

## UNMANNED AERIAL VEHICLES – UAVS (1)



- UAVs are flying vehicles able to autonomously decide their route (different from **drones**, that are remotely piloted)
- Historically, used in the military, mainly deployed in hostile territory to reduce pilot losses



## UNMANNED AERIAL VEHICLES – UAVS (2)

- Now, new applications in civilian and commercial domains:
  - weather monitoring,
  - forest fire detection,
  - traffic control,
  - emergency search and rescue



## MONITORING BY UAVS (1)



- Let be given an AoI whose map is known
- we have a fleet of  $m$  UAVs leaving from a safe location ( $v_0$ ) each with a battery endurance  $B$
- in the AoI there is a set  $S=\{v_1, \dots, v_n\}$  of sites that must be examined (*e.g.* crumbled buildings after a hearthquake)
- each site  $v_i$  needs a time  $t_i$  to be inspected

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## MONITORING BY UAVS (2)

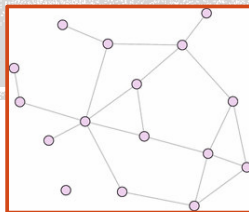


- each UAV must periodically go back to  $v_0$  in order to recharge its battery; this takes time  $R$ , typically 2.5-5 times  $B$
- we want to overfly  $v_1, \dots, v_n$  "as soon as possible" in order to collect data and possibly save people

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## THE GRAPH MODEL

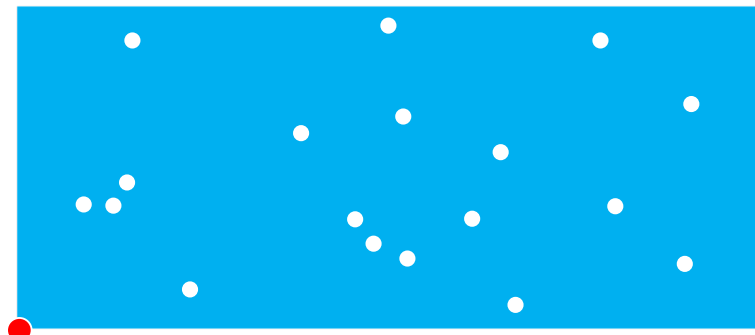


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## THE GRAPH MODEL (1)

It is natural to model this problem as a graph problem:

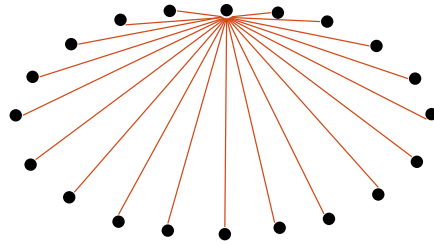
sites  $v_1, \dots, v_n$  + the depot  $v_0$  are the  $n+1$  nodes of  $G$



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## THE GRAPH MODEL (2)

It must be possible to go from each node to every other node, so



there is an edge between each pair of nodes  $\Rightarrow K_{n+1}$



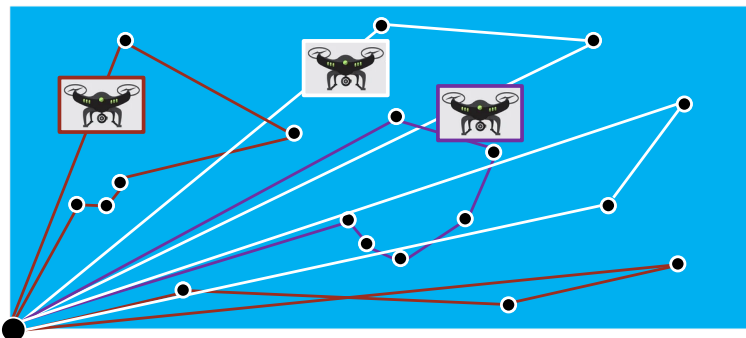
## THE GRAPH MODEL (3)

- Each UAV has a flying+inspection time bounded by  $B$ .
- for each pair of sites  $(v_i, v_j)$  we assume their distance (stored as an edge weight function  $w(u_i, u_j)$ ) as the time a UAV needs to go from  $u_i$  to  $u_j$ .



## THE GRAPH MODEL (4)

- each UAV is characterized by a different color
  - each UAV flies along a cycle (colored with the UAV color) and visits as many sites as it can (w.r.t. its battery constraint  $B$ ), it goes back to the depot to recharge its battery (with time  $R$ ) and it leaves again...
- All sites need to be visited in the "shortest time".



