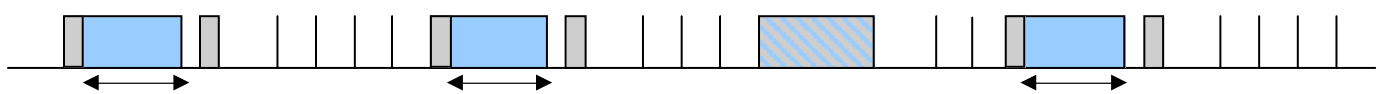
4. Performance Analysis of IEEE 802.11 DCF

Throughput

➔ **Represents the system efficiency (different definitions).**

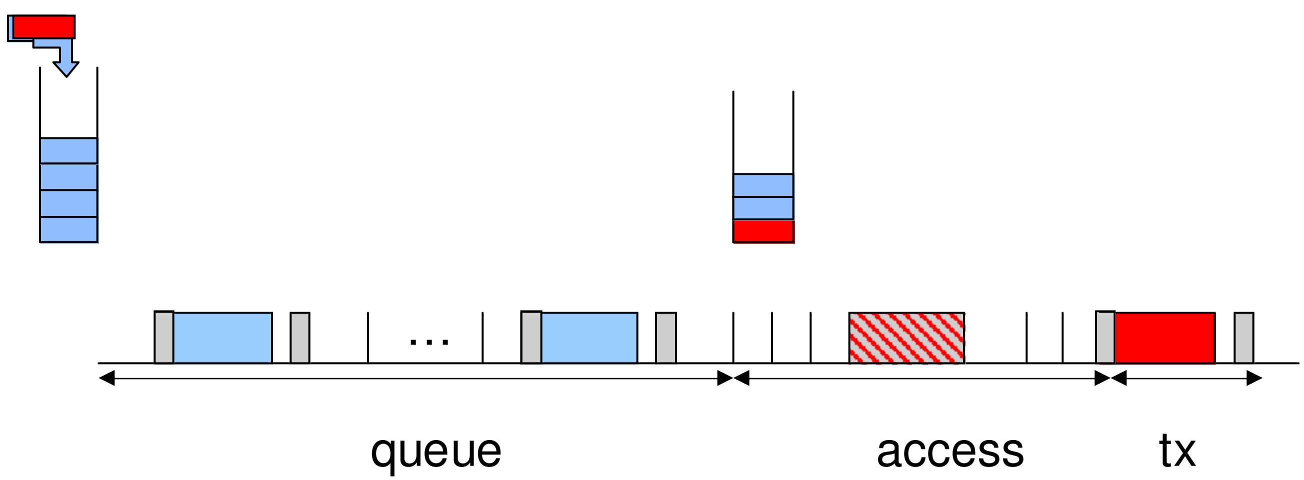
➔ **Overheads: backoff, monitoring times, collisions, headers, ACK/RTS/CTS.**

- ⇨ Payload bits transmitted in average per each second
- ⇨ Fraction of the channel time used for payload transmissions (normalized throughput)

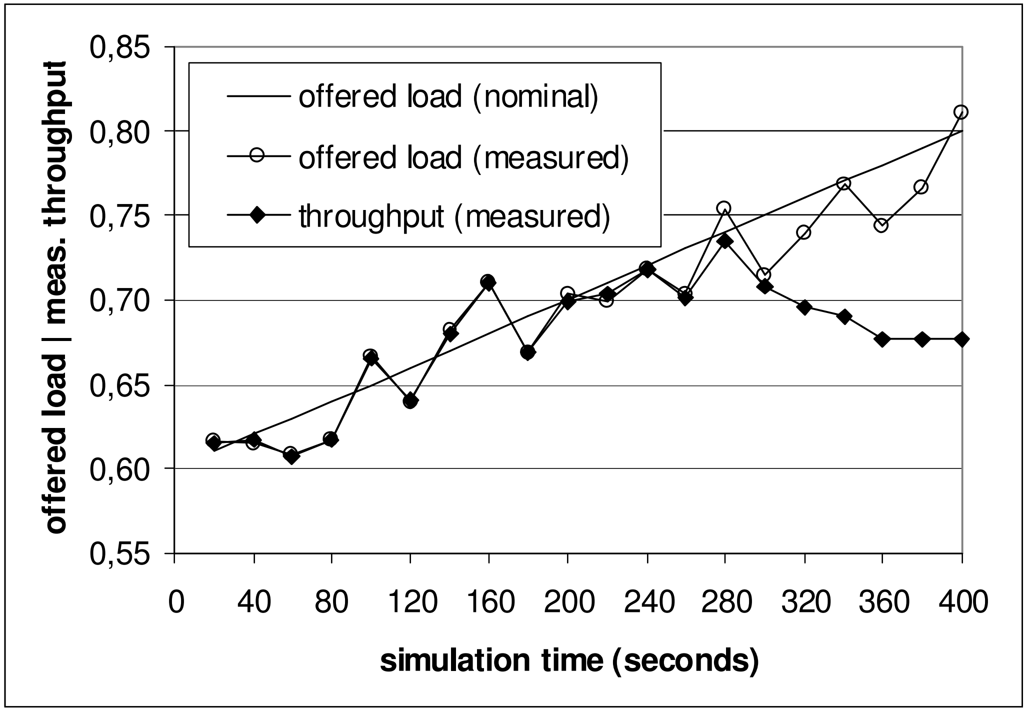


Delay

- ➔ Time required for a packet to reach the destination after it leaves the source.
- ➔ Three different components: queue delay, service access delay, transmission delay.



Saturation Analysis



Fixed number of stations, varying arrival rate

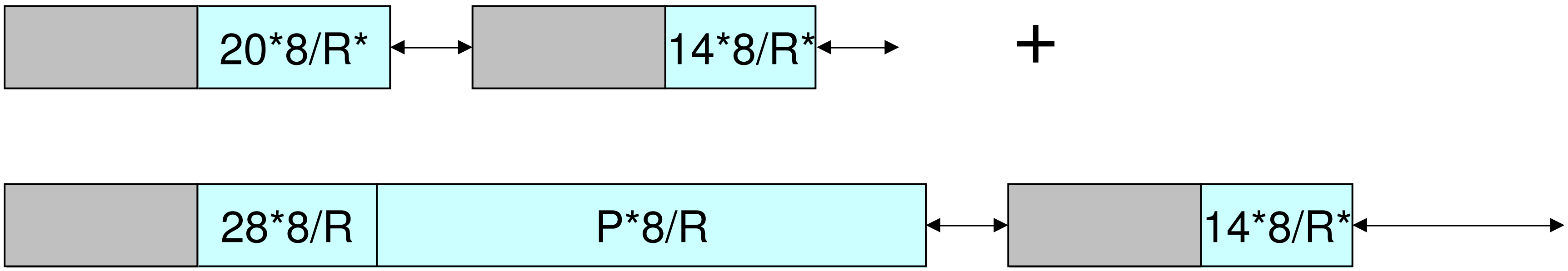
Arrival rate higher than maximum throughput -> transmission queues build up until saturation (always full)

