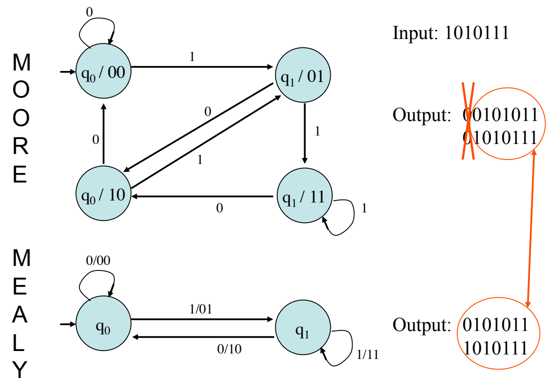
Equivalence between Mealy and Moore models

Prof. Daniele Gorla

Moore vs Mealy (example of reminders MOD 4)



Automata Equivalence

Two automata should be considered equivalent according to their behavior, by abstracting from how they are defined.

This is a *"black box"* approach: the only way to observe an automaton is through experiments:

- Two automata are NOT equivalent if I can find an input sequence that in the two automata produces different outputs
- Otherwise they are equivalent.

Def.: Two automata (of the same kind) are *equivalent* if, for every possible input sequence, they produce the same output sequence.

Def.: two automata (of different kind) are *equivalent* if, for every possible input sequence, they produce output sequences that only differ in the first output character of the Moore automaton.

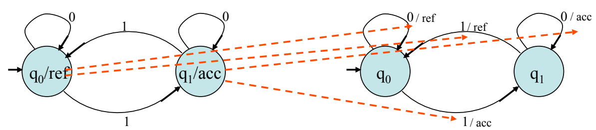
From Moore to Mealy

Theor.: Let $M_1 = (Q,\Sigma,\Delta,q_0,\delta,\lambda)$ be a Moore automaton; then, there exists a Mealy automaton equivalent to it.

Proof. Let $M_2 = (Q, \Sigma,\Delta,q_0,\delta,\lambda')$, where $\lambda'(q,a) = \lambda(\delta(q,a))$.

This means that in M_2 every transition is associated to the output associated to the arrival state in M_1.

Ex. (automaton that accepts all and only the sequences with an odd number of 1s):



Induction proofs

SAPIENZA Università di Roma Dipartimento di Informatica

Problem: to formally proof the equivalence, we have to consider *all* the possible input sequences

How many? If we also assume the smallest possible input alphabeth ($|\Sigma|=1$, e.g. $\Sigma = \{a\}$), we have *infinitely many* possible sequences:

ε, *a*, *aa*, *aaa*, *aaaa*, …

The empty sequence, i.e. without any char

A way to proof a property P(-) over all such strings σ is by using the *induction principle*:

- **Base step:** prove P(ε)
- **Inductive step:** by assuming P(σ) for every σ of size *n*, prove P(σ') for a generic σ' of size *n*+1 (REMARK: *n* is arbitrary!)

In this way, we prove P(σ) for every σ!!

0. P(σ), for |σ| = 0, is proved: P(ε) holds because of the base case
1. P(σ), for |σ| = 1, is proved by using step 0 and the inductive step
2. P(σ), for |σ| = 2, is proved by using step 1 and the inductive step

…

5

Equivalence Moore/Mealy

SAPIENZA Università di Roma Dipartimento di Informatica

Let σ be an input sequence; let's prove, by induction on the length of σ (i.e., on the number of characters contained in it), that

$M_1(\sigma) = c_0\, M_2(\sigma)$

where $c_0 = \lambda(q_0)$.

With M(σ) we denote the output of M with input σ

Base (σ = ε): by definition, $M_1(\varepsilon) = c_0$ and $M_2(\varepsilon) = \varepsilon$.
The thesis holds because $c_0\, \varepsilon = c_0$.

Induction (thesis true for σ of *n* characters, to be proved for σ of length *n*+1):
If σ has *n*+1 characters, then σ = σ'*a*, for some $a \in \Sigma$; so, σ' has *n* characters.
By inductive hypothesis, $M_1(\sigma') = c_0\, M_2(\sigma')$.

According to the behavior of automata, $M_1(\sigma) = M_1(\sigma')\, c$, where $c = \lambda(\delta(q,a))$ and q is the state where M_1 arrives after the input σ'.

Similarly, $M_2(\sigma) = M_2(\sigma')\, c$, since M_2 arrives in q after input σ' (the transition function is the same as M_1) and, by definition of M_2, $\lambda'(q,a) = \lambda(\delta(q,a)) = c$.
Hence, $M_1(\sigma) = M_1(\sigma')\, c = c_0\, M_2(\sigma')\, c = c_0\, M_2(\sigma)$.

Q.E.D.

6

From Mealy to Moore

SAPIENZA Università di Roma Dipartimento di Informatica

Theor.: Let $M_1 = (Q,\Sigma,\Delta,q_0,\delta,\lambda)$ be a Mealy automaton; then, there exists a Moore automaton equivalent to it.

Proof. Let $M_2 = (Q\times\Delta,\Sigma,\Delta,(q_0,b),\delta',\lambda')$, where

- *b* is any character of Δ
- $\delta'(\,(q,c)\,,\, a) = (\,\delta(q,a),\, \lambda(q,a)\,)$
- $\lambda'(\,(q,c)\,) = c$

OBS.: transitions of M_2 are given only by the first element of a pair (q,*c*) and by the input value!

Again, by induction we can prove that M_1 and M_2 are equivalent.

7

Equivalence Mealy/Moore

SAPIENZA Università di Roma Dipartimento di Informatica

Let σ be an input sequence; by induction on the length of σ, we prove that

$M_2(\sigma) = b\, M_1(\sigma)$

where (q_0,b) is the initial state in M_2.

Base (σ = ε): by definition, $M_2(\varepsilon) = b$ and $M_1(\varepsilon) = \varepsilon$.

Induction (thesis true for σ of length *n*, to be proved for σ of length *n*+1):
If σ has *n*+1 characters, then σ = σ'*a*, for some $a \in \Sigma$ and σ' of length *n*. By inductive hypothesis, $M_2(\sigma') = b\, M_1(\sigma')$.

According to how automata behave, $M_1(\sigma) = M_1(\sigma')\, c$, where $c = \lambda(q,a)$ and q is the state reached by M_1 after reading σ'.

Similarly, $M_2(\sigma) = M_2(\sigma')\, c$, since M_2 is in q after the input σ' (recall that the transition function doesn't depend on the output character of a state) and, by definition of M_2, $\lambda'(\delta'(q,a)) = \lambda(q,a) = c$.

Hence, $M_2(\sigma) = M_2(\sigma')\, c = b\, M_1(\sigma')\, c = b\, M_1(\sigma)$.

Q.E.D.

8

From Mealy to Moore: example of the reminders MOD 4

SAPIENZA
UNIVERSITÀ DI ROMA
DIPARTIMENTO DI INFORMATICA