## THE GRAPH MODEL (5)

What does it mean "shortest time"?

Different possibilities for the optimization function:

- Minimize the Total completion Time
- Minimize the Average Waiting Time
- Minimize the number of cycles
- ...
- Note: Minimize the Overall Energy Consumption or the Total Traversed Distance has no meaning...

## THE GRAPH MODEL (6)

Similarities with many problems:

**mTSP** multiple Traveling Salesperson

- $n$  cities, one depot, a cost metric
- $m$  salespersons must overall cover the cities
- objective: determine one tour for each salesperson minimizing the total length
- no visiting times nor battery constraint



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## THE GRAPH MODEL (8)

Similarities with many problems (cntd):

**mTRPD** multiple Traveling Repairperson Problem with Distance Constraints

- $n$  customers, one depot
- $m$  repairpersons to visit all the  $n$  customers
- not allowed to travel a distance longer than a predetermined limit;
- Objective: determining  $m$  cycles... minimizing the total waiting time
- No repairtimes and it is not trivial to extend a solution by just adding them
- number of cycles fixed to  $k$



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## THE GRAPH MODEL (7)

Similarities with many problems (cntd):

**kTRPR**  $k$ -Traveling Repairperson Problem with Repairtimes

- $n$  customers, each with a repairtime, one depot
- $k$  repairpersons to visit all the  $n$  customers



Calling the *latency* of a site the time elapsed before that site is visited by a repairperson:

- objective: determine  $k$  cycles... minimizing the sum of all latencies
- no battery constraint



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## THE GRAPH MODEL (9)

Similarities with many problems (cntd):

variants of **VRP** vehicle routing problem

- VRP is a generic name given to a whole class of problems concerning the optimal design of routes to be used by a fleet of vehicles to serve a set of customers
- There is usually a constraint on the number of visited customers per vehicle



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## THE GRAPH MODEL (10)

Similarities with many problems (cntd):

**TOP** team orienteering problem

- $n$  sites each with a profit, one depot,  $m$  vehicles, with a limited total duration for their routes
- Objective: maximize the total profit
- equivalent to the first round of our problem 😊
- Objective: maximize the no. of covered sites (if the profits are all the same) 😞
- Repeat many times until all sites have been covered does not seem a good idea... 😞

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## THE GRAPH MODEL (11)

Similarities with many problems (cntd):

- **NOTE:** From all these similarities we deduce that the problem is NP-hard and we cannot exploit any known result, among the described ones...

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## CONNECTION WITH RMCCP (1)

[C. & Tavernelli'19]

- $V$  set of locations
- A **cycle cover**  $\mathcal{C} = \{C_1, \dots, C_k\}$  for  $V$  is a set of cycles s.t. each location of  $V$  belongs to at least one cycle.
- Given a fixed value  $x \geq 0$ , an  **$x$ -bounded cycle cover** is a cycle cover in which each cycle  $C$  has  $cost(C) \leq x$ .
- A **rooted cycle**  $C$  is a cycle where  $v_0 \in C$ .
- A **rooted cycle cover** is a cycle cover whose cycles are rooted cycles.
- The **completion time of a (rooted) cycle cover**  $\mathcal{C}$  is  $ct(\mathcal{C}) = \max cost(C)$  on all  $C \in \mathcal{C}$ .

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## CONNECTION WITH RMCCP (2)

- **RMCCP** Minimum Bounded Rooted Cycle Cover Problem requires to find a bounded rooted cycle cover of minimum cardinality.
- **Definition.** RMCCP:  
**Input:**  $\langle G, v_0, d, x \rangle$  where  $G = (V, E)$  is a graph,  $v_0 \in V$  is called root,  $d$  is a distance defined on  $E$  and  $x$  is a positive number  
**Output:** an  $x$ -bounded rooted cycle cover of minimum cardinality, if it exists.

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## CONNECTION WITH RMCCP (3)

- RMCCP has been proved to be approximable first within  $O(\log x)$  [Nagarajan & Ravi '12] and then within  $O(\frac{\log x}{\log \log x})$  [Friggstad & Swamy '14].
- RMCCP and our problem are tightly connected:

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## CONNECTION WITH RMCCP (5)

- In other words, our problem inherits the hardness of RMCCP. Notice that whether RMCCP admits a constant approximation algorithm or not is one of the major open problems in this area.
- **Note.** although RMCCP and our problem are so tightly connected, the first one minimizes the number of cycles, while the second one the completion time.

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## CONNECTION WITH RMCCP (4)

- **Thm.** If RMCCP can be approximated within  $\alpha$ , our problem can be approximated within  $5\alpha + 1$ .  
On the other hand:
- **Thm.** If our problem can be approximated within  $\gamma$ , then RMCCP can be approximated within  $2\gamma + 1$ .

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## A NEW GRAPH MODEL (1)

[C., Corò, Mancini '23]

Since we cannot exploit similar problems, we have to study it as a new problem:

We define a new problem: **MDMT-VRP-TCT** Multi-Depot Multi-Trip Vehicle Routing Problem with Total Completion Times minimization

It perfectly fits the application: 😊

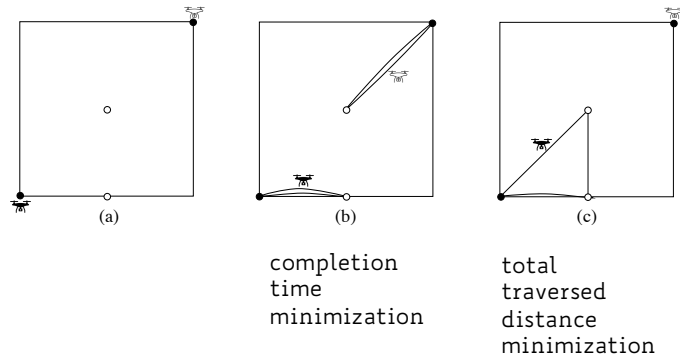
- many depots for many independent rescue teams
- every UAV can perform many trips starting from one among many depots
- Objective: minimize the total completion time

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## A NEW GRAPH MODEL (2)

Note 1. In the multi-depot context, minimizing completion time and the total traversed distance is not the same:



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## A NEW GRAPH MODEL (4)

MDMT-VRP-TCT can be formulated as a MILP.

**Def.** *sequence* = ordered set  $k$  of target nodes.

*duration  $d_k$  of sequence  $k$*  = sum of all traveling times between consecutive target nodes in  $k$  plus the service times of all the target nodes in  $k$ .

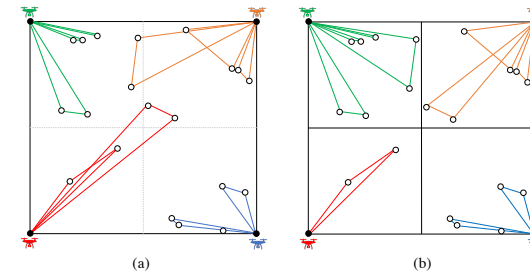
**Def.** *trip* = A sequence  $k$  assigned to a UAV  $u$  with the addition of its depot  $o_u$

*duration  $d_{ku}$  of a trip  $k+o_u$*  = duration of  $k$  plus the traveling distance between  $o_u$  and the first node of  $k$  plus the traveling distance between the last node of  $k$  and  $l_k$

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## A NEW GRAPH MODEL (3)

Note 2. In the multi-depot context, the solution is not correct if we partition the area into as many portions as the number of depots, so that the target nodes falling in a certain portion are automatically assigned



optimal solutions: Completion Time: (a) 41.28; (b) 65.41  
(a) more balanced than (b)

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## A NEW GRAPH MODEL (5)

**Def.** A sequence  $k$  is *compatible* with a UAV  $u$  if the duration of the associated trip is upper bounded by  $B$ .

The main idea consists in generating all possible sequences that are compatible with at least one UAV and choose the best solution...

too large number...

**Matheuristic:** we generate only a subset of the feasible cycles to be passed to the model.

Note: the problem of determining which sequences to generate assumes a crucial importance...

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## VARIANTS OF THE PROBLEM

- introducing priorities (hospitals, schools should be served first) [C. Corò, Mancini '22]
- introducing a double budget (battery + memory)  
[Betti Sorbelli, Navarra, Palazzetti, Pinotti, Prencipe '22]

## ONGOING PROBLEMS

- introducing cooperation
- introducing 'relaxed multi-depot' (that is, a UAV may leave from one depot and come back to another one: battery recharge constraints...)
- introducing some "emergency criteria" able to dynamically change the UAVs' behaviour (what if an injured person is detected? Shall we wait until the UAV is back?)