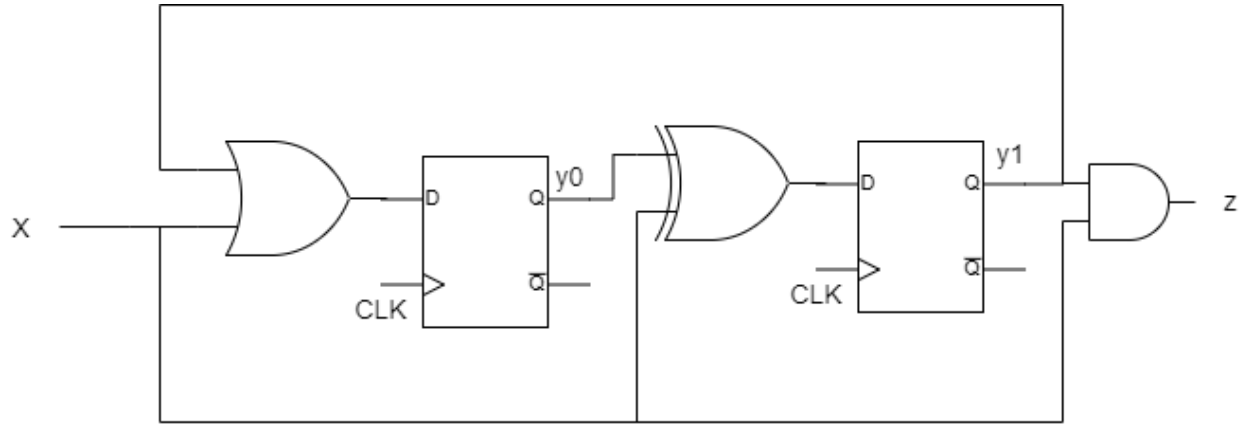
COMPUTER ARCHITECTURE UNIT 1 - 11/01/2022 (A)

Name: _____ **Surname:** _____

Exercise 1 (4+2 points): Analyze the circuit in figure (up to the minimal automaton). Then, provide the temporal diagram of the circuit with the input sequence 110010.



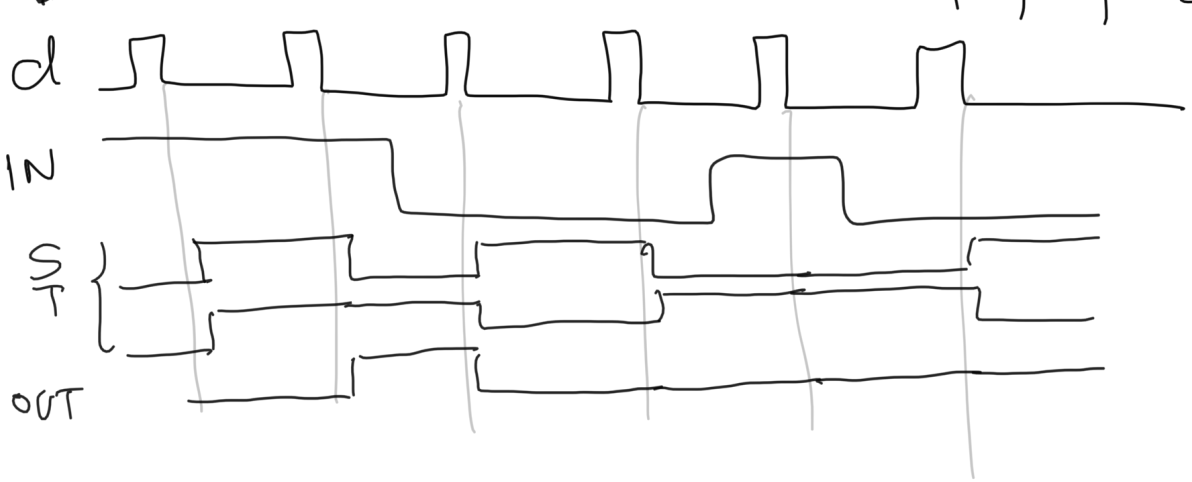
$D_0 = x + y_1 \quad D_1 = y_0 \oplus x \quad z = x y_1$

x	y_1	y_0	$D_1 = Y_1$	$D_0 = Y_0$	z
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	1	1	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	1