“Saturation throughput”: limit reached by the system throughput as the offered load increases

DCF Overheads

Frame Transmission Time

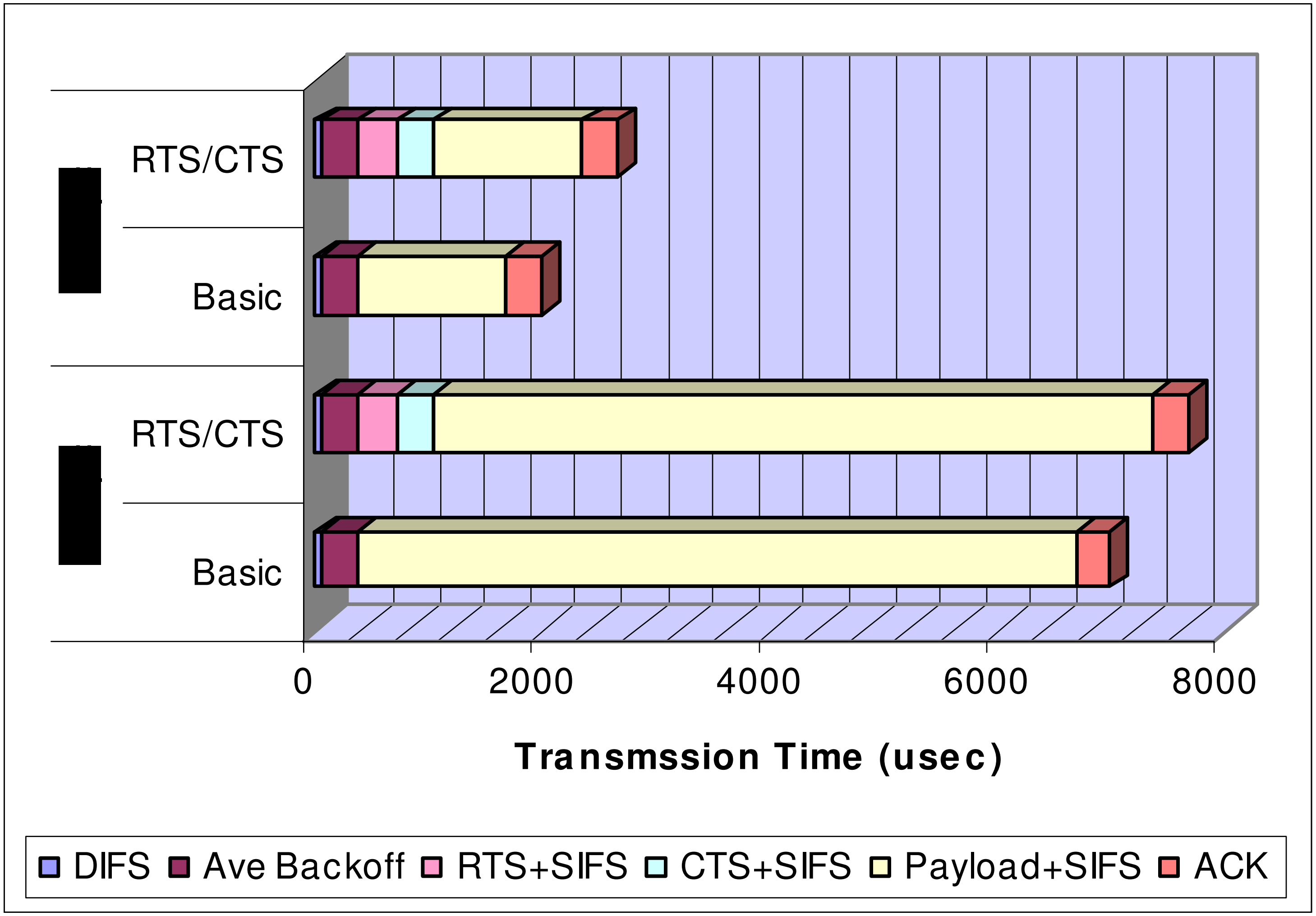
- ➔ **Let R be the data rate and R* the control rate.**
- ➔ **Let P be the MSDU size [byte].**



$$T_{FRAME} = T_{MPDU} + SIFS + T_{ACK} + DIFS$$

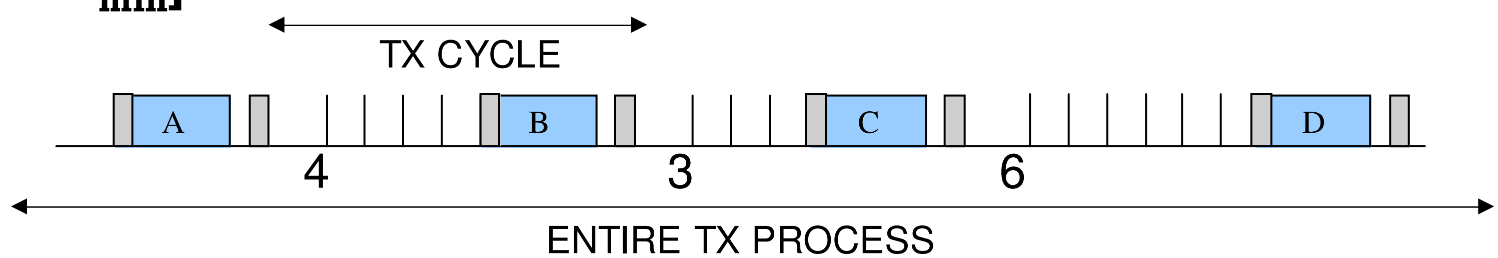
$$T_{FRAME} = T_{RTS} + SIFS + T_{CTS} + SIFS + T_{MPDU} + SIFS + T_{ACK} + DIFS$$

Overheads @ different rates
(P=1500 bytes)



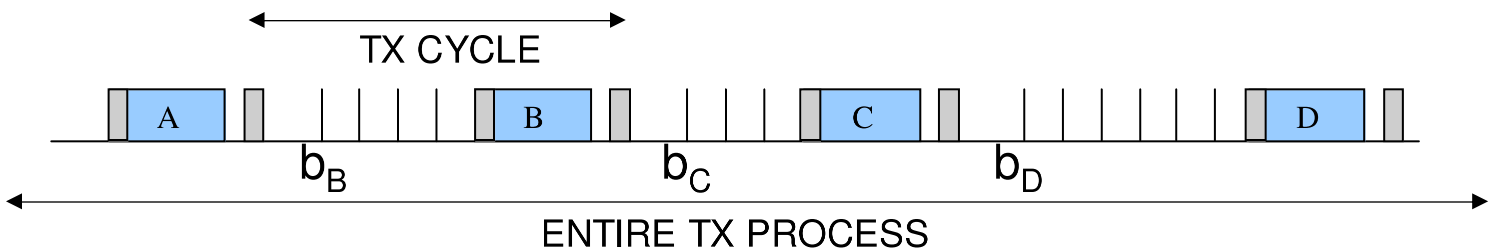
Protocol Overhead

- **Suppose to have just a single station, with a never empty queue**
- **Each transmission is originated after a backoff counter expiration**
- **Since no collision is possible, and no channel error is considered, each backoff is extracted in the range [0, CW_{min}]**



- **Different transmission cycles on the channel, composed of:**
  1) frame transmission time, which depends on the MSDU size;
  2) random delay time, which depends on the backoff extraction.

