Exercises on the Topics of class 11

Exercises with a solution

Ex. 1. Let us consider the following boolean functions *f(x, y, z, t)* and *g(x, y, z, t)* such that:

- *f* returns 1 if and only if the string *xyzt* contains an even number of 1s;
- *g* returns 1 if and only if the string *xyzt*, seen as an integer number, is divisible by 2.

For *f* and *g* give: the canonical forms (conjunctive and disjunctive), the minimal SOP and POS forms and finally a minimal boolean expression (obtained by using the NAND, NOR, XOR and XNOR gates)

SOLUTION:

The tabular representation of *f* and *g* is:

x	*y*	*z*	*t*	*f*	*g*
0	0	0	0	1	1
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	1	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	0	1
1	0	0	1	1	0
1	0	1	0	1	1
1	0	1	1	0	0
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	1	0

Let's denote with mi and Mi (for i = 0, …, 15) the minterm and the maxterm associated to the i-th row of the table. For example,

m2 = $\bar{x} \cdot \bar{y} \cdot z \cdot \bar{t}$ and M2 = $x + y + \bar{z} + t$

With this formalism, the canonical forms are:

FCD(*f*) = m0 + m3 + m5 + m6 + m9 + m10 + m12 + m15
FCC(*f*) = M1 · M2 · M4 · M7 · M8 · M11 ·M13 · M14

FCD(*g*) = m0 + m2 + m4 + m6 + m8 + m10 + m12 + m14
FCC(*g*) = M1 · M3 · M5 · M7 · M9 ·M11 ·M13 · M15

For the minimal normal forms, we can easily check (by writing down the KM) that the minimal SOP and POS for f are the canonical forms. For g we have that:

xy \ zt	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

and so minSOP(g) = $\bar{t}$. For the minPOS, we can use g's KM and cover the 0s or observing that

$$\text{minPOS}(\,g\,) = \overline{\text{minPOS}(\,\neg g\,)}$$

that is, we have to cover the 1s of the following map:

xy \ zt	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

In both cases, we obtain that minPOS(g) = $\bar{t}$.

Finally, for the minimal BEs, it is easy to see that the minimal expression for g is the minSOP (that coincides with the minPOS). For f , by using XORs and NXORs (respectively denoted by $\oplus$ and $\otimes$) and by properly factoring the CCF, we have that

$$\begin{aligned}
\text{FCC}(f) &= (x+y+z+\underline{t})\,(x+y+\underline{z}+t)\,(x+\underline{y}+z+t)\,(x+\underline{y}+\underline{z}+\underline{t})\,(\underline{x}+y+z+t)\,(\underline{x}+y+\underline{z}+\underline{t})\,(\underline{x}+\underline{y}+z+\underline{t})\,(\underline{x}+\underline{y}+\underline{z}+t) \\
&= (x+y+(z+\underline{t})(\underline{z}+t))\,(x+\underline{y}+(z+t)(\underline{z}+\underline{t}))\,(\underline{x}+y+(z+t)(\underline{z}+\underline{t}))\,(\underline{x}+\underline{y}+(z+\underline{t})(\underline{z}+t)) \\
&= (x+y+(z\otimes t))\,(x+\underline{y}+(z\oplus t))\,(\underline{x}+y+(z\oplus t))\,(\underline{x}+\underline{y}+(z\otimes t)) \\
&= ((x+y)(\underline{x}+\underline{y})+(z\otimes t))\,((x+\underline{y})(\underline{x}+y)+(z\oplus t)) \\
&= ((x\oplus y)+(z\otimes t))\,((x\otimes y)+(z\oplus t)) \\
&= (x\oplus y)\otimes(z\oplus t)
\end{aligned}$$

Indeed, it is easy to check that $(a+\underline{b})(\underline{a}+b) = (a\otimes b)$ and $(a+b)(\underline{a}+\underline{b}) = (a\oplus b)$. For example,

$$(a+\bar{b})(\bar{a}+b) = \overline{\overline{(a+\bar{b})}+\overline{(\bar{a}+b)}} = \overline{\bar{a}b+a\bar{b}} = \overline{a\oplus b} = a\otimes b$$

Hence,

minEB(f) = $(\,x\oplus y\,)\otimes(\,z\oplus t\,)$

Exercises without solutions

Ex. 1. Let us consider the function that returns 1 only when its (4 bit) input represents an integer in 2's complement that is a multiple of 3. Do not consider 1000 (i.e., on that sequence the function is undefined). Give a minimal SOP and POS associated to this function.

Ex. 2. Find the minimal SOP associated to the expression $(x \oplus (y \text{ NAND } z)) \text{ NOR } \bar{x}$