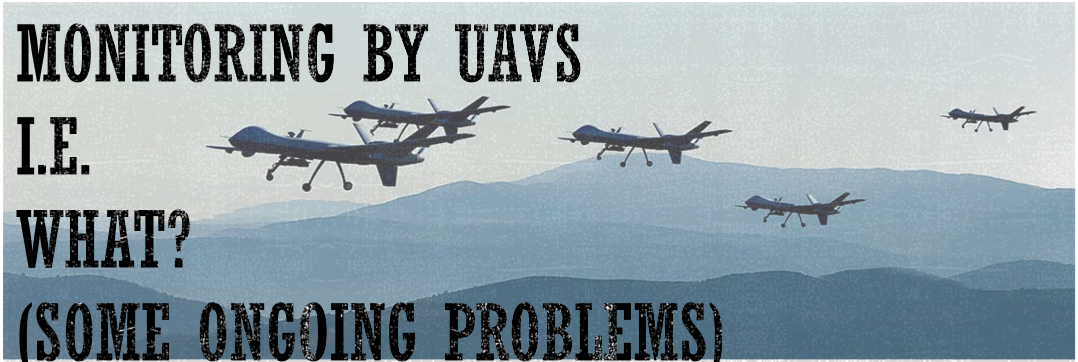
MONITORING BY UAVS
I.E.
WHAT?
(SOME ONGOING PROBLEMS)



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Network Algorithms

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UNMANNED AERIAL VEHICLES – UAVS (1)

- UAVs are flying vehicles able to autonomously decide their route (different from **drones**, that are remotely piloted)
- Historically, used in the military, mainly deployed in hostile territory to reduce pilot losses

UNMANNED AERIAL VEHICLES – UAVS (2)

- Now, new applications in civilian and commercial domains:
  - weather monitoring,
  - forest fire detection,
  - traffic control,
  - emergency search and rescue

THE PROBLEM

MONITORING BY UAVS (1)

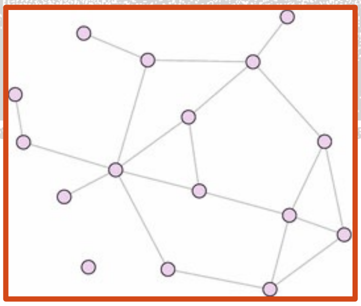
- Let be given an AoI whose map is known
- we have a fleet of m UAVs leaving from a safe location (v_0) each with a battery endurance B
- in the AoI there is a set $S=\{v_1, \ldots, v_n\}$ of sites that must be examined (*e.g.* crumbled buildings after a hearthquacke)
- each site v_i needs a time t_i to be inspected

MONITORING BY UAVS (2)

- each UAV must periodically go back to v_o in order to recharge its battery; this takes time R, typically 2.5-5 times B
- we want to overfly $v_1, \ldots, v_n$ "as soon as possible" in order to collect data and possibly save people

??

THE GRAPH MODEL



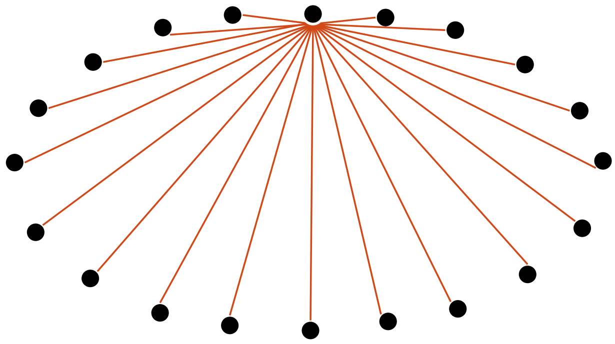
THE GRAPH MODEL (1)

It is natural to model this problem as a graph problem:

sites $v_1, \ldots, v_n$ + the depot v_0 are the $n+1$ nodes of G

THE GRAPH MODEL (2)