	0	1
S_0	S_0 / 0	S_3 / 0
S_1	S_2 / 0	S_1 / 0
S_2	S_1 / 0	S_3 / 1
S_3	S_3 / 0	S_1 / 1

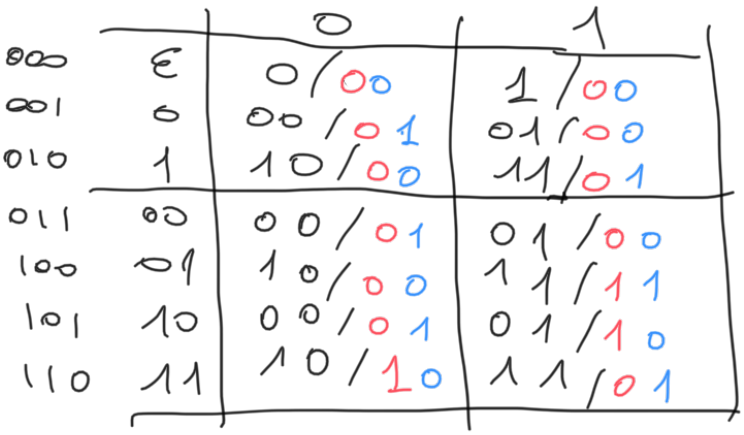
S_1	X		
S_2	X	X	
S_3	X	X	X
	S_0	S_1	S_2

Initial state is assumed to be S_0

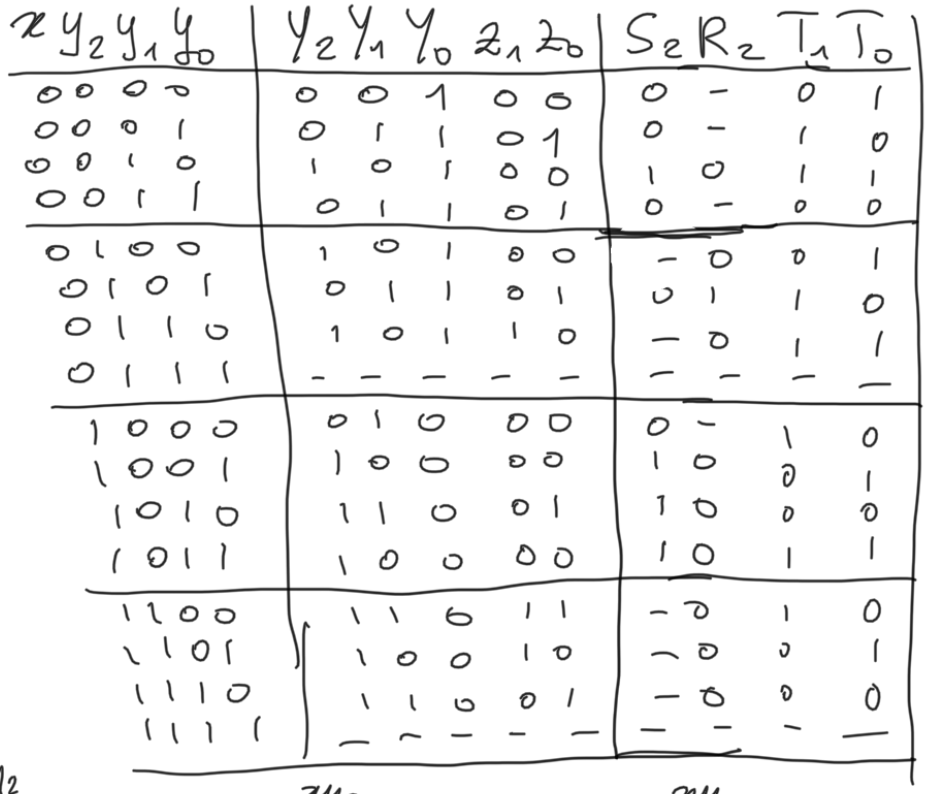


Exercise 2 (7 points): Design a sequential circuit with one input x and two outputs z1 and z0. Output z1 must be 1 if the last three input bits exactly contain two 1s, while z0 must be 1 if the last two bits are equal. Use an SR FF for the MSB and T FFs for the remaining bits.

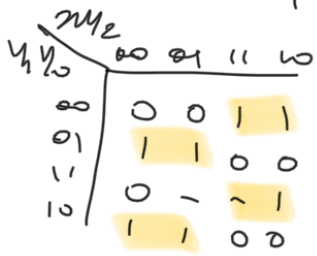
```
Example:  x    0101110010000011
          z1   0001101000000001
          z0   0000110100111101
```

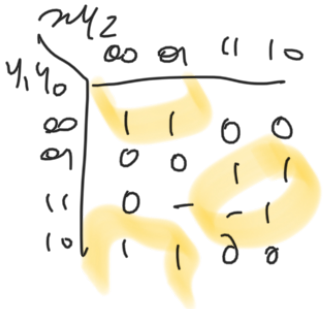


$$z_1 = x y_2 \bar{y}_1 + \bar{x} y_2 y_1$$
$$= y_2 (x \oplus y_1)$$



$$z_0 = \bar{x} y_0 + x y_2 \bar{y}_0 + x y_1 \bar{y}_0$$
