Analysis of sequential nets

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Analysis of sequential circuits

Given a sequential circuit, describe its behavior through an automaton

- Given the circuit schema, we should first have to identify the memory elements.
- In every moment, the memory of the system (that is, the binary values stored in all FFs) given the status of the circuit.
- For every possible state and input combination, we have to compute the output values and the next state by looking at the combinatorial part of the circuit.

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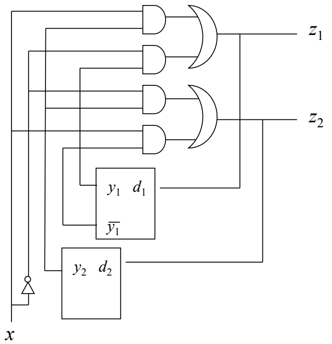
Analysis Procedure (1)

We first analyze the combinatorial part of the circuit: for every input of every FF and for every output line, give the BE in terms of the input lines of the circuit and of the values stored in the FFs.



$$d_1 = z_1 = xy_2 + \overline{x}y_1$$
$$d_2 = z_2 = \overline{x}y_2 + x\overline{y}_1$$

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Analysis Procedure (2)

Write down the TT corresponding to the BEs found at the previous step.

$$d_1 = z_1 = xy_2 + \overline{x}y_1$$
$$d_2 = z_2 = \overline{x}y_2 + x\overline{y}_1$$

x	y_2	y_1	z_2	z_1	d_2	d_1
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	0	0	0	0
1	1	0	1	1	1	1
1	1	1	0	1	0	1

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Analysis Procedure (3)

According to the kind of FFs, we can establish the future states, by considering the current states and the inputs of the FFs.

x y_2 y_1	z_2 z_1	d_2 d_1	Y_2 Y_1
0 0 0	0 0	0 0	0 0
0 0 1	0 1	0 1	0 1
0 1 0	1 0	1 0	1 0
0 1 1	1 1	1 1	1 1
1 0 0	1 0	1 0	1 0
1 0 1	0 0	0 0	0 0
1 1 0	1 1	1 1	1 1
1 1 1	0 1	0 1	0 1

Analysis Procedure (4)

Give symbolic name to every possible combination of bits stored in the FFs, to every possible input sequence and to every possible output sequence. Then, infer the transition and the output functions of the automaton.

REMARK: actually, giving symbolic names is not strictly necessary: everything could be left in binary, but this would make the automaton less readable.

Ex.:

$y_2\,y_1$: $00 \rightarrow q_0$, $01 \rightarrow q_1$, $10 \rightarrow q_2$, $11 \rightarrow q_3$
x: $0 \rightarrow 0$, $1 \rightarrow 1$
$z_2\,z_1$: $00 \rightarrow 0$, $01 \rightarrow 3$, $10 \rightarrow 1$, $11 \rightarrow 2$

x y_2 y_1	z_2 z_1	d_2 d_1	Y_2 Y_1
0 0 0	0 0	0 0	0 0
0 0 1	0 1	0 1	0 1
0 1 0	1 0	1 0	1 0
0 1 1	1 1	1 1	1 1
1 0 0	1 0	1 0	1 0
1 0 1	0 0	0 0	0 0
1 1 0	1 1	1 1	1 1
1 1 1	0 1	0 1	0 1

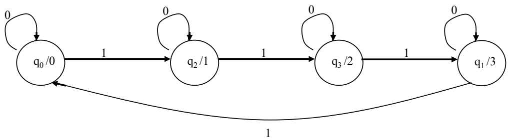
	0	1
q_0	q_0 / 0	q_2 / 1
q_1	q_1 / 3	q_0 / 0
q_2	q_2 / 1	q_3 / 2
q_3	q_3 / 2	q_1 / 3

Analysis Procedure (5)

Minimize the obtained automaton, draw it and give a verbal description of its behavior (if possible).

REMARK: the initial state is arbitrary, unless it is not explicitly specified which values are initially stored in the FFs (tipically all 0s).

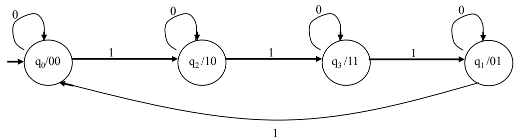
In our example, the automaton is already minimal: we can draw it as a Moore one (since the output is a characteristic of every state).



By taking q_0 as starting state, this automaton is a counter of 1s modulo 4

Remark

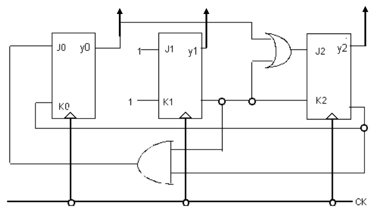
Without the symbolic names, the behavior of the obtained automaton Would have been much more difficult to understand:



This automaton cyclically returns 00,…,00,10,…,10,11,…,11,01,…,01
Where the passage from one sequence to the other happens at every "1" in input and the repetitions of such sequences correspond to the number of "0s" in input.

→ with some experience, also in this way we can (obviously) recognize a "1-counter" modulo 4, but it is harder to see!

A second example (1)



$j_0 = \bar{y}_2 \bar{y}_1$

$k_0 = \bar{y}_2$

$j_1 = k_1 = 1$

$j_2 = y_0 + \bar{y}_1$

$k_2 = \bar{y}_1$

Sequential Circuit without inputs: state transitions and outputs at every clock signal (descending wave fronts)

y_2 y_1 y_0	j_2 k_2	j_1 k_1	j_0 k_0	Y_2 Y_1 Y_0
0 0 0	1 1	1 1	1 1	1 1 1
0 0 1	1 1	1 1	1 1	1 1 0
0 1 0	0 0	1 1	0 1	0 0 0
0 1 1	1 0	1 1	0 1	1 0 0
1 0 0	1 1	1 1	0 0	0 1 0
1 0 1	1 1	1 1	0 0	0 1 1
1 1 0	0 0	1 1	0 0	1 0 0
1 1 1	1 0	1 1	0 0	1 0 1

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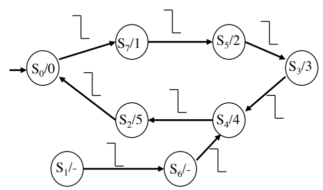
A second example (2)

Encoding: S0 → 000, S1 → 001, S2 → 010, S3 → 011, S4 → 100, S5 → 101, S6 → 110, S7 → 111

y_2 y_1 y_0	Y_2 Y_1 Y_0
0 0 0	1 1 1
0 0 1	1 1 0
0 1 0	0 0 0
0 1 1	1 0 0
1 0 0	0 1 0
1 0 1	0 1 1
1 1 0	1 0 0
1 1 1	1 0 1

State(t)	State(t+1)
S_0	S_7
S_1	S_6
S_2	S_0
S_3	S_4
S_4	S_2
S_5	S_3
S_6	S_4
S_7	S_5



Clock signals counter modulo 6

Outputs are the bits stored in the FFs.
We can use the following encoding of outputs:
000 = 0, 111 = 1, 101 = 2, 011 = 3, 100 = 4, 010 = 5, 001 = -, 110 = -

Moreover, let's assume that at the outset FFs all store a 0

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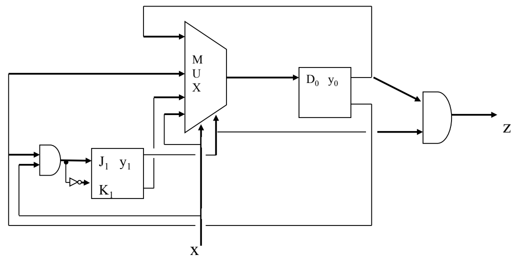
A third example (1)

Analyze the following sequential circuit, with FFs initially at 0.



$J_1 = x\bar{y}_0 \qquad K_1 = \bar{x} + y_0 \qquad z = y_1 y_0$

$$D_0 = \bar{x}\ \bar{y}_1 y_0 + \bar{x} y_1 \bar{y}_0 + x\bar{y}_1\bar{y}_1 + x y_1 x = \bar{x}\ \bar{y}_1 y_0 + \bar{x} y_1 \bar{y}_0 + x\bar{y}_1 + x y_1$$

$$= \bar{x}(\bar{y}_1 y_0 + y_1 \bar{y}_0) + x(\bar{y}_1 + y_1) = \bar{x}(y_0 \oplus y_1) + x$$

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A third example (2)

y_1	y_0	x	J_1	K_1	D_0	Y_1	Y_0	Z
0	0	0	0	1	0	0	0	0
0	0	1	1	0	1	1	1	0
0	1	0	0	1	1	0	1	0
0	1	1	0	1	1	0	1	0
1	0	0	0	1	1	0	1	0
1	0	1	1	0	1	1	1	0
1	1	0	0	1	0	0	0	1
1	1	1	0	1	1	0	1	1

$J_1 = x\bar{y}_0$

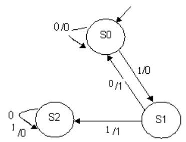
$K_1 = \bar{x} + y_0$

$D_0 = \bar{x}(y_0 \oplus y_1) + x$

$z = y_1 y_0$

00 → S0 , 11 → S1 , 01 → S2

	0	1
S0	S0/0	S1/0
S1	S0/1	S2/1
S2	S2/0	S2/0



The automaton returns "1" every time it receives "10" without any preceding "11";
As soon as it reads "11", returns "1" and the gives "0" for ever.

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