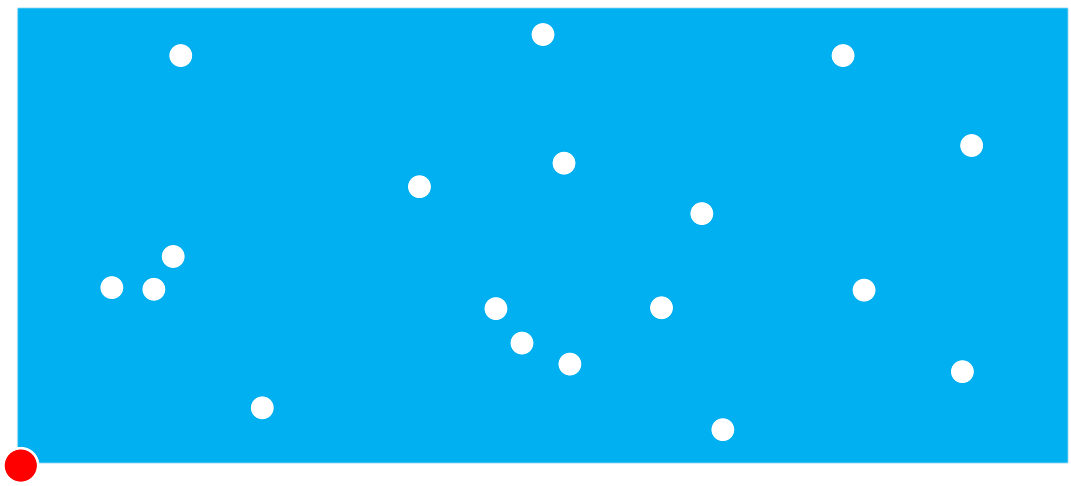
It must be possible to go from each node to every other node, so



there is an edge between each pair of nodes ⇒ K_{n+1}

THE GRAPH MODEL (3)

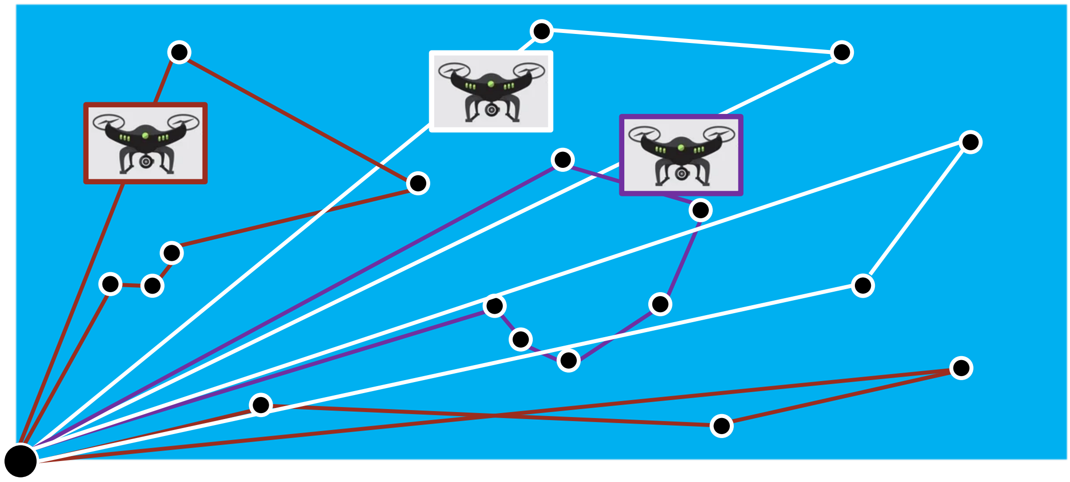
- Each UAV has a flying+inspection time bounded by B.
- for each pair of sites (v_i, v_j) we assume their distance (stored as an edge weight function $w(u_i, u_j)$) as the time a UAV needs to go from u_i to u_j.



THE GRAPH MODEL (4)

- each UAV is characterized by a different color
- each UAV flies along a cycle (colored with the UAV color) and visits as many sites as it can (w.r.t. its battery constraint *B*), it goes back to the depot to recharge its battery (with time *R*) and it leaves again...

All sites need to be visited in the "shortest time".



THE GRAPH MODEL (5)

What does it mean "shortest time"?

Different possibilities for the optimization function:

- Minimize the Total completion Time
- Minimize the Average Waiting Time
- Minimize the number of cycles
- ...
- Note: Minimize the Overall Energy Consumption or the Total Traversed Distance has no meaning...