$$= \bar{x} y_0 + x \bar{y}_0 (y_2 + y_1)$$





$$S_2 = y_1 \bar{y}_0 + x y_0 \qquad R_2 = \bar{x} y_0$$

$$T_1 = x \bar{y}_1 \bar{y}_0 + x y_1 y_0 + \bar{x} \bar{y}_1 y_0 + \bar{x} y_1 \bar{y}_0$$
$$= x (y_1 \otimes y_0) + \bar{x} (y_1 \oplus y_0)$$
$$= x \oplus (y_1 \oplus y_0)$$

$$T_0 = \bar{x} \bar{y}_0 + x y_0$$
$$= x \otimes y_0$$

Exercise 3 (3 points): By using Boolean algebra, prove that:

$$(\bar{a} \oplus b) + \overline{(ac + b)}(a + bc) = \bar{a} + b + c$$

$$= ab + \bar{a}\bar{b} + ac + b + \overline{a + bc}$$

$$= ab + \bar{a}\bar{b} + ac + b + \bar{a}(\bar{b} + \bar{c})$$

$$= ab + \bar{a}\bar{b} + ac + b + \bar{a}\bar{b} + \bar{a}\bar{c} = b + \bar{a}\bar{b} + ac + \bar{a}\bar{c}$$

$$= (b + \bar{a})(b + \bar{b}) + ac + \bar{a}\bar{c} = b + \bar{a} + ac + \bar{a}\bar{c} = b + \bar{a} + ac$$