Max Throughput Computation



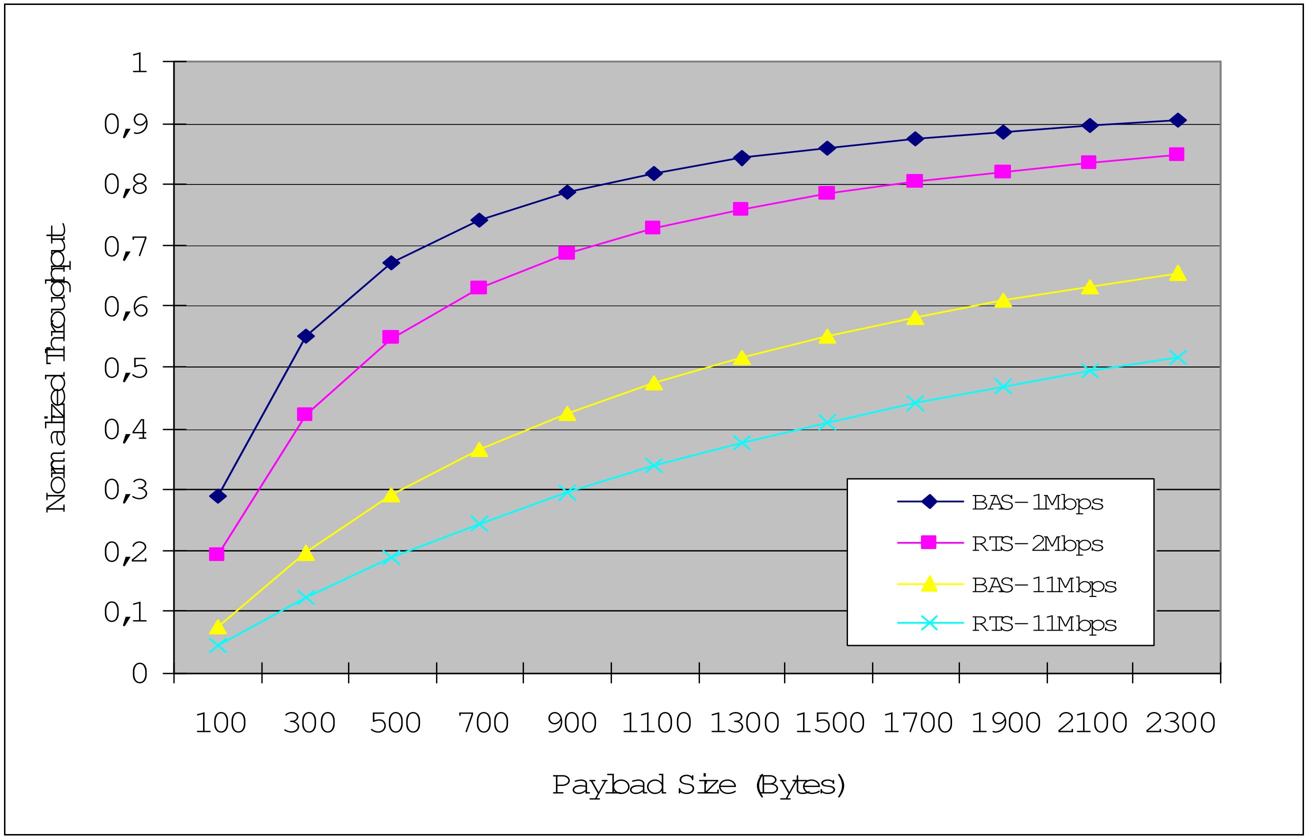
➔ **From the throughput definition:** $S = \dfrac{\sum P_i}{\sum \left(T_{FRAME_i} + b_i\right)}$

➔ **After each transmission, backoff counters are regenerated.**
From Renewal Theory: $S = \dfrac{E[P]}{E[T_{FRAME}] + E[b]}$

➔ **In the case of fixed packet size, given CW_{min}:**

$$S = \frac{P}{T_{FRAME} + \sigma CW_{\min}/2}$$

Max Throughput (normalized)



Saturation Throughput Analysis

802.11 DCF Bianchi's model approach

Step 1: Discrete-Time model of backoff for tagged STA

⇨ background STAs "summarize" into a unique collision probability value *p*

Step 2: find transmission prob. t

⇨ Result: t versus *p* non-lin function

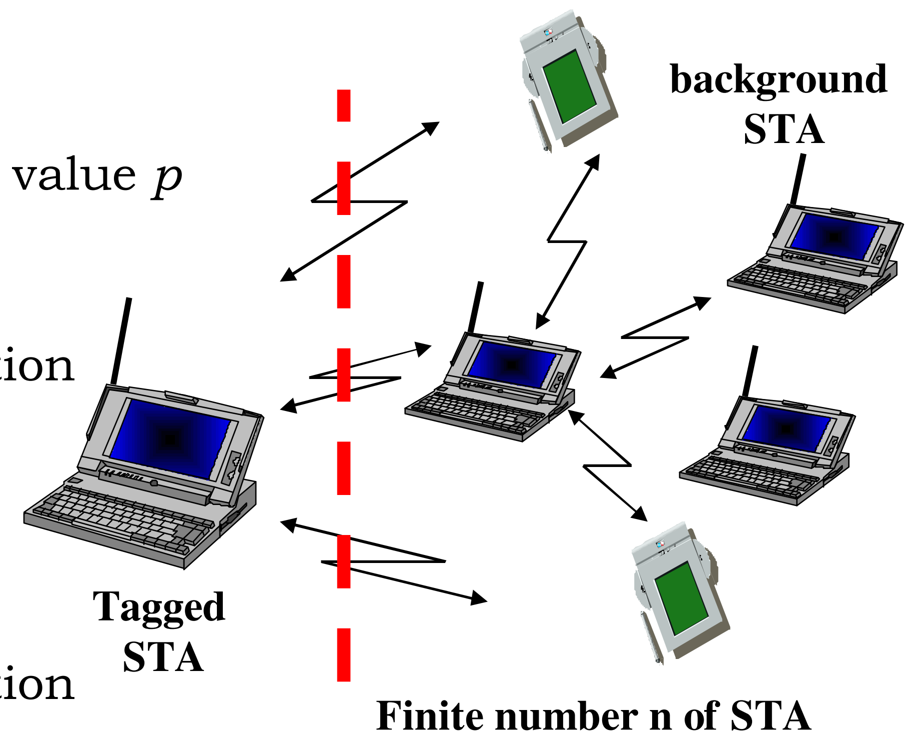
Step 3: assume background STAs behave as tagged STA, i.e. transmit with probability t

