Exercises on the topics of class 8

Exercises with solutions

Ex. 1. By using the laws and axioms of Boolean algebra, prove that $\bar{x} + yz = \bar{x} + z(\bar{x} + y)$.

SOLUTION:

$\bar{x} + z(\bar{x} + y) = \bar{x} + z\bar{x} + zy = \bar{x}(1 + z) + zy = \bar{x} + zy$

by using distributivity, neutral element, again distributivity, annihilator and finally neutral.

Ex. 2. By using the laws and axioms of Boolean algebra, prove that

$$ab + bc + ca = (a+b)(b+c)(c+a)$$

SOLUTION:

$(a+b)(b+c)(c+a)$	$= a(b+c)(c+a) + b(b+c)(c+a)$	distr.
	$= ab(c+a) + ac(c+a) + bb(c+a) + bc(c+a)$	distr.
	$= abc + aab + acc + aac + bbc + abb + bcc + abc$	distr.
	$= abc + ab + ac + ac + bc + ab + bc$	idempot.
	$= abc + ab + ac + bc$	idempot.
	$= ab(c+1) + ac + bc$	distr.
	$= ab + ac + bc$	annihil. & neutral

Ex. 3. Prove the following equality (where, for graphical reasons, negations are underneath the variable):

$$a\underline{b} + b\underline{c} + c\underline{a} = \underline{a}b + \underline{b}c + \underline{c}a$$

SOLUTION:

$a\underline{b} + b\underline{c} + c\underline{a}$	$= a\underline{b}(c+\underline{c}) + b\underline{c}(a+\underline{a}) + c\underline{a}(b+\underline{b})$	neutral & complem.
	$= a\,\underline{b}\,c + a\,\underline{b}\,\underline{c} + a\,b\,\underline{c} + \underline{a}\,b\,\underline{c} + \underline{a}\,b\,c + \underline{a}\,\underline{b}\,c$	distr.
	$= \underline{a}b(c+\underline{c}) + \underline{b}c(a+\underline{a}) + \underline{c}a(b+\underline{b})$	distr.
	$= \underline{a}b + \underline{b}c + \underline{c}a$	complem. & neutral

Exercises without solutions

Ex. 1. By using the Boolean axioms and laws, prove that $x + y = x + \bar{x}y$.

Ex. 2. By using the Boolean axioms and laws, prove that $x + \bar{y}z = x + z(x + \bar{y})$.

Ex. 3. By using the Boolean axioms and laws, prove that $\overline{(x + yz)}(\bar{x} + y)(\bar{x}z + \bar{y}) = \overline{x + y}$.