$$= b + (\bar{a} + a)(\bar{a} + c)$$

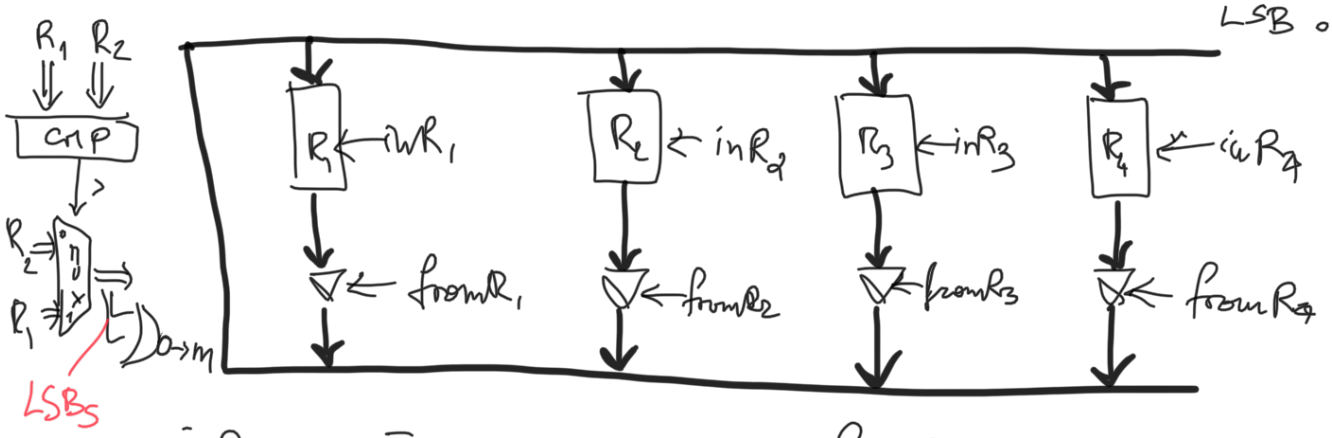
$$= b + \bar{a} + c$$

Exercise 4 (4 points): Interconnect 4 registers R1…R4 through a bus so that:

- If R1 is even, then the content of R2 goes into R3 and R4
- otherwise, the content of R3 goes into R1 and R2.

Movements are enabled only if the maximum between R1 and R2 is multiple of 4.

b is the LSB of R_1



$inR_1 = m\,\bar{b}$ $\quad$ $fromR_1 = 0$

$inR_2 = m\,\bar{b}$ $\quad$ $fromR_2 = b$

$inR_3 = m\,b$ $\quad$ $fromR_3 = \bar{b}$

$inR_4 = m\,b$ $\quad$ $fromR_4 = 0$

Exercise 5 (4 points): Given X = < 0 ; 10000 ; 0010000000 > and Y = < 1 ; 01111 ; 0100000000 > in IEEE half-precision format, calculate X+Y and represent the result both in IEEE half-precision and in decimal notation.

Y' = < 1 ; 00000 ; 1010000000 >

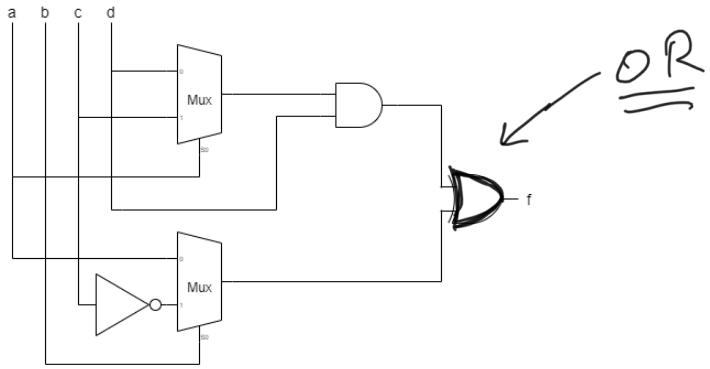
1,0010000000 −
0,1010000000 =
0,1000000000

< 0 ; 01111 ; 0000000000 >

Result in base 10 is 1

Exercise 6 (2+1+1+2 points)

- Consider the circuit in figure and write down the BE of function f
- Write the truth table for f
- Write the minimal POS expression for f
- Write f in ALLNOR form.



