum spanning tree T (MST) is a lower bound on the cost of an optimal traveling salesman tour. Indeed: let H^* be an optimal tour. A ST P can be deduced from H^* by deleting an edge. Moreover, P is a path. It holds that:

$$w(T) \leq w(P) \leq w(H^*).$$

APPROXIMATE ALGORITHMS (3)

In the hypothesis of triangle inequality (metric TSP):

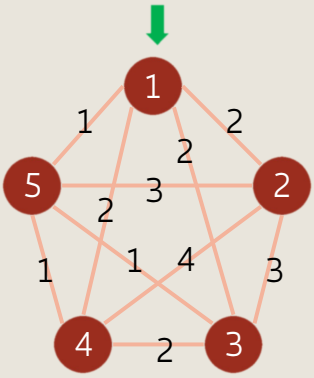
2-Approx-mTSP
Input: $K_n(V,E)$
Output: a Hamiltonian cycle

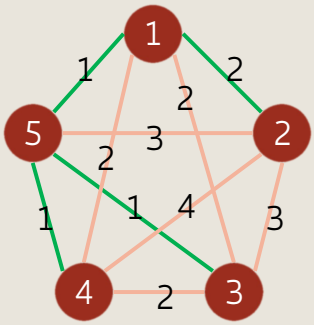
1. Select a "root" node r of K_n
2. Compute an MST T from r
3. let L be the sequence of nodes visited in a preorder walk of T
4. Return the HC that visits the nodes in the order L without repetitions

APPROXIMATE ALGORITHMS (4)

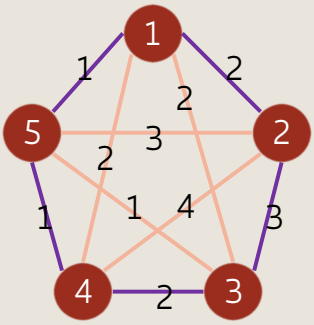
1. Select a "root" node r of K_n
2. Compute an MST T from r
3. let L be the sequence of nodes visited in a preorder walk of T
4. Return the HC that visits the nodes in the order L without repetitions

example





Weight=5
L=1 5 4 5 3 5 1 2 1

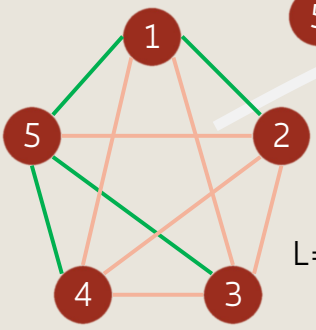


Weight=9

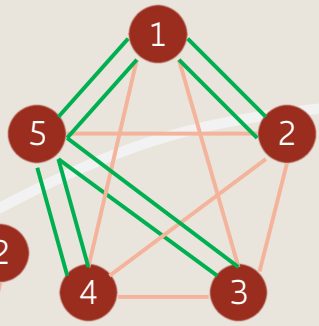
to pass from L to a cycle...

APPROXIMATE ALGORITHMS (5)

example



L=1 5 4 5 3 5 1 2 1



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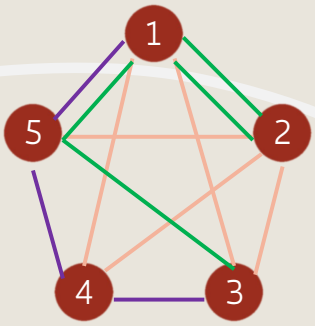

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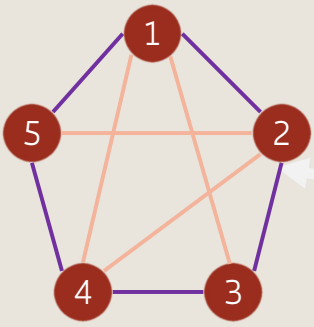
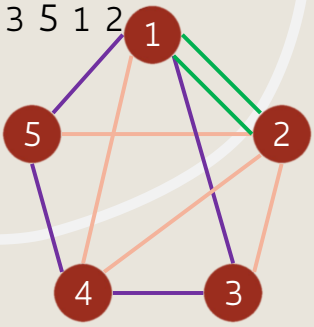


L=1 5 4 3 2 1



L=1 5 4 3 1 2 1

APPROXIMATE ALGORITHMS (6)

Th. 2-Approx-TSP is a 2-approximation algorithm for TSP if w satisfies the triangle inequality.

Proof. Let H^* be an optimal tour and T a MST. We know that $w(T) \leq w(H^*)$.