THE GRAPH MODEL (6)

Similarities with many problems:

mTSP multiple Traveling Salesperson

- n cities, one depot, a cost metric
- m salespersons must overall cover the cities
- objective: determine one tour for each salesperson minimizing the total length
  no visiting times nor battery constraint

THE GRAPH MODEL (7)

Similarities with many problems (cntd):

kTRPR k-Traveling Repairperson Problem with Repairtimes

- n customers, each with a repairtime, one depot
- k repairpersons to visit all the n customers

Calling the *latency* of a site the the time elapsed before that site is visited by a repairperson:

- objective: determine k cycles…
  minimizing the sum of all latencies
- no battery constraint

THE GRAPH MODEL (8)

Similarities with many problems (cntd):

mTRPD multiple Traveling Repairperson Problem with Distance Constraints

- n customers, one depot
- m repairpersons to visit all the n customers
- not allowed to travel a distance longer than a predetermined limit;
- Objective: determining m cycles...

  minimizing the total waiting time
- No repairtimes and it is not trivial to extend a solution by just adding them
- number of cycles fixed to k

THE GRAPH MODEL (9)

Similarities with many problems (cntd):

variants of **VRP** vehicle routing problem

- VRP is a generic name given to a whole class of problems concerning the optimal design of routes to be used by a fleet of vehicles to serve a set of customers
- There is usually a constraint on the number of visited customers per vehicle

THE GRAPH MODEL (10)

Similarities with many problems (cntd):

TOP team orienteering problem

- n sites each with a profit, one depot, m vehicles, with a limited total duration for their routes
- Objective: maximize the total profit
- equivalent to the first round of our problem 
- Objective: maximize the no. of covered sites (if the profits are all the same) 
- Repeat many times until all sites have been covered does not seem a good idea... 

THE GRAPH MODEL (11)

Similarities with many problems (cntd):

- NOTE: From all these similarities we deduce that the problem is NP-hard and we cannot exploit any known result, among the described ones...

CONNECTION WITH RMCCP (1)

[C. & Tavernelli'19]

- V set of locations
- A cycle cover $\mathscr{C} = \{C_1,\dots,C_k\}$ for V is a set of cycles s.t. each location of V belongs to at least one cycle.
- Given a fixed value $x \geq 0$, an x-bounded cycle cover is a cycle cover in which each cycle C has $cost(C) \leq x$.
- A rooted cycle C is a cycle where $v_0 \in C$.
- A rooted cycle cover is a cycle cover whose cycles are rooted cycles.
- The completion time of a (rooted) cycle cover $\mathscr{C}$ is

$$ct(\mathscr{C}) = max\ cost(C) \text{ on all } C \in \mathscr{C}.$$

CONNECTION WITH RMCCP (2)

- **RMCCP** Minimum Bounded Rooted Cycle Cover Problem requires to find a bounded rooted cycle cover of minimum cardinality.
- Definition. RMCCP:
  Input: $\langle G, v_0, d, x \rangle$ where $G = (V,E)$ is a graph, $v_0 \in V$ is called root, d is a distance defined on E and x is a positive number
  Output: an x-bounded rooted cycle cover of minimum cardinality, if it exists.

CONNECTION WITH RMCCP (3)

- RMCCP has been proved to be approximable first within *O(log x)* [Nagarajan & Ravi '12] and then within $O(\frac{\log x}{\log \log x})$ [Friggstad & Swamy '14].
- RMCCP and our problem are tightly connected:

CONNECTION WITH RMCCP (4)

- Thm. If RMCCP can be approximated within α, our problem can be approximated within 5 α +1.

On the other hand:

- Thm. If our problem can be approximated within γ, then RMCCP can be approximated within 2 γ +1.

CONNECTION WITH RMCCP (5)

- In other words, our problem inherits the hardness of RMCCP. Notice that whether RMCCP admits a constant approximation algorithm or not is one of the major open problems in this area.
- Note. although RMCCP and our problem are so tightly connected, the first one minimizes the number of cycles, while the second one the completion time.

