Exercises on the topics of class 10

Exercises with solutions

Ex. 1. Answer to the following questions, by properly justifying your answers:
1. Is it true that every BE is either a CCF or a DCF?
2. Is it true that there are BEs that are simultaneously both a CCF and a DCF?
3. Is it true that, for every BE, there exists one single DCF?

SOLUTION:

1. No. Consider $(x + 1)y + z$ that is niether a CCF nor a DCF.
2. Yes. The boolean expressions 0 , 1 , x , ... are all both CCFs and DCFs.
3. Unicity of DCFs is up-to the order of sums and products. Indeed, both

$$\bar{x}y + x\bar{y} + xy \qquad \text{and} \qquad \bar{y}x + \bar{x}y + xy$$

are DCFs for the logical OR of x and y. It suffices to introduce a sorting among the variables to have one unique DCF.

Ex. 2. Write the DCF and the CCF of the function that returns 1 *iif* it receives in input an even number of 1s (consider a 3-arguments BF)

SOLUTION:

The truth table of f is:

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Hence, $\text{DCF}(f) = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3 + \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 \cdot \bar{x}_2 \cdot x_3 + x_1 \cdot x_2 \cdot \bar{x}_3$

$\text{CCF}(f) = (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + x_3) \cdot (\bar{x}_1 + x_2 + x_3) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$

Exercises without solutions

Ex. 1. Turn $x + \underline{x}\,\underline{y}\,z$ in CCF and DCF, by specifying the boolean axioms used. Then, chose one of the two expressions and derive from it the associated truth table.

Ex. 2. Write the DCF and the CCF of the BF that returns 1 whenever 3 of its 4 inputs hold 0. Then, write at least one DNF and one CNF for it.