In L every edge appears exactly twice, so the tour C deduced by L is such that $w(C)=2\ w(T)$.

…

APPROXIMATE ALGORITHMS (7)

2-Approx-TSP is a 2-approximation algorithm for TSP if w satisfies the triangle inequality (proof contd)

Unfortunately, C is not a tour, since some nodes are repetead. We can erase some visits without increasing the cost: if a node a appears for the second time in the full path between b and c, we can go from b to c directly. In this way we get a tour H. By the triangle inequality:

$$w(H) \leq w(C) = 2\ w(T) \leq 2\ w(H^*).$$ ■

APPROXIMATE ALGORITHMS (8)

Euclidean TSP

- When the cities that the salesman has to visit lie in the Euclidean plane, the problem is called Euclidean TSP.
- Like the general TSP, Euclidean TSP is NP-hard.
- It is a special case of metric TSP, since distances in a plane obey the triangle inequality

  → 2-approximation algorithm.
- Christofides [C76] improves the previous approximation to 3/2:

APPROXIMATE ALGORITHMS (9)

Euclidean TSP(cntd)

Christofides Algorithm:

Input: $K_n(V,E)$

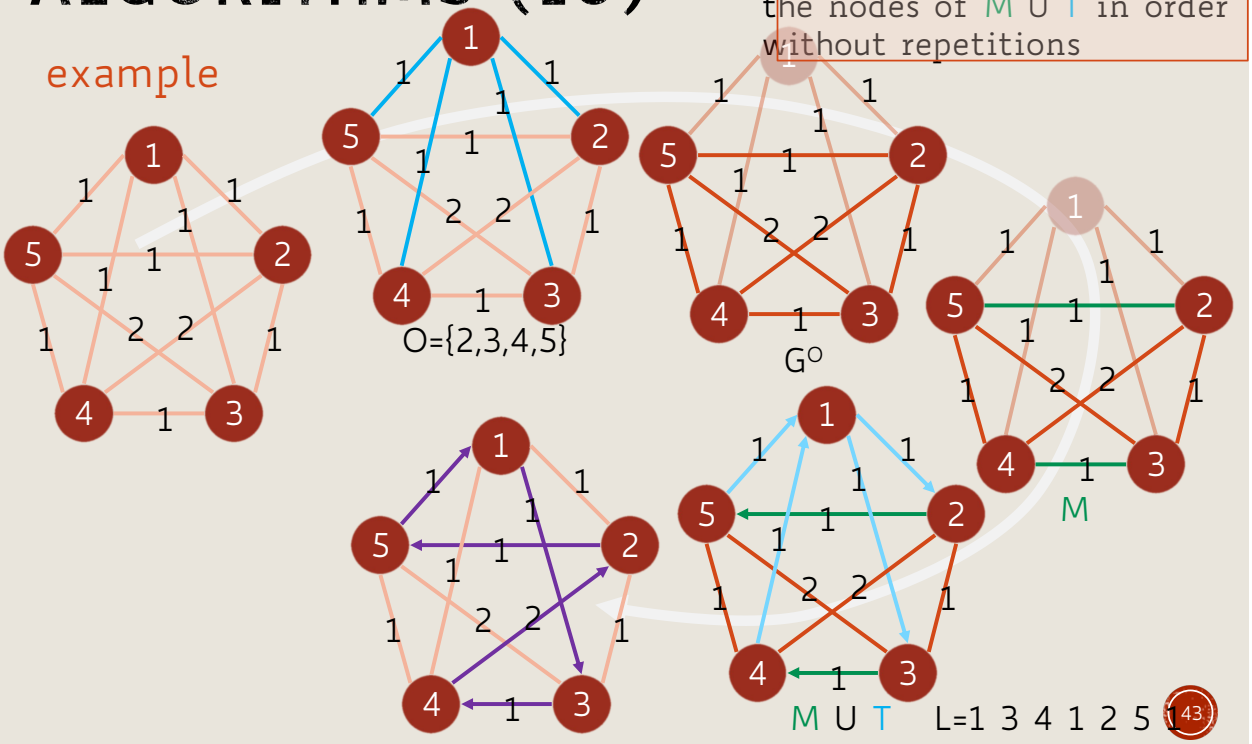
Output: a Hamiltonian cycle

- Compute a MST T of G
- O = set of nodes with odd degree in T. |O| is even
- G^O = subgraph induced by O
- Find a min-weight perfect matching M in G^O
- The graph induced by M ∪ T is a connected multigraph and each node has even degree
- Form an Eulerian circuit
- Return the HC that visits the nodes without repetitions

APPROXIMATE ALGORITHMS (10)

1. Compute a MST T
2. Compute G^O
3. Find a min-weight PM M
4. Return the HC that visits the nodes of M U T in order without repetitions

example



APPROXIMATE ALGORITHMS (11)

Euclidean TSP(contd)

Th. Christofides Algorithm is a 3/2-approximation algorithm for TSP if w satisfies the triangle inequality.