$$f = \left[(\bar{a}d + ac)d\right] + (\bar{b}a + b\bar{c})$$

$$= \bar{a}d + acd + a\bar{b} + b\bar{c}$$

ab	cd	f
00	00	0
00	01	1
00	10	0
00	11	1
01	00	1
01	01	1
01	10	0
01	11	1
10	00	1
10	01	1
10	10	1
10	11	1
11	00	1
11	01	1
11	10	0
11	11	1

cd \ ab	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	0	0	0	1

$$\text{minPOS} = (a+b+d)(\bar{b}+\bar{c}+d)$$

$$= \overline{\overline{a+b+d} + \overline{\bar{b}+\bar{c}+d}}$$

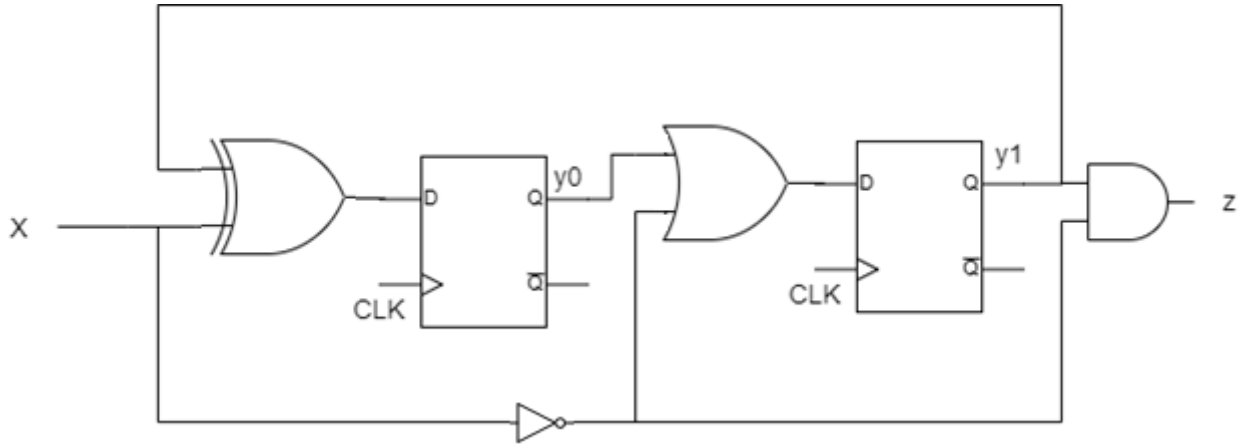
$$= \overline{\overline{a+b+d} + \overline{\overline{b+b} + \overline{c+c}+d}}$$

ALLNOR

COMPUTER ARCHITECTURE UNIT 1 - 11/01/2022 (B)

Name: _____ **Surname:** _____

Exercise 1 (4+2 points): Analyze the circuit in figure (up to the minimal automaton). Then, provide the temporal diagram of the circuit with the input sequence 100110.



