## Exercises on the topic of class 20

## Exercises with solutions

Ex. 1. Given the following sequential circuit:


Find the corresponding automaton, minimize it and describe its behavior at words. Assume that at the outset all FFs store 0 .

## SOLUTION:

The BEs are:

$$
\mathrm{S}=\mathrm{Q} 1 \overline{\mathrm{x}} \quad \mathrm{R}=\mathrm{x} \quad \mathrm{~T}=\mathrm{x} \oplus \mathrm{Q} 0 \quad \mathrm{Z}=\mathrm{x} \mathrm{Q} 0
$$

And so the table of the future states is:

|  | $\begin{aligned} & \text { Q1 } \\ & \text { ( } \mathrm{t}) \end{aligned}$ |  |  | R t) |  | z ( t ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

From it, the automaton table (whose states are, as usual, called S0 if Q1Q0 $=00, \mathrm{~S} 1$ if $01, \mathrm{~S} 2$ if 10 and S3 if 11), with initial state S0.

|  | 0 | $\mathbf{1}$ |
| :---: | :---: | :---: |
| S0 | $\mathrm{S} 0 / 0$ | $\mathrm{~S} 1 / 0$ |
| S1 | $\mathrm{S} 0 / 0$ | $\mathrm{~S} 1 / 1$ |
| S2 | $\mathrm{S} 2 / 0$ | $\mathrm{~S} 1 / 0$ |
| S3 | $\mathrm{S} 2 / 0$ | $\mathrm{~S} 1 / 1$ |

Let's first observe that S2 and S3 are unreachable when the starting state is S0; it is then easy to check that the remaining automaton is minimal and can be drawn as follows:


The automaton return 1 if it reads at least two ' 1 s ' in sequence.

Ex. 2. Analyze the following sequential circuit, by assuming that at the outset the FFs are set to $\mathrm{q} 2 \mathrm{q} 1 \mathrm{q} 0=110$.


## SOLUTION:

The BEs associated to the inputs of the FFs and to the output of the circuit are:

$$
\begin{aligned}
& \mathrm{T} 0=\mathrm{x} \\
& \mathrm{~J} 1=\mathrm{x} \text { Q2 } \\
& \mathrm{K} 1=\mathrm{Q} 2 \\
& \mathrm{D} 2=\mathrm{Q} 0 \mathrm{Q} 1 \\
& \mathrm{z}=\mathrm{x} \text { Q2 }
\end{aligned}
$$

From them, we can build the future states table:

| $\boldsymbol{x}$ Q2 Q1 Q0 | TO | J1 $\mathbf{K 1}$ | $\boldsymbol{D 2}$ | $\mathbf{Q 2}^{\prime} \mathbf{Q 1}^{\prime}$ Q0' $^{\prime}$ | $\boldsymbol{z}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

Since the initial configuration is the one with Q2 Q1 Q0 $=110$, we obtain the following automaton (REMARK: some states are unreachable starting from 110; hence, they can be safely discarded):

|  | $\boldsymbol{0}$ | $\mathbf{1}$ |
| :---: | :---: | :---: |
| $\mathbf{1 1 0}$ | $000 / 0$ | $001 / 1$ |
| $\mathbf{0 0 0}$ | $000 / 0$ | $001 / 0$ |
| $\mathbf{0 0 1}$ | $001 / 0$ | $000 / 0$ |

We can notice that the automaton is not minimal: we can merge 000 and 001 , and obtain

|  | $\boldsymbol{0}$ | $\mathbf{1}$ |
| :---: | :---: | :---: |
| $\boldsymbol{S 0}$ | $\mathrm{S} 1 / 0$ | $\mathrm{~S} 1 / 1$ |
| $\boldsymbol{S 1}$ | $\mathrm{~S} 1 / 0$ | $\mathrm{~S} 1 / 0$ |

This circuit gives 1 upon reception of input sequences of the form 1000... 0

## Exercises without solutions

Ex. 1. Analyze the following sequential circuit:


Ex. 2. Given the following circuit, analyze it by assuming that both FFs initially store 0 :


Ex. 3. Analyze the following circuit, whose FF initially stores 0 :


