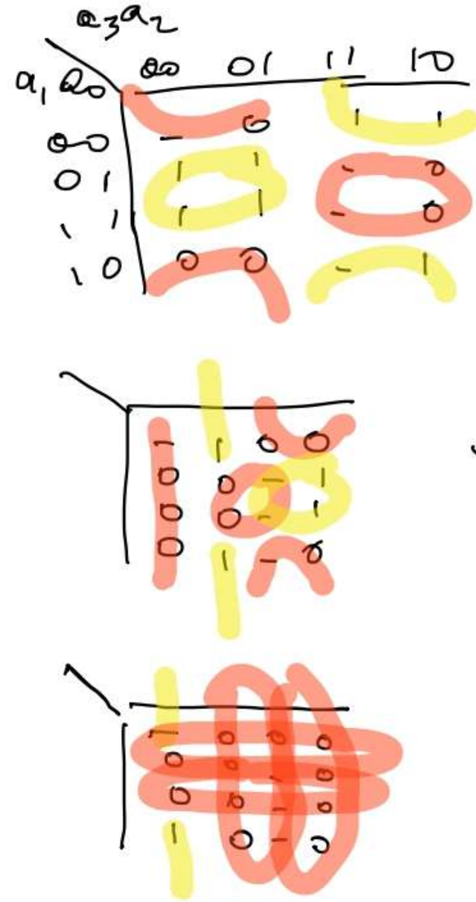
Exam of Computer Architectures – UNIT 1 - July 13th, 2021

Exercise 1 (5 points) Design a circuit that provides how many days there are in a month. The month is specified by a 4 bits input, $a_3a_2a_1a_0$. For example, with input 000 the month is January; if the input is 1100 the month is December. The circuit output Y_2 must be 1 only when the input month has 31 days; Y_1 is 1 when the month has 30 days; and Y_0 is 1 when the month has 28 days. Write the minimal SOP and POS formulae. Then, implement Y2 with a 4-to-1 multiplexer.

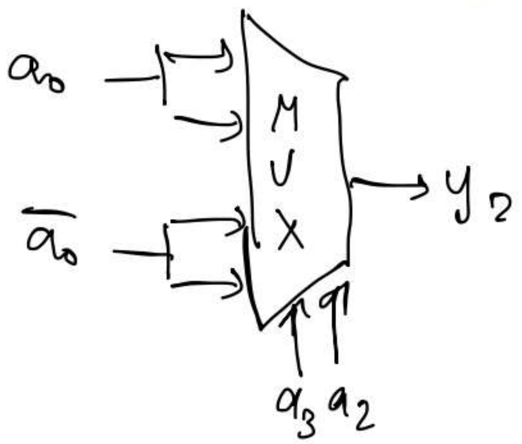
$a_3a_2a_1a_0$	Y_2	Y_1	Y_0
0000	-	-	-
0001	1	0	0
0010	0	0	1
0011	1	0	0
0100	0	1	0
0101	1	0	0
0110	0	1	0
0111	1	0	0
1000	1	0	0
1001	0	1	0
1010	1	0	0
1011	0	1	0
1100	1	0	0
1101	-	-	-
1110	-	-	-
1111	-	-	-