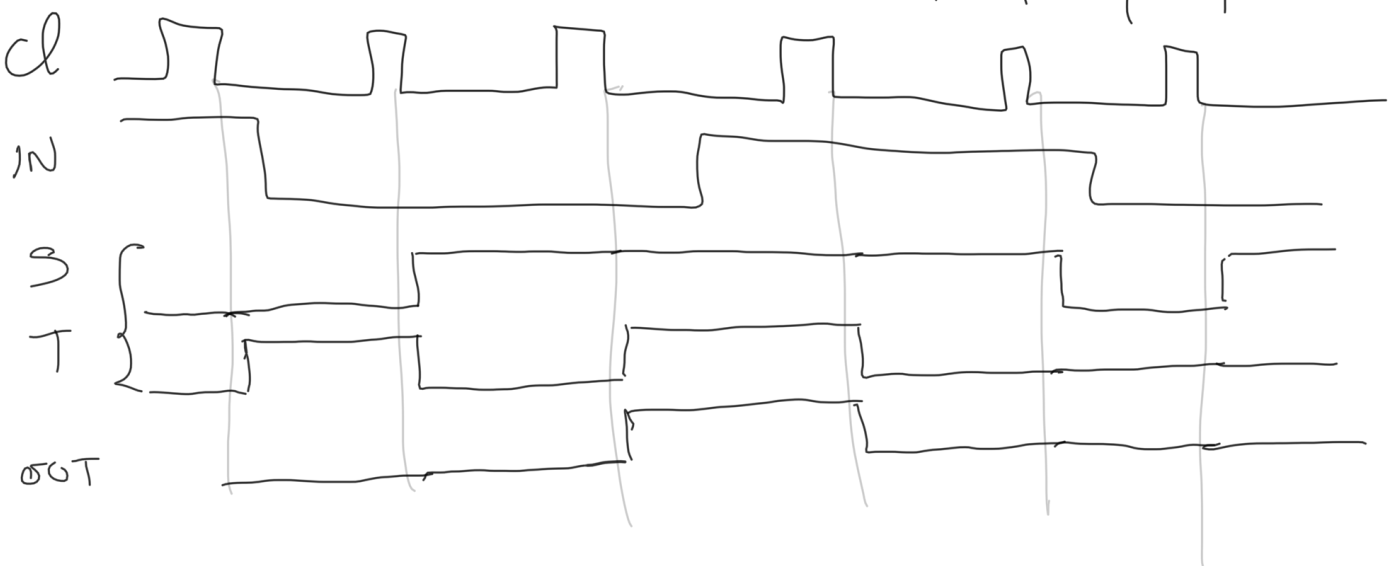
$D_0 = x \oplus y_1 \qquad D_1 = \bar{x} + y_0 \qquad z = \bar{x}\, y_1$

x	y_1	y_0	$D_1 = Y_1$	$D_0 = Y_0$	z
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	1	0	0

	0	1
S_0	$S_2/0$	$S_1/0$
S_1	$S_2/0$	$S_3/0$
S_2	$S_3/1$	$S_0/0$
S_3	$S_3/1$	$S_2/0$

S_1	X		
S_2	X	X	
S_3	X	X	X
	S_0	S_1	S_2

Initial state is S_0 $(=00)$



Exercise 2 (7 points): Design a sequential circuit with one input x and two outputs z1 and z0. Output z1 must be 1 if the last three input bits represent an odd negative number, while z0 must be 1 if the last two bits are different. Use an SR FF for the MSB and T FFs for the remaining bits.

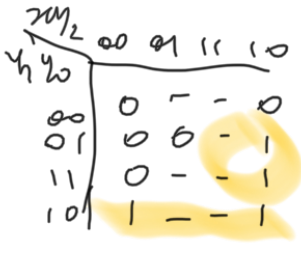
```
Example:  x    0111100100000011
          z1   0001100000000000
          z0   0100010110000010
```

		0	1
000	ε	0 / 00	1 / 00
001	0	00 / 00	01 / 01
010	1	10 / 01	11 / 00
011	00	00 / 00	01 / 01
100	01	10 / 01	11 / 00
101	10	00 / 00	01 / 11
110	11	10 / 01	11 / 10

x	y_2	y_1	y_0	Y_2	Y_1	Y_0	z_1	z_0	S_2	R_2	T_1	T_0
0	0	0	0	0	0	1	0	0	0	–	0	1
0	0	0	1	0	1	1	0	0	0	–	1	0
0	0	1	0	1	0	1	0	1	1	0	1	1
0	0	1	1	0	1	1	0	0	0	–	0	0
0	1	0	0	1	0	1	0	1	–	0	0	1
0	1	0	1	0	1	1	0	0	0	1	1	0
0	1	1	0	1	0	1	0	1	–	0	1	1
0	1	1	1	–	–	–	–	–	–	–	–	–
1	0	0	0	0	1	0	0	0	0	–	1	0
1	0	0	1	1	0	0	0	1	1	0	0	1
1	0	1	0	1	1	0	0	0	1	0	0	0
1	0	1	1	1	0	0	0	1	1	0	1	1
1	1	0	0	1	1	0	0	0	–	0	1	0
1	1	0	1	1	0	0	0	1	–	0	0	1
1	1	1	0	1	1	0	0	0	–	0	0	0
1	1	1	1	–	–	–	–	–	–	–	–	–