Proof. Let H^* be an optimal tour and T a MST. We know that $w(T) \leq w(H^*)$.

Number the nodes of G^O in cyclic order around H^*, and partition H^* into two sets of edges: the ones with an odd number on the first node and the ones with an even number on the first node. Each set is a perfect matching: $M_o \cup M_e \subseteq H^*$.

APPROXIMATE ALGORITHMS (12)

Euclidean TSP: Christofides Algorithm is a 3/2-approximation algorithm for TSP if w satisfies the triangle inequality (proof contd)

…

So, $w(M_o)+w(M_e) \leq w(H^*)$; w.l.o.g. $w(M_o) \leq w(H^*)/2$.

M is a min-weight perfect matching of G^O so $w(M) \leq w(M_o)$.

Putting everything together:

$$w(H) \leq w(M)+w(T) \leq w(H^*)/2+w(H^*) \leq 3/2\ w(H^*).$$ ■

APPROXIMATE ALGORITHMS (13)

Euclidean TSP(contd)

- There exist inputs to TSP that cause the Christofides algorithm to find a solution whose approximation ratio is arbitrarily close to 3/2.
- Can we do better?

APPROXIMATE ALGORITHMS (14)

Euclidean TSP(cntd)

- Yes: in general, there is a polynomial time approx scheme (PTAS) i.e. for any c > 0, if d is the dimension of the Euclidean space, there is a polynomial time algorithm that finds a tour of length at most (1 + 1/c) times the optimal for geometric instances of TSP in time

$$O(n(\log n)^{\left(O(c\sqrt{d})\right)^{d-1}})$$

- Arora and Mitchell were awarded with the Gödel Prize in 2010 for their concurrent discovery of a PTAS for the Euclidean TSP.

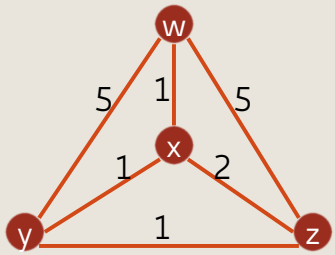
APPROXIMATE ALGORITHMS (15)

Euclidean TSP(cntd)

- …
- In 2013, Bartal and Gottlieb improved the time complexity of the PTAS
- But in practice, simpler heuristics with weaker guarantees continue to be used…

SOME GENERALIZATIONS (1)

- We can drop the condition that the travelling salesman should visit each city exactly once, so that we now consider not Hamiltonian cycles anymore, but simply closed walks containing each node at least once.
- If the problem is metric, any optimal tour will be also an optimal solution, but this is not true in general:



Example:
w-x-y-z-x-w is a shortest closed walk (of length 6), but the shortest tour w-x-y-z-w has length 8.

SOME GENERALIZATIONS (2)

Asymmetric TSP

- Instead of K_n, we consider the complete directed graph K'_n on n nodes. Weights in the two directions may be different.
- This problem contains the usual TSP as a special case, and hence it is likewise NP-hard.

SOME GENERALIZATIONS (3)

- We may also consider an arbitrary connected graph G with some length function w instead of K_n.
- In this case, it is neither clear whether any tours exist: we need to check first whether G is a Hamiltonian graph (i.e., if it contains a HC). This feasibility question is already an NP-complete problem in itself.

A CURIOSITY

- The TSP, in particular the Euclidean variant of the problem, has attracted the attention of researchers in cognitive psychology:

  It has been observed that humans are able to produce good quality solutions quickly [Macgregor, Ormerod '96].

- These results suggest that computer performance on the TSP may be improved by understanding and emulating the methods used by humans for these problems and have also led to new insights into the mechanisms of human thought.

ANOTHER APPLICATION (1)
FROM CRYSTALLOGRAPHY

Analysis of the structure of crystals

An X-ray diffractometer is used to obtain information about the structure of crystalline material: a detector measures the intensity of X-ray reflections of the crystal from various positions.

…

ANOTHER APPLICATION (2)
FROM CRYSTALLOGRAPHY

The measurement can be accomplished quite fast, but there is a considerable overhead in positioning time since up to hundreds of thousands positions have to be realized for some experiments.