$$Y_2 = \bar{a}_3 a_0 + a_3 \bar{a}_0 = (a_3 + a_0)(\bar{a}_3 + \bar{a}_0)$$

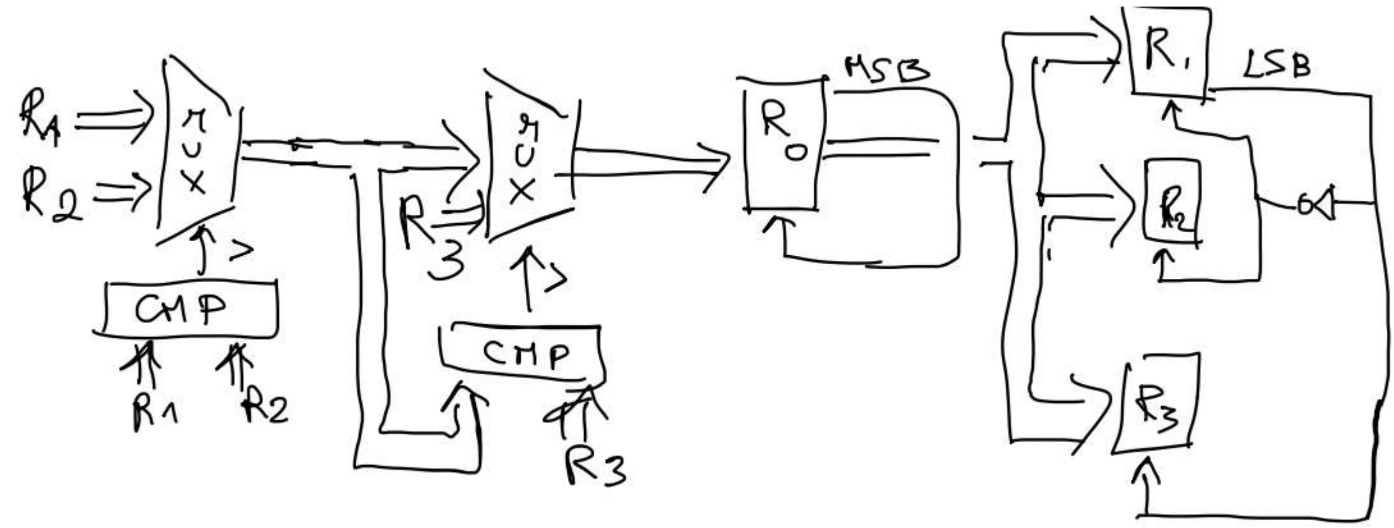
$$Y_1 = \bar{a}_3 a_2 \bar{a}_0 + a_3 a_0 = (a_3 + a_2)(\bar{a}_2 + \bar{a}_0)(\bar{a}_3 + a_0)$$

$$Y_0 = \bar{a}_3 \bar{a}_2 \bar{a}_0 = a_1 \cdot \bar{a}_0 \cdot \bar{a}_2 \cdot \bar{a}_3$$



Exercise 2 (5 points): Design an interconnection of 4 registers R0, .., R3 such that:

- R0 receives the minimum value among the remeining three registers; this transfer is enabled only if R0 is positive;
- R0 is moved into R1 and R2, if R1 is even, in R3, otherwise.



Exercise 3 (4 points)

a. Turn the decimal numbers X=111 and Y=78 in 2-complement with 8 bits and calculate Z=X-Y and W=X+Y. Then, turn the results in hesixadecimal.

b. Sum $3EAB_{16}$ and $2E73_{16}$, turn the result in base 4 and subtract 31321_4.

111 = 64 + 32 + 8 + 4 + 2 + 1 ⟶ 01101111

78 = 64 + 8 + 4 + 2 ⟶ 01001110 —Ca2→ 10110001 + 1

= 10110010

Z: 01101111 +
10110010 =
00100001
2 1

W: 01101111 +
01001110 =
10111101 ← overflow
B D

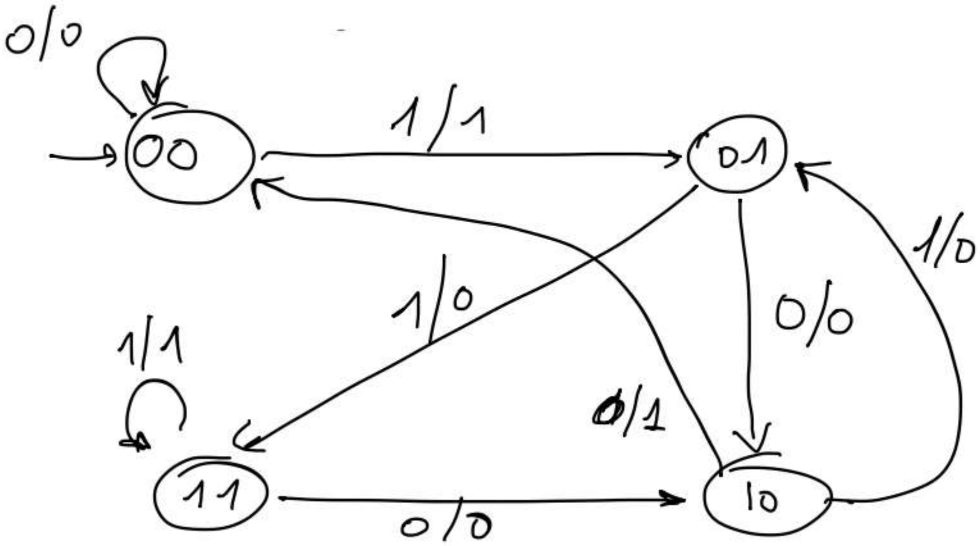
3EAB +
2E73 =
6D1E

6D1E
12310132 −
31321 =
12212211

001 100 111

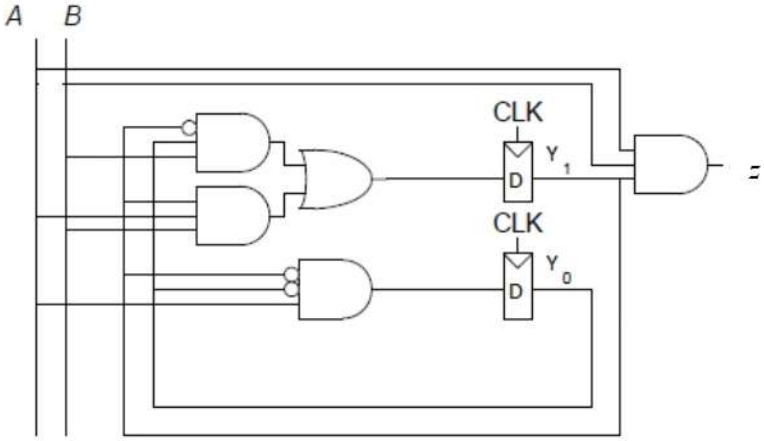
Exercise 4 (5 points): Design an automaton that receives in input x and produces in output z. The output is 1 if and only if the natural number given by the last 3 bits received so far has reminder 1 when divided by 3. You can accept overlappings. Ignore the first two outputs (that can be any value).

Example: INPUT: 1101100011110
OUTPUT: - - 00001010110



	0	1
00	00/0	01/1
01	10/0	11/0
10	00/1	01/0
11	10/0	11/1

Exercise 5 (4 points): Analyze the following sequential circuit and give the associated automaton.



$D_1 = \overline{y_1} y_0 b + y_1 ab$ $D_0 = \overline{y_1}\,\overline{y_0} a$ $z = aby_1$

$a\,b\,y_1\,y_0$	D_1 ($=Y_1$)	D_0 ($=Y_0$)	z
0000	0	0	0
0001	0	0	0
0010	0	0	0
0011	0	0	0
0100	0	0	0
0101	1	0	0
0110	0	0	0
0111	0	0	0
1000	0	1	0
1001	0	0	0
1010	0	0	0
1011	0	0	0
1100	0	1	0
1101	1	0	0
1110	1	0	1
1111	1	0	1

	00	01	10	11
00	00/0	00/0	01/0	01/0
01	00/0	10/0	00/0	10/0
10	00/0	00/0	00/0	10/1
11	00/0	00/0	00/0	10/1

Exercise 6 (3 points) Given the expression $f = \left(\bar a + \overline{b(b+\overline{cde})}\right) \oplus (\bar a + cd)$, simplify it and write it in canonical SOP form. Then, realize f in ALL-NAND form.

$$\left(\bar a + \overline{b(b+\overline{cde})}\right) \oplus (\bar a + cd) = (\bar a + \bar b + \bar b cde) \oplus (\bar a + cd)$$

$$= (\bar a + \bar b) \oplus (\bar a + cd) = \overline{ab} \oplus (\bar a + cd) = \overline{ab}\,\overline{(\bar a + cd)} + ab(\bar a + cd)$$

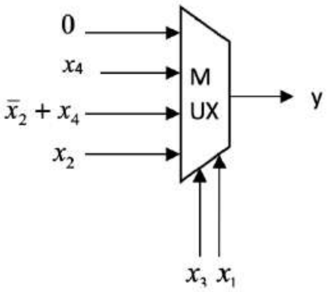
$$= (\bar a + \bar b)\, a(\bar c + \bar d) + abcd = a\bar b(\bar c + \bar d) + abcd = a\bar b\bar c + a\bar b\bar d + abcd$$

$$= a\bar b\bar c\bar d + a\bar b\bar c d + a\bar b\bar c\bar d + a\bar b c\bar d + abcd = a\bar b\bar c\bar d + a\bar b\bar c d + a\bar b c\bar d + abcd$$

NAND: $$a\bar b\bar c + a\bar b\bar d + abcd = \overline{\overline{a\bar b\bar c}\cdot\overline{a\bar b\bar d}\cdot\overline{abcd}}$$

$$= \overline{\overline{a\cdot\overline{b\cdot b}\cdot\overline{c\cdot c}}\cdot\overline{a\cdot\overline{b\cdot b}\cdot\overline{d\cdot d}}\cdot\overline{a\cdot b\cdot c\cdot d}}$$

Exercise 7 **(4 points)** Consider the following combinatorial circuit:



$$y = \bar x_3\bar x_1\cdot 0 + \bar x_3 x_1\cdot x_4 + x_3\bar x_1(\bar x_2 + x_4) + x_3 x_1 x_2$$

$$= x_4\bar x_3 x_1 + x_3\bar x_2\bar x_1 + x_4 x_3\bar x_1 + x_3 x_2 x_1$$

$$x_4(x_3 \oplus x_1) \qquad x_3(x_2 \otimes x_1)$$