⇨ Result: *p* versus t non-lin function

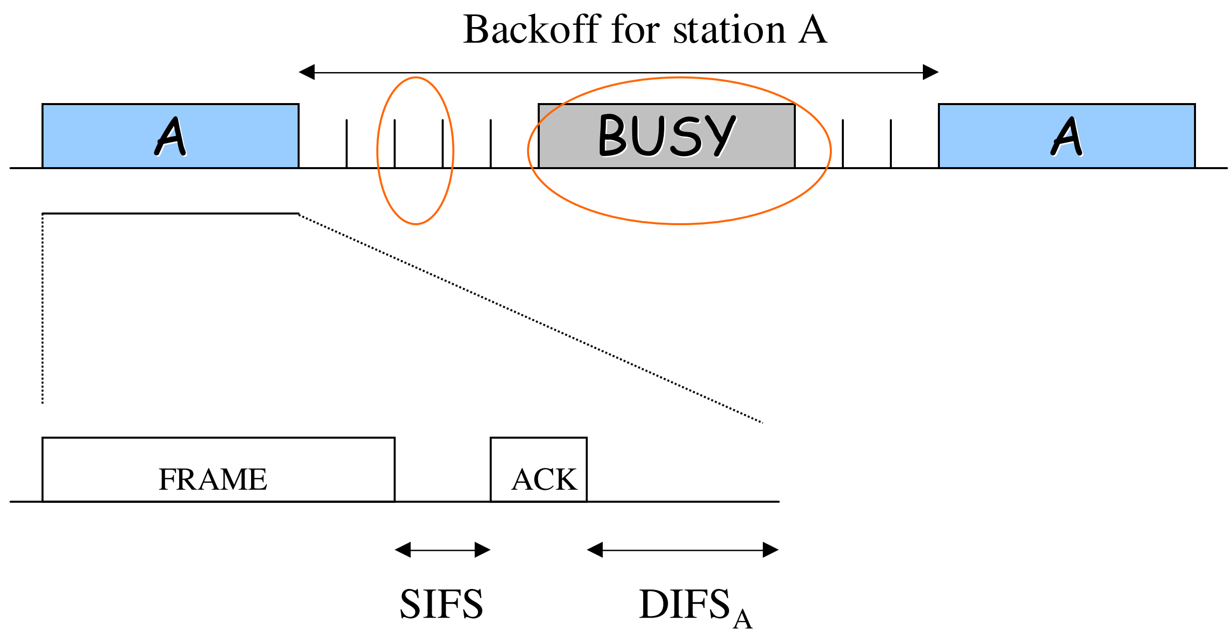
Step 4: solve non linear 2eqs system

Step 5: find performance figures

⇨ Throughput, Delay



Discrete Not Uniform Time scale

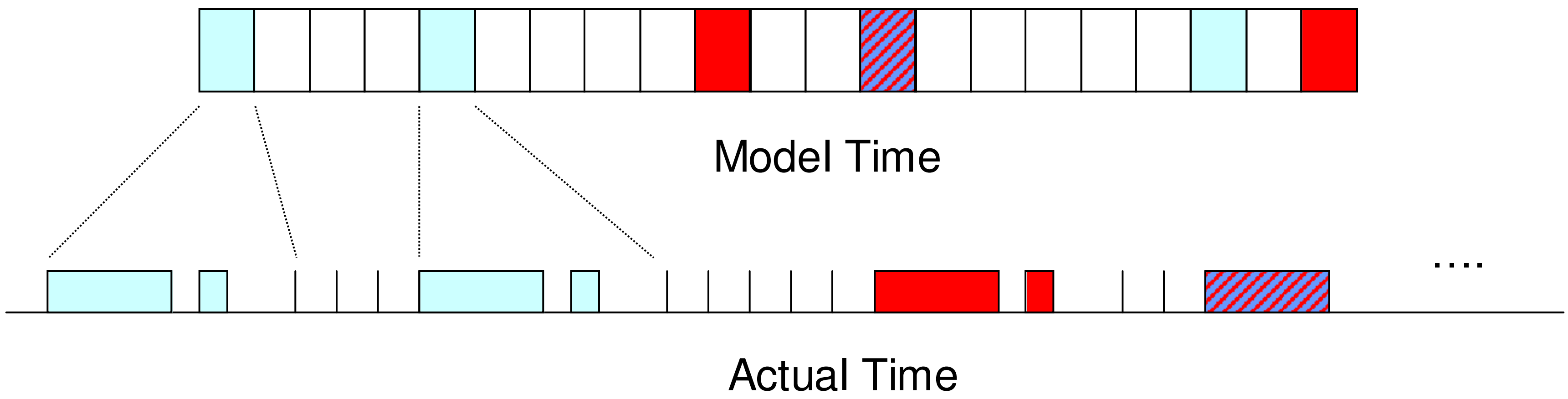


Slot time = Time interval between two consecutive backoff time counter decrements:

•idle backoff slot

•busy time + DIFS + 1 backoff slot

DCF as τ-persistent CSMA



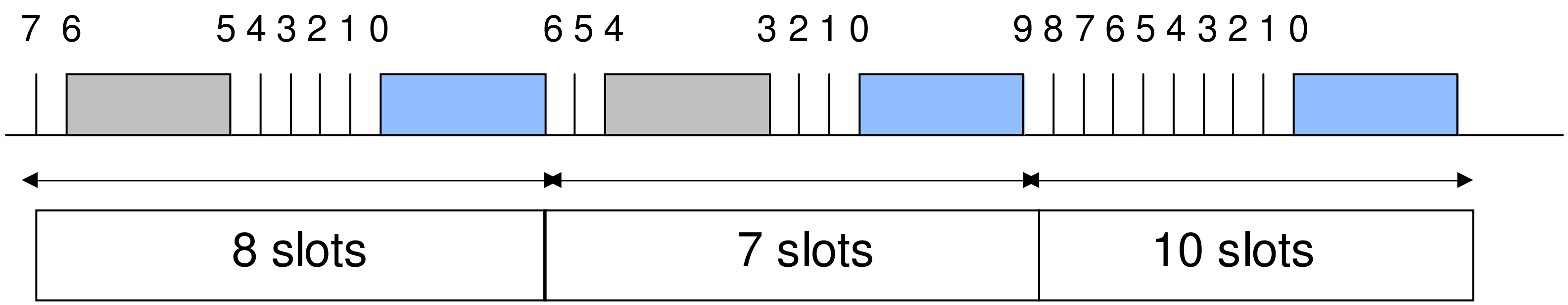
-In each system slot, each station accesses with probability τ (and does not access with probability $1-\tau$).