The time needed to move from one position to the other can be computed very accurately and the result of the experiment does not depend on the sequence of the measurements.

…

ANOTHER APPLICATION (3)
FROM CRYSTALLOGRAPHY

… However, the total time needed for the experiment depends on the sequence. Therefore, the problem consists of finding a sequence that minimizes the total positioning time.

This leads to the TSP.

ANOTHER APPLICATION (1)
FROM BIOLOGY

DNA sequencing problem

Given an unknown DNA fragment, it can be deduced via the sequencing by hybridization method.

It consists of two phases, the biological phase and the computational phase.

ANOTHER APPLICATION (2)
FROM BIOLOGY

Biological phase:
a library of oligonucleotides (oligos for short), i.e. short sequences of nucleotides of a given length, say l, is built and placed on a DNA chip.
Such a chip contains all possible 4^l oligonucleotides, each one of them characterized by a unique set of coordinates on the chip.
…

ANOTHER APPLICATION (3)
FROM BIOLOGY

Biological phase (contd):
… the double-stranded DNA fragment (in fact many clones of it) is separated into two single-stranded DNA molecules (denaturing process) by heating the double-stranded molecule.
Once the single-stranded DNA fragment is cooled again, it reacts, or hybridizes to complementary fragments.
…

ANOTHER APPLICATION (4)
FROM BIOLOGY

Biological phase (contd):
… Since each oligo on the chip has unique coordinates, the fluorescent image on the chip leads to the identification of which oligos appear in the unknown DNA fragment.
The set of oligos from the library that hybridize with the DNA fragment makes up the spectrum of the fragment object of the study and are given as input to the computational phase.

ANOTHER APPLICATION (5)
FROM BIOLOGY

Computational phase:
It is unknown where and how many times each oligo appears in the sequence. The unknown DNA sequence is, therefore, reconstructed by finding the best permutation of oligos from the spectrum.
If the hybridization experiment is ideal, i.e., with no experimental errors, the original DNA sequence can be reconstructed in polynomial time [Pevznev'89].

ANOTHER APPLICATION (6)
FROM BIOLOGY

Computational phase (contd):
Let n be the length of the original sequence (n can be identified with gel electrophoresis); the spectrum is ideal when it contains uniquely all the $n-l+1$ different fragments (of length l) of the sequence.
The assembled sequence is the projection of the ordered oligos where each overlaps the previous one by l-1 oligos.
…

ANOTHER APPLICATION (7)
FROM BIOLOGY

Computational phase (contd):
… However, when the experiment incorporates errors, then the computational problem of reconstructing the sequence becomes NP-hard [Blazewicz & Kasprzak '03].
Realistic hybridization experiments produce spectra with errors, due to false hybridization and false reading from the DNA chip.
…

ANOTHER APPLICATION (8)
FROM BIOLOGY

Computational phase (contd):

… Two classes of error can be identified:

- positive errors, i.e., oligos not present in the original sequence are included in the spectrum,
- negative errors, i.e., oligos that are present in the original sequence are not included in the spectrum.

…

ANOTHER APPLICATION (9)
FROM BIOLOGY

Computational phase (contd):

…

NOTE: Since an oligo is included in the spectrum only once even if multiple fragments of the sequence hybridize with it, multiple repetitions of oligos are excluded and, therefore, treated as negative errors.

ANOTHER APPLICATION (10)
FROM BIOLOGY

Computational phase (contd):

…

the DNA sequence is constructed by determining the best possible permutation of oligos, taking into account the maximum overlaps between consecutive nucleotides.

The maximum overlap between two oligos is obviously less than or equal to l.

ANOTHER APPLICATION (11)
FROM BIOLOGY

Computational phase (contd):

… this problem is modeled as an asymmetric TSP [Nikolakopoulos & Sarimveis'08], where the nodes of the complete graph are the m oligos, and the distance between u and v may be different from the one between v and u.

Any permutation of the original sequence among the $m!$ can be considered as a candidate solution, and the optimal solution minimizes the sum of the distances of sequential nodes.

ANOTHER APPLICATION (12)
FROM BIOLOGY

Computational phase (contd):

… Given two oligos, s_i and s_j, let $ov(s_i, s_j)$ be the cardinality of their overlap.

The distance between s_i and s_j in the graph is defined as $2l - ov(s_i, s_j)$.

NOTE. it is clear that the distance between any two oligonucleotides is at least $l+1$.