$$z_1 = xy_2(y_1 + y_0)$$

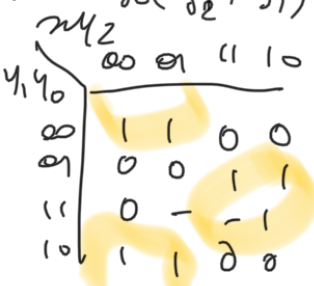
$$z_0 = xy_0 + \bar{x}y_2\bar{y}_0 + \bar{x}y_1\bar{y}_0 = xy_0 + \bar{x}\bar{y}_0(y_2 + y_1)$$



$$S_2 = y_1\bar{y}_0 + xy_0$$

$$R_2 = \bar{x}y_0$$

$$T_1 = x\bar{y}_1\bar{y}_0 + xy_1y_0 + \bar{x}\bar{y}_1y_0 + \bar{x}y_1\bar{y}_0 = x(y_1 \otimes y_0) + \bar{x}(y_1 \oplus y_0) = x \oplus (y_1 \oplus y_0)$$



$$T_0 = \bar{x}\bar{y}_0 + xy_0 = x \otimes y_0$$

Exercise 3 (3 points): By using Boolean algebra, prove that:

$$\overline{\overline{(ab + c)}(a + bc)} + (a \oplus \bar{c}) = \bar{a} + b + c$$

$ab + c + \bar{a}(\bar{b} + \bar{c}) + ac + \bar{a}\bar{c} = ab + c + \bar{a}\bar{b} + \bar{a}\bar{c} + ac$

$= ab + \bar{a}\bar{b} + c + \bar{a}\bar{c} = ab + \bar{a}\bar{b} + (c + \bar{a})(c + \bar{c}) = ab + \bar{a}\bar{b} + c + \bar{a}$

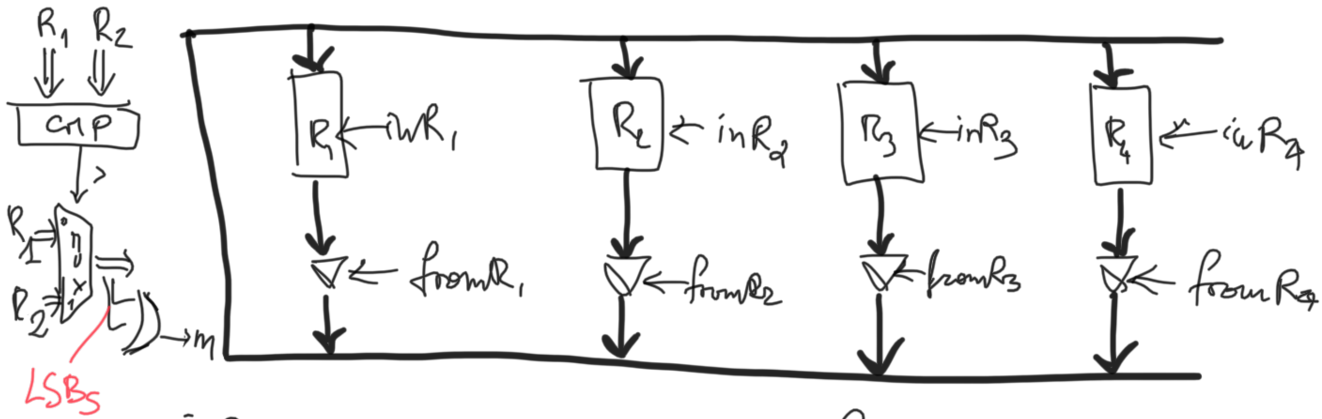
$= ab + \bar{a} + c = (a + \bar{a})(b + \bar{a}) + c = b + \bar{a} + c$

Exercise 4 (4 points): Interconnect 4 registers R1…R4 through a bus so that:

- If R1 is odd, then the content of R3 goes into R2 and R4
- otherwise, the content of R2 goes into R1 and R3.