-Each system slot can assume 3 different sizes: idle slot, successful slot, collision slot.

-The key assumption is that τ is fixed slot by slot (and then also the collision probability p).

Channel Access Probability τ

Given the time scale, 1 transmission every backoff + 1 slot



Note that the access cycle length in discrete time slot is not related to the actual channel time (e.g., the last cycle is the shortest cycle according the channel time, but the longest one according to the model time)

Whenever $W_0=W_1=..=W_R=W$, from renewal theory: $\tau = 1 / (W/2+1) = 1/ (E[W] + 1)$

In general, the access time depends on the *backoff stage,* i.e. on the number of consecutive collisions..

N.B. Access cycle = transmission cycle for a given station

Formal derivation

$$P(TX)P(s=i/TX)=P(s=i)P(TX/s=i)$$

$$P(s=i)=P(TX)\frac{P(s=i/TX)}{P(TX/s=i)}$$

$$\sum_{i=o}^{R}P(s=i)=1\rightarrow\tau=P(TX)=\frac{1}{\sum_{i=0}^{R}\frac{P(s=i/TX)}{P(TX/s=i)}}$$

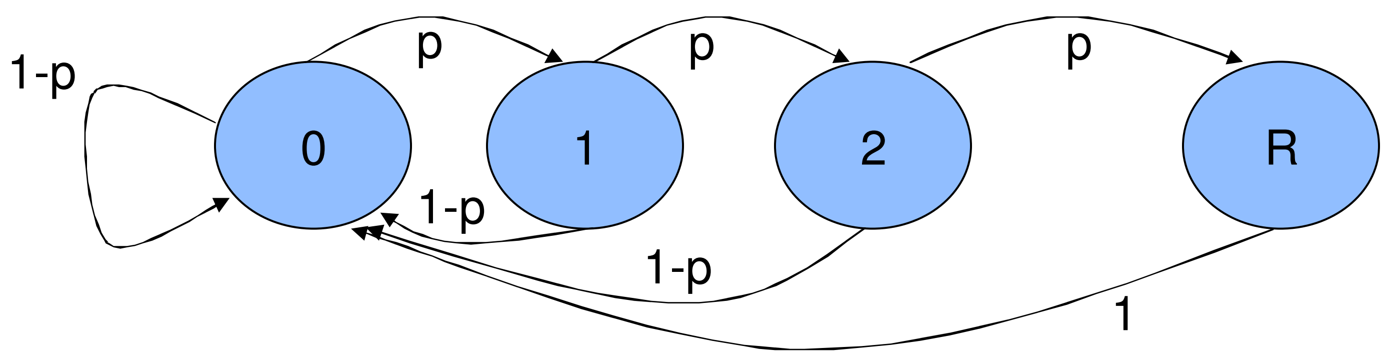
Backoff stage Probablity

Suppose to know the collision probability p:

We are in stage **i** if we were in stage **i-1** in the previous attempt and we experienced a collision: Pr(s=i/TX)=Pr(s=i-1/TX) p;

After a success or after R collisions, we come back to stage 0. Then, this probability has a geometric distribution:

$$P(s = i \mid TX) = \frac{(1-p)p^i}{1-p^{R+1}} \quad i \in (0,1,\ldots R)$$

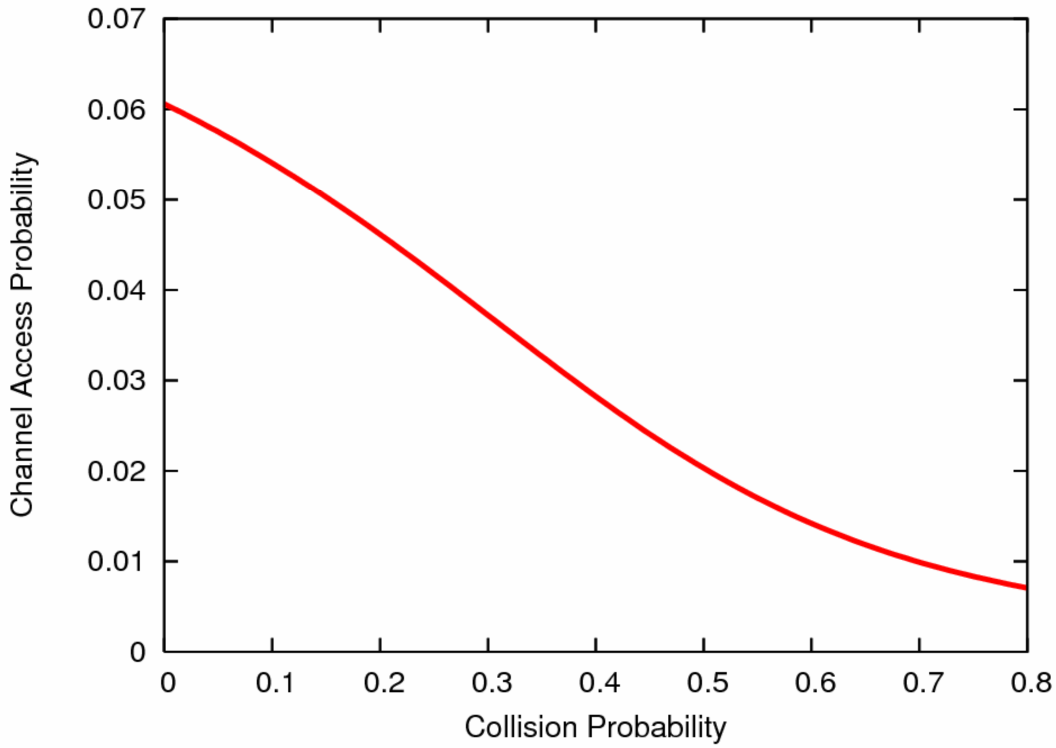


Channel Access Probability τ (2)

We can finally express the channel access probability as a function of the collision probability p and of the average backoff values $W_i/2$:

$$\tau = \frac{1}{\frac{1-p}{1-p^{R+1}} \sum_{i=0}^{R} \frac{W_i}{2} p^i + 1}$$

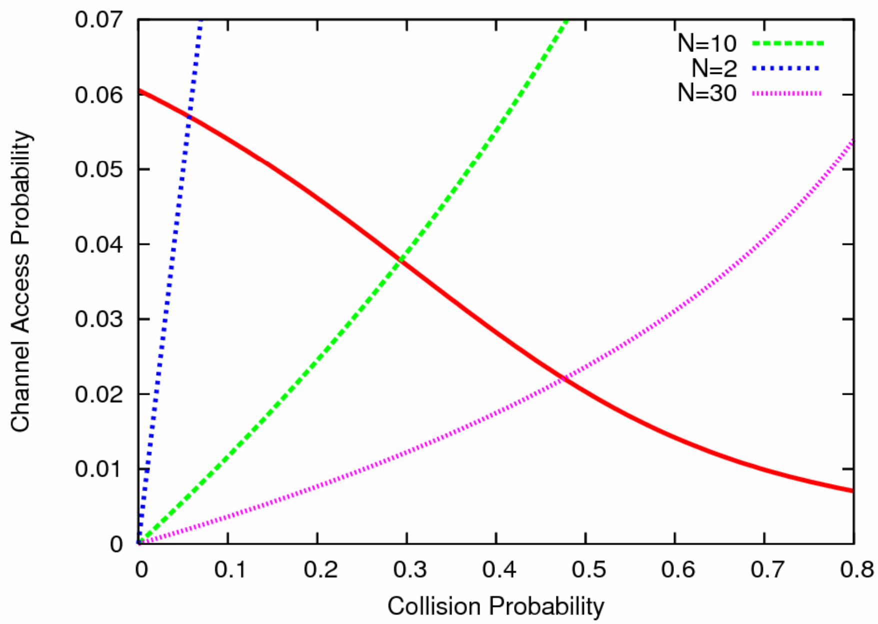
Note: τ does not depend on the backoff value distribution, but only on the average value!!!



How much is p???

The conditional collision probability p, i.e. the probability to experience a collision in a given slot, given that the tagged station is transmitting, is the probability that at least one of the other N-1 stations is accessing the channel.

If we assume that all the stations have the same behavior, and then access the channel with probability τ, it is easily expressed as: $p=1-(1-\tau)^{N-1}$



From τ&p to Throughput Performance

$$S = \frac{P_s E[P]}{P_s E[T_{SUCC}] + P_c E[T_{COLL}] + P_{idle}\sigma} = \frac{P_s E[P]}{E[slot]}$$

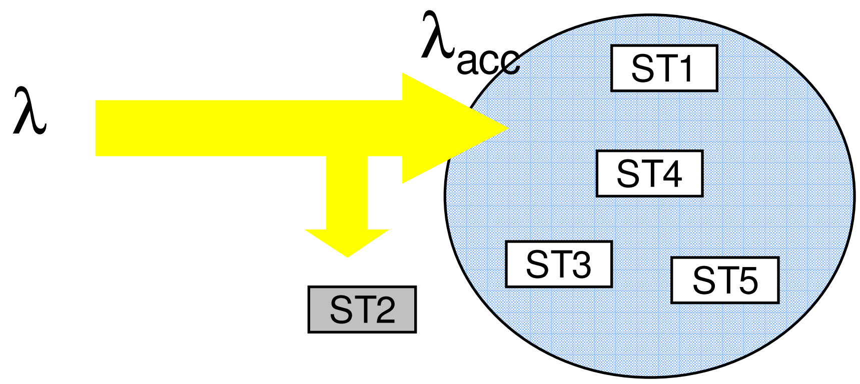
P_s: Pr success in a contention slot = $N\tau(1-\tau)^{N-1}$=N $\tau(1-p)$

$P_{idle} = 1-(1-\tau)^N$

$P_c = 1-P_S-P_{idle}$

Delay Computation

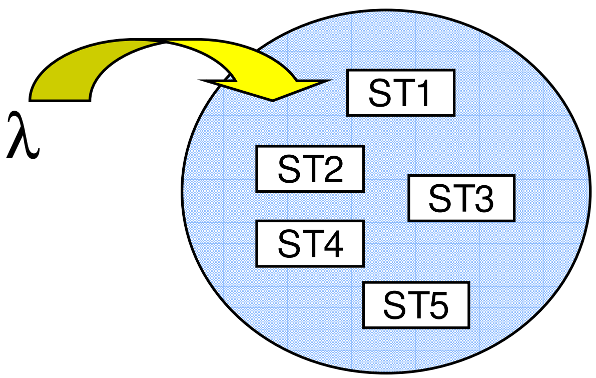
- ➔ **Very easy, via Little's Result!**
  - ⇨ Clients: Contending packets which will be ultimately delivered
  - ⇨ Server: DCF protocol
  - ⇨ Delay: system permanence time **D = E[N] /** λ_{acc}
- ➔ **In static scenarios and saturation:**
  - ⇨ A new client is accepted if the packet is transmitted before the retry limit expiration
  - ⇨ Arrival rate λ: e new packet arrives to the system :
    - →When a packet is successfully transmitted
    - →When a packet is dropped because of a retry limit expiration
  - ⇨ Accepted traffic λ_{acc}=throughput!

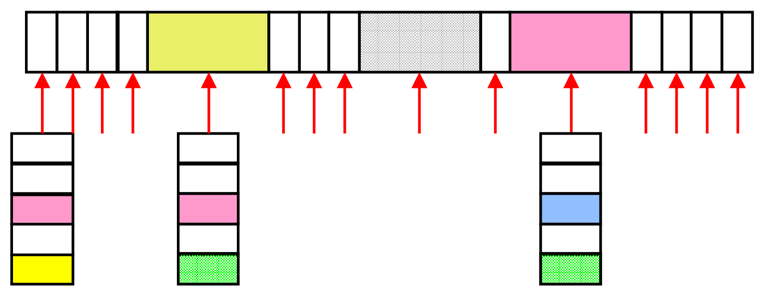


Retry Limit = ∞

➔λ = Throughput [pk/s]