A NEW GRAPH MODEL (1)

[C., Corò, Mancini '23]

Since we cannot exploit similar problems, we have to study it as a new problem:

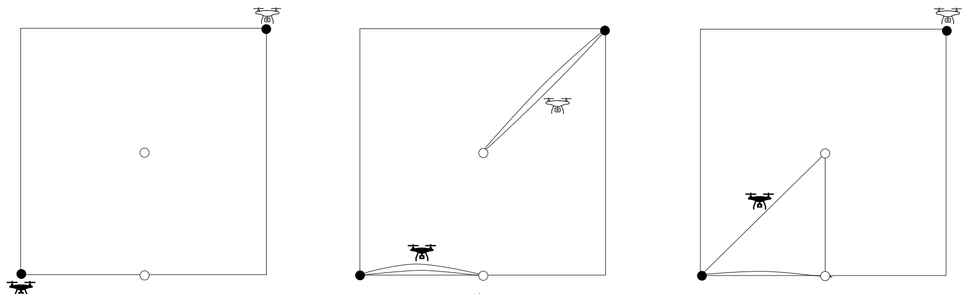
We define a new problem: **MDMT-VRP-TCT** Multi-Depot Multi-Trip Vehicle Routing Problem with Total Completion Times minimization

It perfectly fits the application:

- many depots for many independent rescue teams
- every UAV can perform many trips starting from one among many depots
- Objective: minimize the total completion time

A NEW GRAPH MODEL (2)

Note 1. In the multi-depot context, minimizing completion time and the total traversed distance is not the same:



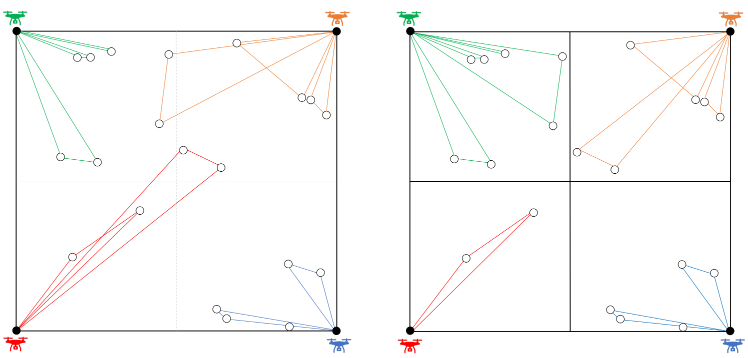
(a) (b) (c)

completion time minimization

total traversed distance minimization

A NEW GRAPH MODEL (3)

Note 2. In the multi-depot context, the solution is not correct if we partition the area into as many portions as the number of depots, so that the target nodes falling in a certain portion are automatically assigned



(a) (b)

optimal solutions: Completion Time: (a) 41.28; (b) 65.41
(a) more balanced than (b)

A NEW GRAPH MODEL (4)

MDMT-VRP-TCT can be formulated as a MILP.

Def. *sequence* = ordered set k of target nodes.

duration d_k *of sequence* k = sum of all traveling times between consecutive target nodes in k plus the service times of all the target nodes in k.

Def. *trip* = A sequence k assigned to a UAV u with the addition of its depot o_u

duration d_{ku} of a trip k+o_u = duration of k plus the traveling distance between o_u and the first node of k plus the traveling distance between the last node of k and l_k

A NEW GRAPH MODEL (5)

Def. A sequence k is *compatible* with a UAV u if the duration of the associated trip is upper bounded by B.

The main idea consists in generating all possible sequences that are compatible with at least one UAV and choose the best solution...

too large number...

Matheuristic: we generate only a subset of the feasible cycles to be passed to the model.

Note: the problem of determining which sequences to generate assumes a crucial importance...

VARIANTS OF THE PROBLEM

- introducing priorities (hospitals, schools should be served first) [C. Corò, Mancini '22]
- introducing a double budget (battery + memory)

  [Betti Sorbelli, Navarra, Palazzetti, Pinotti, Prencipe '22]

ONGOING PROBLEMS

- introducing cooperation
- introducing 'relaxed multi-depot' (that is, a UAV may leave from one depot and come back to another one: battery recharge constraints…)
- introducing some "emergency criteria" able to dynamically change the UAVs' behaviour (what if an injured person is detected? Shall we wait until the UAV is back?)