Movements are enabled only if the minimum between R1 and R2 is not multiple of 4.

b is the LSB of R_1



$inR_1 = m\bar{b}$ $fromR_1 = 0$

$inR_2 = mb$ $fromR_2 = \bar{b}$

$inR_3 = m\bar{b}$ $fromR_3 = b$

$inR_4 = mb$ $fromR_4 = 0$

Exercise 5 (4 points): Given X = < 1 ; 10001 ; 0100000000 > e Y = < 0 ; 10000 ; 1100000000 > in IEEE half-precision format, calculate X+Y and represent the result both in IEEE half-precision and in decimal notation.

Y' = <0; 00000; 1110000000>

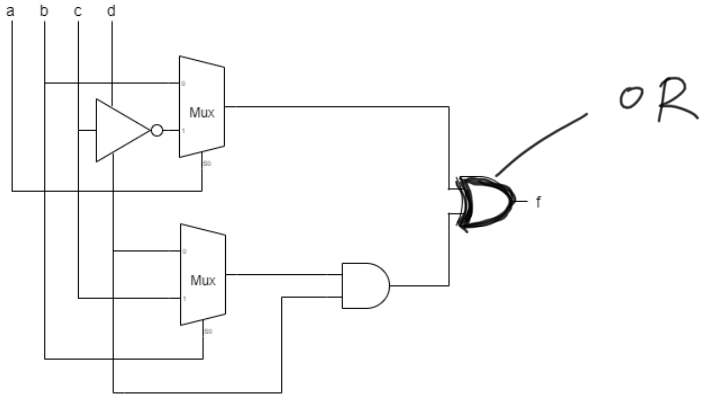
```
  1,0100000000 -
  0,1110000000 =
  ------------
  0,0110000000
```

< 1; 01111; 1000000000>

In base 10: −1,5

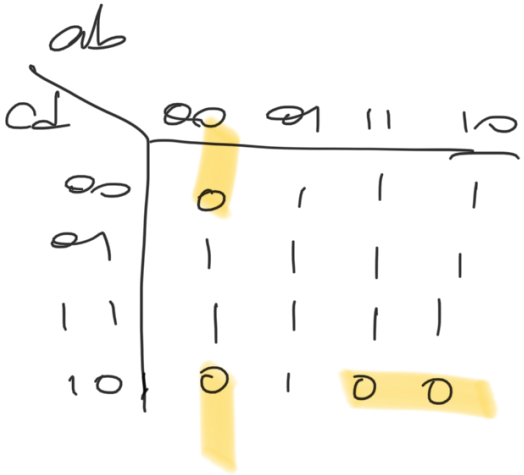
Exercise 6 (2+1+1+2 points)

- Consider the circuit in figure and write down the BE of function f
- Write the truth table for f
- Write the minimal POS expression for f
- Write f in ALLNOR form.



$$f = \bar{a}b + a\bar{c} + (\bar{b}d + bc)d = \bar{a}b + a\bar{c} + \bar{b}d + bcd$$

a b c d	f
0 0 0 0	0
0 0 0 1	1
0 0 1 0	0
0 0 1 1	1
0 1 0 0	1
0 1 0 1	1
0 1 1 0	1
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	0
1 0 1 1	1
1 1 0 0	1
1 1 0 1	1
1 1 1 0	0
1 1 1 1	1



$$\text{minPOS} = (a+b+d)(\bar{a}+\bar{c}+d)$$

$$= \overline{\overline{a+b+d} + \overline{\bar{a}+\bar{c}+d}}$$

$$= \overline{\overline{a+b+d} + \overline{\overline{a+a} + \overline{c+c} + d}}$$

ALLNOR