Exercises on the topic of class 19

Exercises with solutions

Ex. 1. Minimize the Mealy automaton specified by the following table:

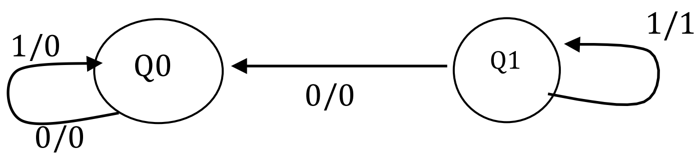
	0	1
S0	S1/0	S0/0
S1	S2/0	S4/0
S2	S2/0	S1/0
S3	S1/0	S3/1
S4	S1/0	S0/0
S5	S2/0	S3/1

SOLUTION:

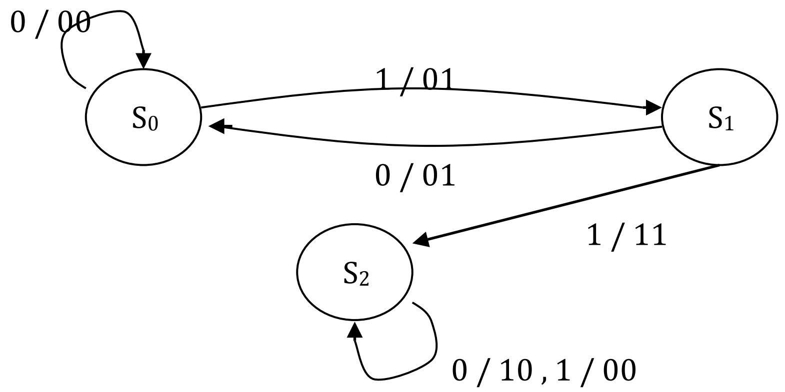
Since the starting state is not specified, we cannot speak about unreachable states. Let us them build the triangular table:

S1	(S1,S2) (S0,S4)				
S2	(S1,S2) (S0,S1)	(S1,S4)			
S3	X	X	X		
S4	O	(S1,S2) (S0,S4)	(S1,S2) (S0,S1)	X	
S5	X	X	X	(S1,S2)	X
	S0	S1	S2	S3	S4

Since S0 and S4 are equivalent, the pointers to such pairs have to be deleted. In the pointers left, we note that (S1,S2) depends from (S1,S4), and vice versa; hence, also these pairs are equivalent. This entails that also (S0,S1) and (S3,S5) are equivalent, so as allthe remaining pairs. Hence, the minimal automaton is made up from states Q0 = {S0, S1, S2, S4} and Q1 = {S3, S5} and can be drawn as follows:



Ex. 2. Find the minimal Moore automaton equivalent to the following Mealy automaton, with starting state S_0



SOLUTION:

We first consider all pairs state/output (for transforming the Mealy automaton into the Moore one). Then, the resulting Moore automaton has table

		Input = 0	**Input = 1**
State $_{Me}$ Output $_{Me}$	State $_{Mo}$ Output $_{Mo}$		
S_0 0 0	T_0 00	T_0	T_5
S_0 0 1	T_1 01	T_0	T_5
S_0 1 0	T_2 10	T_0	T_5
S_0 1 1	T_3 11	T_0	T_5
S_1 0 0	T_4 00	T_1	T_{11}
S_1 0 1	T_5 01	T_1	T_{11}
S_1 1 0	T_6 10	T_1	T_{11}
S_1 1 1	T_7 11	T_1	T_{11}
S_2 0 0	T_8 00	T_{10}	T_8
S_2 0 1	T_9 01	T_{10}	T_8
S_2 1 0	T_{10} 10	T_{10}	T_8
S_2 1 1	T_{11} 11	T_{10}	T_8

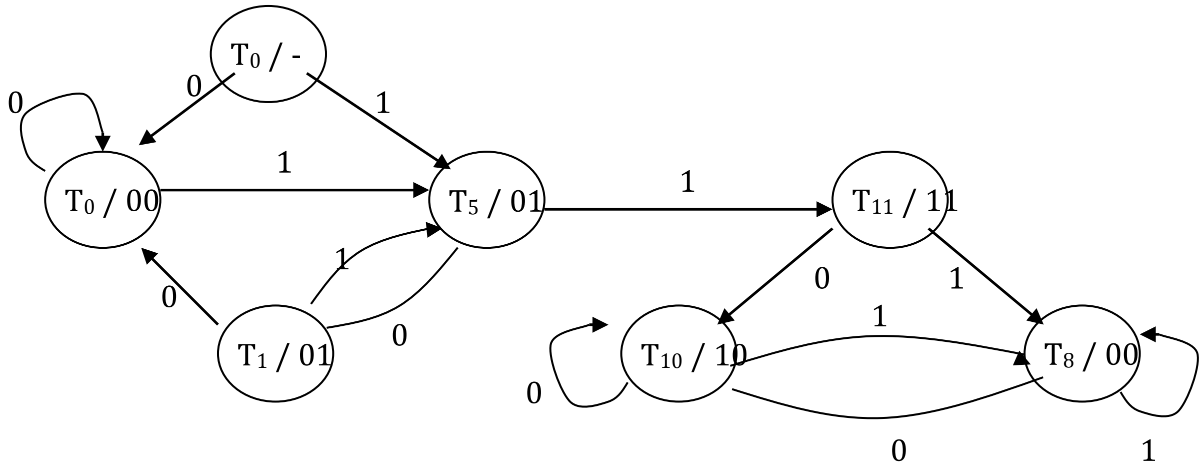
We can assume any clone of S0 to be the starting state; for example, let's pick T0. Hence, T_2 , T_3 , T_4 , T_6 , T_7 , T_9 can be canceled because unreachable. Then, the automaton becomes

