AGAIN ON MINIMUM ENERGY BROADCAST

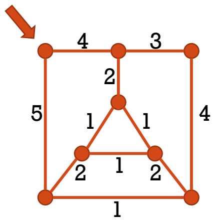
HEURISTICS (1)

In [Wieselthier, Nguyen, Ephremides, 00]: three heuristics all based on the greedy technique:

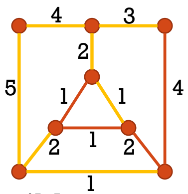
- SPT (spanning path tree): it runs Dijkstra algorithm to get the minimum path tree, then it directs the edges of the tree from the root to the leaves.
- BAIP (Broadcast Average Incremental Power): it is a modification of the Dijkstra algorithm based on the nodes (i.e. a new node is added to the tree on the basis of its minimum average cost).
- MST (min spanning tree): it runs Prim algorithm to get a MST, then it directs the edges of the tree from the root to the leaves.

A PARENTHESIS

SPT (spanning path tree) and MST (min spanning tree) can be different:

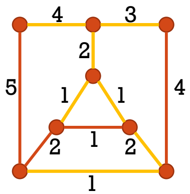


Dijkstra algorithm (SPT)



Possibly many trees, not all of the same weight

Prim algorithm (MST)



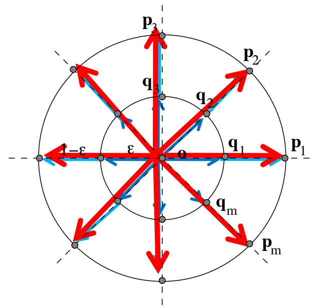
Possibly many trees, all of the same (minimum) weight

HEURISTICS (2)

GREEDY IS NOT ALWAYS GOOD

Greedy is not always good [Wan, Calinescu, Li, Frieder '02]:

- SPT: it runs Dijkstra algorithm to get the minimum path tree, then it directs the edges of the tree from the root to the leaves



(let $\alpha=2$)

- SPT outputs a tree with total energy: $\varepsilon^2+n/2(1-\varepsilon)^2$
- If the root transmits with radius 1 the energy is 1
- When $\varepsilon \to 0$ SPT is far $n/2$ from the optimal solution.

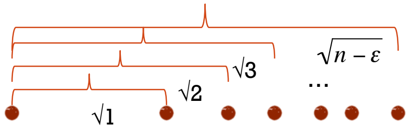
HEURISTICS (3)

GREEDY IS NOT ALWAYS GOOD

- BAIP (Broadcast Average Incremental Power): it is a modification of the Dijkstra algorithm based on the nodes: a new node is added to the tree on the basis of the min average cost=energy increasing/# of added nodes.
- It has been designed to solve the problems of SPT.

HEURISTICS (4)

GREEDY IS NOT ALWAYS GOOD

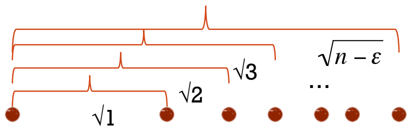


(let α=2):

- The min transmission power of the source to reach k receiving nodes is $\sqrt{k^2}=k$ and thus the average power efficiency is $k/k=1$
- On the other hand, the min transmission power of the source to reach all receiving nodes is $(\sqrt{n-\varepsilon})^2=n-\varepsilon$ and thus the average power efficiency is $(n-\varepsilon)/n=1-\varepsilon/n$...

HEURISTICS (5)

GREEDY IS NOT ALWAYS GOOD



- BAIP will let the source to transmit at power $\sqrt{n-\varepsilon}$ to reach all nodes in a single step.
- However, the opt. routing is a path consisting of all nodes from left to right. Its min power is:

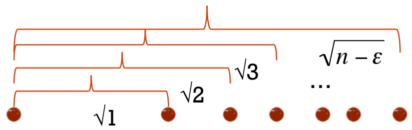
$$\sum_{i=1}^{n-1}(\sqrt{i}-\sqrt{i-1})^2+(\sqrt{n-\varepsilon}-\sqrt{n-1})^2<\sum_{i=1}^{n}(\sqrt{i}-\sqrt{i-1})^2=$$

$$\sum_{i=1}^{n}(\sqrt{i}-\sqrt{i-1})^2\frac{(\sqrt{i}+\sqrt{i-1})^2}{(\sqrt{i}+\sqrt{i-1})^2}=\sum_{i=1}^{n}\frac{((\sqrt{i}-\sqrt{i-1})(\sqrt{i}+\sqrt{i-1}))^2}{(\sqrt{i}+\sqrt{i-1})^2}=$$

$$=\sum_{i=1}^{n}\frac{(i-(i-1))^2}{(\sqrt{i}+\sqrt{i-1})^2}=\sum_{i=1}^{n}\frac{1}{(\sqrt{i}+\sqrt{i-1})^2}=1+\sum_{i=2}^{n}\frac{1}{(\sqrt{i}+\sqrt{i-1})^2}\le$$

HEURISTICS (6)

GREEDY IS NOT ALWAYS GOOD



(computation of the performance ratio of BAIP – cntd)

$$\le 1+\sum_{i=2}^{n}\frac{1}{i+(i-1)+2\sqrt{i}\sqrt{i-1}}\le 1+\sum_{i=2}^{n}\frac{1}{2i-1+2(i-1)}\le$$

$$\le 1+\sum_{i=2}^{n}\frac{1}{2i-1+2(i-1)}=1+\sum_{i=2}^{n}\frac{1}{4i-3}\le 1+\sum_{i=2}^{n}\frac{1}{4(i-1)}\le$$

Substituting $i=j+1$:

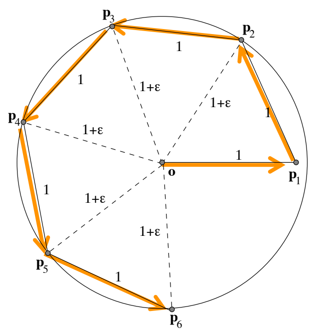
$$\le 1+\sum_{j=1}^{n-1}\frac{1}{4j}\le 1+\frac{1}{4}\sum_{j=1}^{n-1}\frac{1}{j}\le 1+\frac{1}{4}(\ln(n-1)+1)=\frac{\ln(n-1)+5}{4}$$

Thus the approx ratio of BAIP is at least:

$$\frac{n-\varepsilon}{\frac{\ln(n-1)+5}{4}}\to(\varepsilon\to 0)\frac{4n}{\ln(n-1)+5}=\frac{4n}{\ln n}+o(1)$$

HEURISTICS (7)
GREEDY IS NOT ALWAYS GOOD

MST: it runs Prim algorithm to get a MST, then it directs the edges of the tree from the root to the leaves



- Path $op_1 \ldots p_6$ is the unique MST, and its total energy is 6.
- On the other hand, the opt. routing is the star centered at o, whose energy is $(1+\varepsilon)^{\alpha}$.
- The approx. ratio converges to 6, as ε goes to 0.

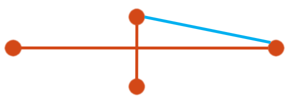
HEURISTICS (8)

- We have just shown a lower bound on the approximation ratio of MST.
- This ratio is a constant and an upper bound is 12.
- The proof involves complicated geometric arguments, and therefore we only sketch some of them:
  - …(not this year: directly go to page 56)

HEURISTICS (9)

- Any pair of edges do not cross each other

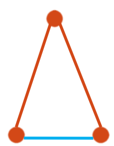
The blue edge is necessarily shorter than at least one of the two crossing edges



HEURISTICS (10)

(properties of the geometric MST – cntd)

- The angles between any two edges incident to a common node is at least π/3



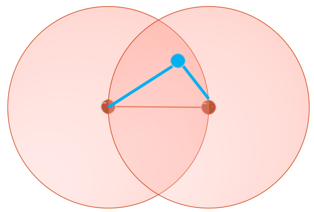
The blue edge is necessarily shorter than at least one of the two orange edges

HEURISTICS (11)

(properties of the geometric MST - cntd)

- The *lune* determined by each edge does not contain any other nodes.

The lune through points p_1 and p_2 is the intersection of the two open disks of radius $dist(p_1,p_2)$ centered at p_1 and p_2, respectively, hence an internal node would create a cycle

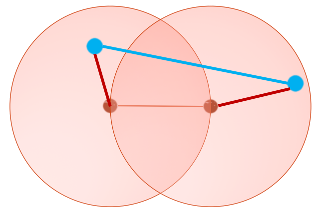


HEURISTICS (12)

(properties of the geometric MST – cntd)

- Let p_1p_2 be any edge. Then the two endpoints of any other edge are either both outside the open disk $D(p_1, dist(p_1, p_2))$ or both outside the open disk $D(p_2, dist(p_1, p_2))$

The red edges are added before than the blue edge because they are shorter. The blue edge would create a cycle.



HEURISTICS (13)

- **Obs.** The proof in [Wan, Calinescu, Li, Frieder '02] contains a small flaw that can be solved, arriving to an approximation ratio of 12,15 [Klasing, Navarra, Papadopoulos, Perennes '04]
- Indipendently, an approximation ratio of 20 has been stated in [Clementi, Crescenzi, Penna, Rossi, Vocca '01]
- Approx. ratio improved to 7,6 [Flammini, Klasing, Navarra, Perennes '04]
- Approx. ratio improved to 6,33 [Navarra '05]
- Optimal bound 6 [Ambüehl '05]

HEURISTICS (14)

- For realistic instances, experiments suggest that the tight approximation ratio is not 6 but 4 [Flammini, Navarra, Perennes '06] -> possible lesson