

## Exercises on the topics of class 3

### Exercises with solutions

**Ex. 1.** Turn in base 2 the decimal numbers 14 and 11; then, multiply them in base 2. How many bits are needed to express the result? Could you reply to the latter question even without explicitly doing the product?

SOLUTION:

$14 : 2 = 7 \text{ rem. } 0$        $7 : 2 = 3 \text{ rem. } 1$        $3 : 2 = 1 \text{ rem. } 1$        $1 : 2 = 0 \text{ rem. } 1$   
 Hence,  $14_{10}$  is turned into  $1110_2$ .

$11 : 2 = 5 \text{ rem. } 1$        $5 : 2 = 2 \text{ rem. } 1$        $2 : 2 = 1 \text{ rem. } 0$        $1 : 2 = 0 \text{ rem. } 1$   
 Hence,  $11_{10}$  is turned into  $1011_2$ .

Their product is

$$\begin{array}{r}
 1110 \times \\
 1011 = \\
 \hline
 1110 \\
 1110 - \\
 0000 - \\
 1110 - \\
 \hline
 10011010
 \end{array}$$

So, we need 8 bits, as it was expectable, since the products of two n-bits number requires  $2n$  bits.

**Ex. 2:** Turn in base 4 the decimal numbers 24 and 18. Then, add, subtract and multiply them (still in base 4). Finally, check that your results are correct by converting the results back to base 10.

SOLUTION:

$24 : 4 = 6 \text{ rem. } 0$                    $6 : 4 = 1 \text{ rem. } 2$                    $1 : 4 = 0 \text{ rem. } 1$   
 $18 : 4 = 4 \text{ rem. } 2$                    $4 : 4 = 1 \text{ rem. } 0$                    $1 : 4 = 0 \text{ rem. } 1$

Hence,  $24_{10} = 120_4$  and  $18_{10} = 102_4$ .

Sum:

$$\begin{array}{r}
 120 + \\
 102 = \\
 \hline
 222
 \end{array}$$

The result is correct, since  $222_4 = (2 \times 16 + 2 \times 4 + 2)_{10} = 42_{10} = (24 + 18)_{10}$

Difference:

$$\begin{array}{r} 120 - \\ 102 = \\ \hline 12 \end{array}$$

The result is correct, since  $12_4 = (4+2)_{10} = 6_{10} = (24-18)_{10}$

Product:

$$\begin{array}{r} 120 \times \\ 102 = \\ \hline 300 \\ 000 - \\ 120 - \\ \hline 12300 \end{array}$$

The result is correct, since  
 $12300_4 = (256 + 2 \times 64 + 3 \times 16)_{10} = (256 + 128 + 48)_{10}$   
 $= 432_{10} = (24 \times 18)_{10}$

### Exercises without solutions

**Ex. 1.** Consider the binary numbers 1110010 and 1010001. Sum and subtract them. The results can be expressed in the format used for the operands?

**Ex. 2.** Consider the binary numbers 11001 and 110. Multiply them (in base 2) and then check the correctness of the result by turning in base 10 both the operands and the result.

**Ex. 3.** Turn in base 2 the number 242; then, divide it by 2 in such a base. What can you notice by looking at the dividend and the quotient?