Stato $_t$	**Input** $_t$ **= 0**	**Input** $_t$ **= 1**
T_0 / 00	T_0	T_5
T_1 / 01	T_0	T_5
T_5 / 01	T_1	T_{11}
T_8 / 00	T_{10}	T_8
T_{10} / 10	T_{10}	T_8
T_{11} / 11	T_{10}	T_8

At a first sight, a minimization would seems possible (e.g., by grouping together <T_8, T_{10}, T_{11} > and <T_0, T_1 >, since they have the same row in the above table). This is however not possible since the groupable states have different outputs, whereas those with same output lead to distinguishable states (and so are not groupable). Indeed, this is confirmed by the triangular table:

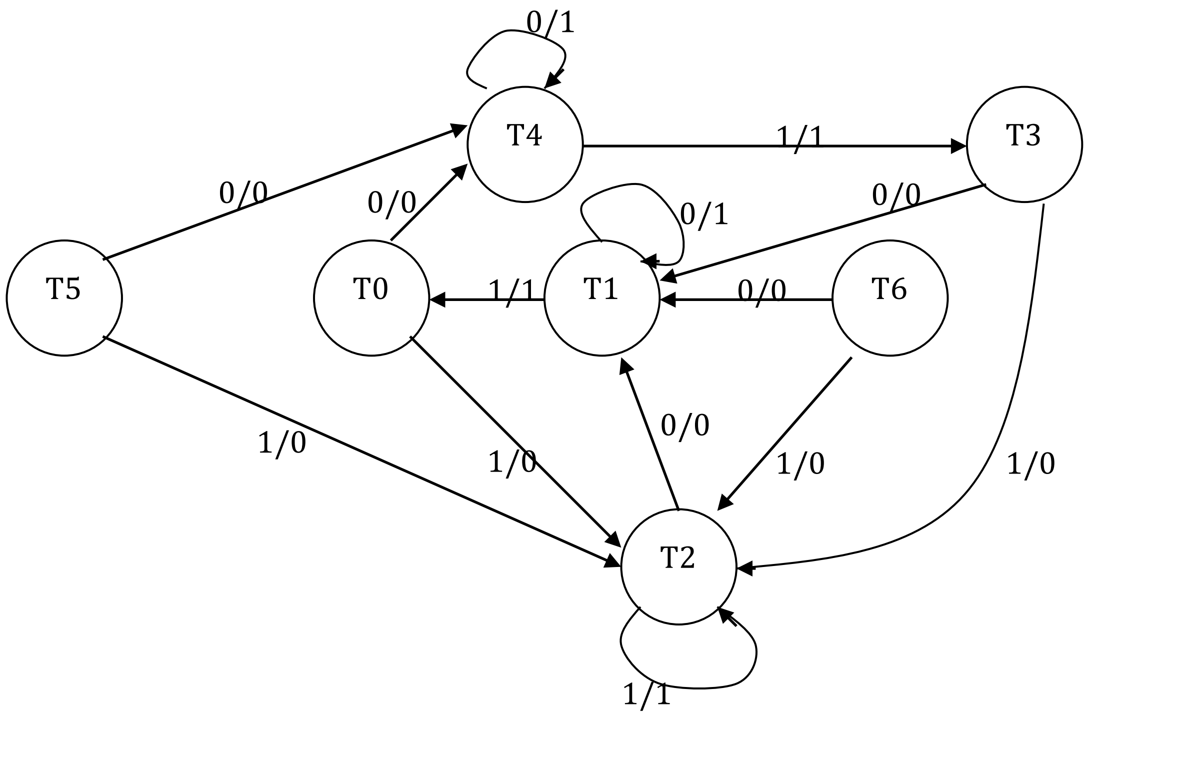
T_1	X				
T_5	X				
T_8		X	X		
T_{10}	X	X	X	X	
T_{11}	X	X	X	X	X
	T_0	T_1	T_5	T_8	T_{10}

Then, (T_0,T_8) must be marked with X since, by reading 0, we reach (T_0,T_{10}) that is marked with X; similarly, (T_1,T_5) must be marked with X becasue always with 0 it reaches (T_0,T_1).
So, all states are distinguishable and the minimal Moore automaton is:



Exercises without solutions

Ex. 1. Given the following automaton with initial state T0:



Minimize it and give, through temporal diagrams, output and state transitions obtained with input 110001001001.

Ex. 2. Minimize the following automaton: