Exercises on the topics of class 5

Exercises with solutions

Ex. 1. Turn in base 16 the following decimal number 1364,37 by using 4 digits for the integer part and 6 for the fractional one. By having more digits for the fractonal part, the procedure will terminate? Why?

SOLUTION:

The integer part is obtained by the iterated divisions procedure:

$1364 : 16 = 85$ rem. 4
$85 : 16 = 5$ rem. 5
$5 : 16 = 0$ rem. 5

This gives us the hexadecimal number 554. The fractional part is obtained by the iterated multiplications procedure:

$0{,}37 \times 16 = 5{,}92$
$0{,}92 \times 16 = 14{,}72$
$0{,}72 \times 16 = 11{,}52$
$0{,}52 \times 16 = 8{,}32$
$0{,}32 \times 16 = 5{,}12$
$0{,}12 \times 16 = 1{,}92$

This gives us the hexadecimal number 0,5EB851. Hence, the number required is 554,5EB851.

By having an unbounded number of available digits for the fractional part, the procedure will not terminate anyway, since the representation of the fractional part is periodic (indeed, notice that a seventh multiplication would give usa gain $0{,}92 \times 16$).

Ex. 2. Consider the two positive numbers in base 4: 32,012 and 0,0123. Turn them in base 2 under the IEEE half-precision format. Did you loose significant digits in this conversion?

SOLUTION:

We can directly convert the numbers

$32{,}012_4 \rightarrow 1110{,}000110_2 \rightarrow < 0 , 10010 , 1100001100 >$
$0{,}0123_4 \rightarrow 0{,}00011011_2 \rightarrow < 0 , 01011 , 1011000000 >$

without loosing any significant bit.

Ex. 3. Condìsider $321{,}041_5$. Turn it in base 2, by using 8 bits for the integer part and 8 bits for the fractional one. Then, turn the representation in the half precision standard IEEE.

SOLUTION:

Let’s first turn the number in base 10:

$321{,}041_5 = 3 \times 25 + 2 \times 5 + 1 \times 1 + 4 \times 5^{-2} + 1 \times 5^{-3} = 86{,}168$

that, turned in base 2, is:

$$\begin{aligned} 86 : 2 &= 43 \text{ rem. } 0 \\ 43 : 2 &= 21 \text{ rem. } 1 \\ 21 : 2 &= 10 \text{ rem. } 1 \\ 10 : 2 &= 5 \text{ rem. } 0 \\ 5 : 2 &= 2 \text{ rem. } 1 \\ 2 : 2 &= 1 \text{ rem. } 0 \\ 1 : 2 &= 0 \text{ rem. } 1 \end{aligned}$$

$$\begin{aligned} 0{,}168 \times 2 &= 0{,}336 \\ 0{,}336 \times 2 &= 0{,}672 \\ 0{,}672 \times 2 &= 1{,}344 \\ 0{,}344 \times 2 &= 0{,}688 \\ 0{,}688 \times 2 &= 1{,}376 \\ 0{,}376 \times 2 &= 0{,}752 \\ 0{,}752 \times 2 &= 1{,}504 \\ 0{,}504 \times 2 &= 1{,}008 \end{aligned}$$

Hence: 01010110 , 00101011. This number in floating point representation becomes

$$<0\ ,\ 10101\ ,\ 0101100010>$$

Notice that with the IEEE half precision standard, the representation is not exact.

Exercises without solutions

Ex. 1. Turn in base 5 in the fixed point representation the decimal number 214,1362 by using 5 digits for the integer part and 6 for the fractional one. Is the obtained representation exact or an approximation of the given number?

Ex. 2. Turn $61{,}81_9$ in the IEEE half precision standard.