$$D = \frac{N}{\lambda} = \frac{N}{S / E[P]} = \frac{N}{P_S} E[slot]$$

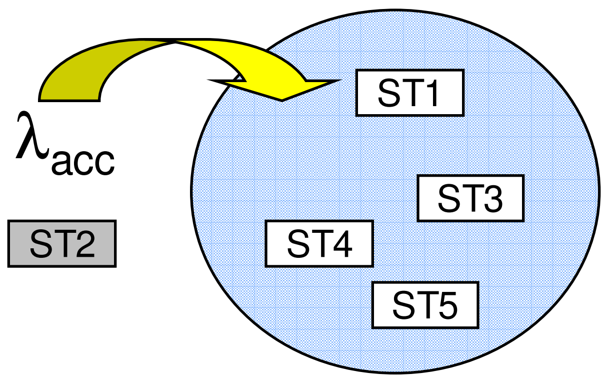


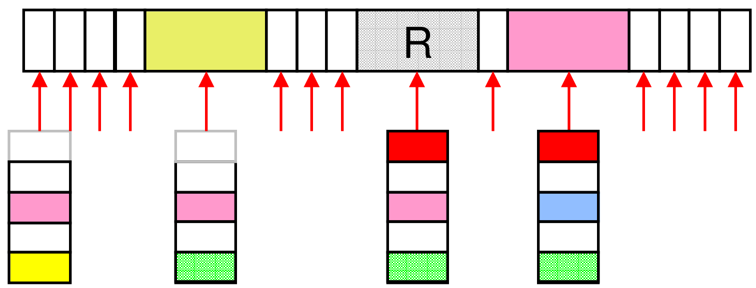


Retry Limit = R

➔ $E[N] = N(1-p_{lost})$

$$D = \frac{E[N]}{\lambda_{acc}} = \frac{N(1-p_{lost})}{S/E[P]}$$





Packet loss probability

- In a generic contention slot, with access probability τ, the packet loss probability depends on the number of already suffered collisions:

$$p_{lost} = \sum_{i=0}^{R} P(lost / s = i) P(s = i)$$

$$P(lost / s = i) = p^{R+1-i}$$

$$P(s = i) = P(TX) \frac{P(s = i / TX)}{P(TX / s = i)} = \tau \frac{(1 - p) p^{i}}{1 - p^{R+1}} (1 + W_i / 2)$$

How to take into account the 2-way or 4-way access mode?

➔ **By simply defining opportunely frame transmission times and collision times. Assuming fixed MPDU size:**

BASIC ACCESS:

$$T_{FRAME} = T_{MPDU} + SIFS + T_{ACK} + DIFS$$

$$T_{COLL} = T_{MPDU} + DIFS$$

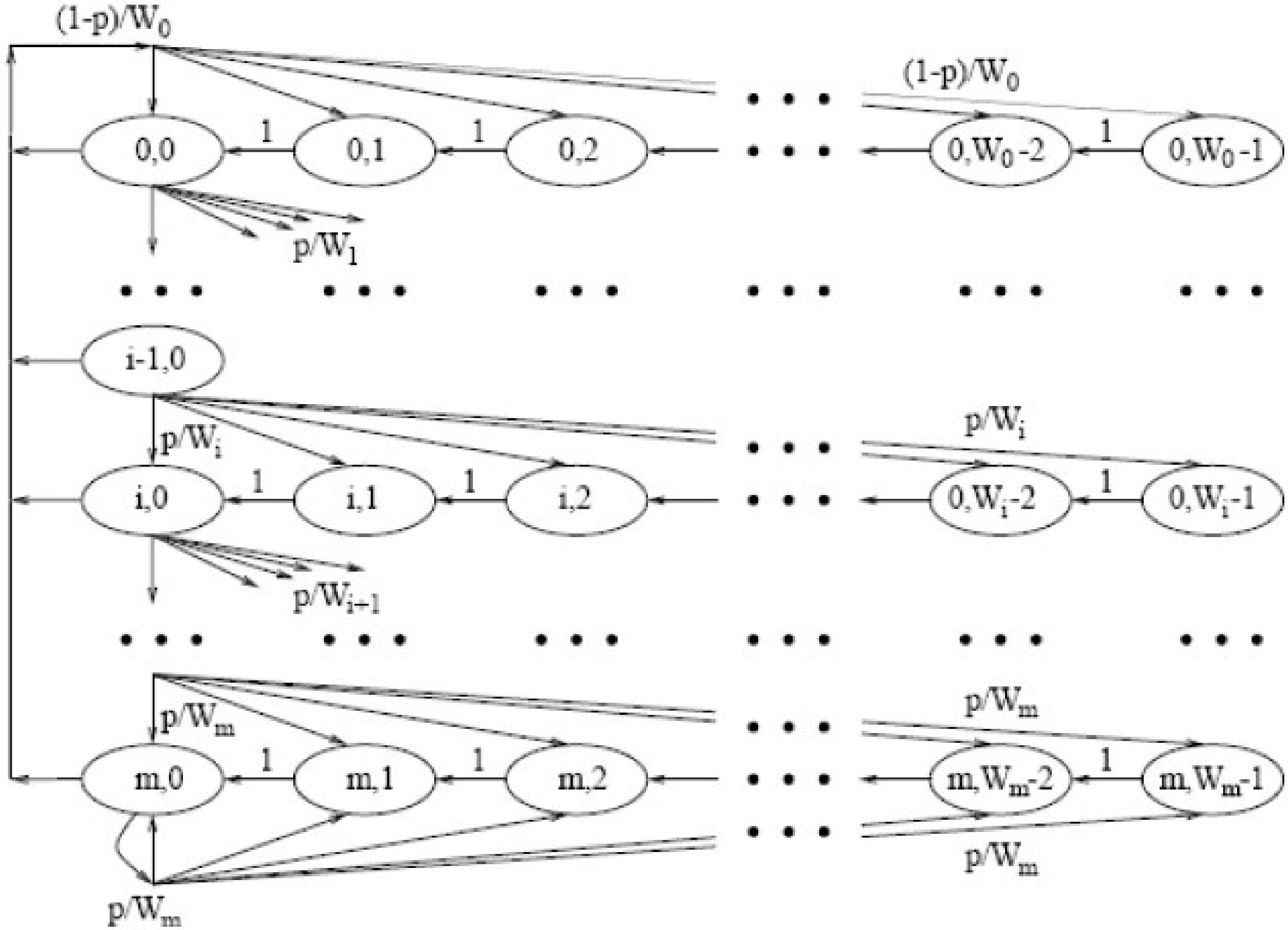
RTS/CTS:

$$T_{FRAME} = T_{RTS} + SIFS + T_{CTS} + SIFS\ T_{MPDU} + SIFS + T_{ACK} + DIFS$$

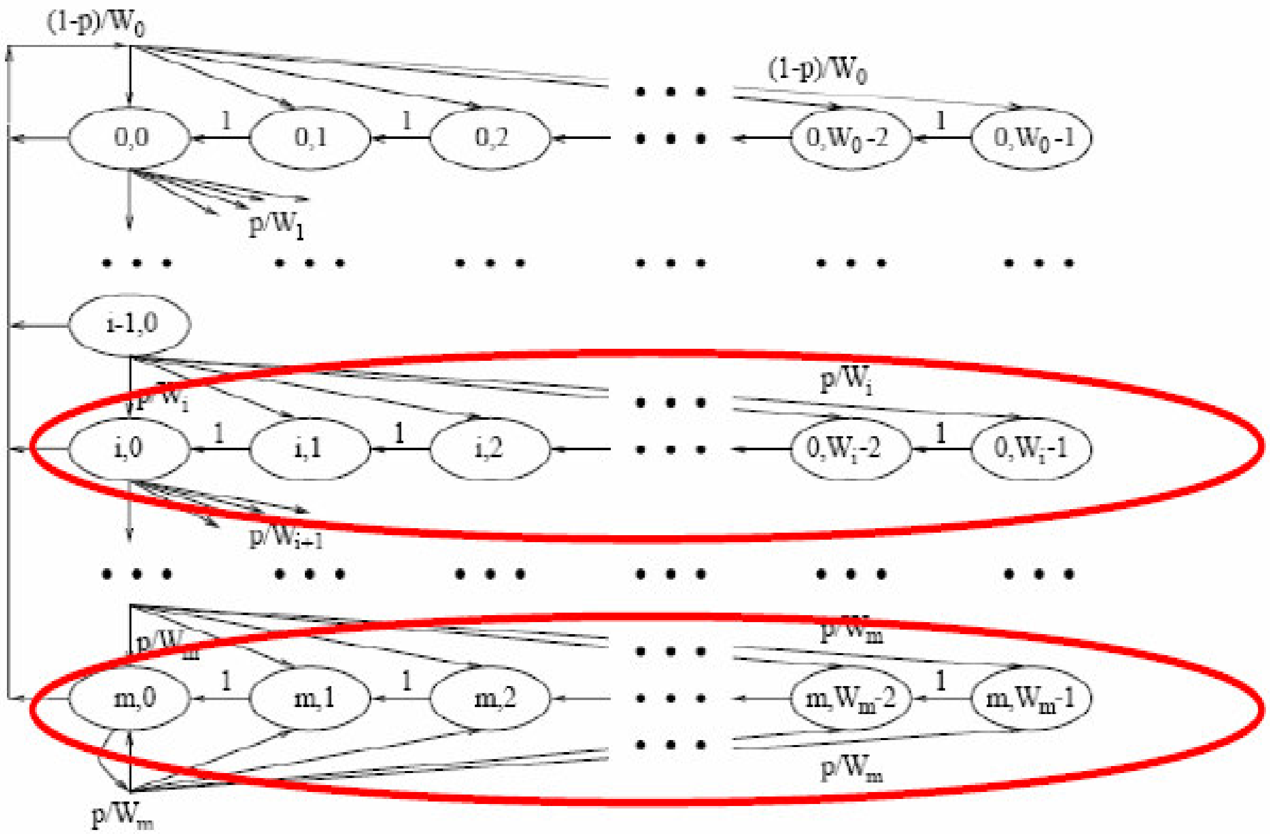
$$T_{COLL} = T_{RTS} + DIFS$$

Note: The channel access probability and the collision probability do not depend on the employed access mode!

Alternative (τ, p) derivation



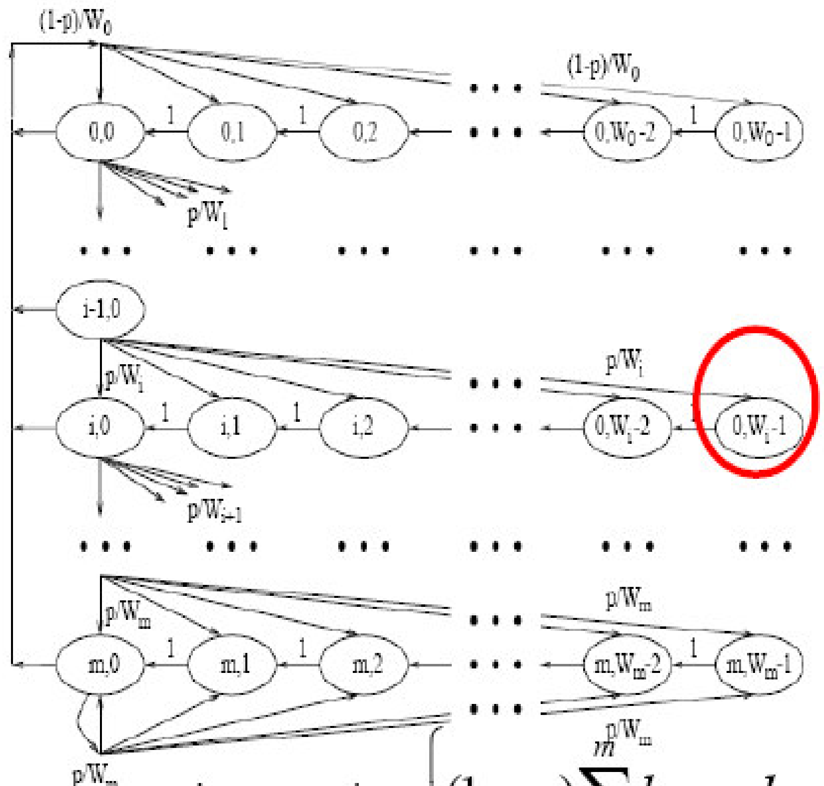
Markov Chain Solution (1)



$$b_{i-1,0}\,p = b_{i,0}\left(p + (1-p)\right) = b_{i,0} \quad \rightarrow \quad b_{i,0} = p^i b_{0,0}$$

$$b_{m-1,0}\,p = b_{m,0}(1-p) \quad \rightarrow \quad b_{m,0} = \frac{p^m}{1-p} b_{0,0}$$

Markov Chain Solution (2)



$$b_{i,W_i-1} = p\frac{1}{W_i}b_{i-1,0} = p\frac{W_i-(W_i-1)}{W_i}b_{i-1,0}$$

$$b_{i,W_i-2} = p\frac{2}{W_i}b_{i-1,0} = p\frac{W_i-(W_i-2)}{W_i}b_{i-1,0}$$

$$b_{i,k} = p\frac{k}{W_i}b_{i-1,0} = p\frac{W_i-k}{W_i}b_{i-1,0}$$

In general:

$$b_{i,k} = \frac{W_i-k}{W_i}\cdot\begin{cases}(1-p)\sum_{j=0}^{m} b_{j,0} = b_{0,0} & i=0\\ pb_{i-1,0} = b_{i,0} & 0<i<m \quad \Rightarrow b_{i,k} = \dfrac{W_i-k}{W_i}b_{i,0}\\ pb_{m-1,0}+pb_{m,0} = b_{m,0} & i=m\end{cases}$$

Markov Chain Solution (3)

$$1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k} = \sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{W_i-1} \frac{W_i - k}{W_i} = \sum_{i=0}^{m} b_{i,0} \frac{W_i + 1}{2} =$$

$$= \frac{b_{0,0}}{2} \left[W \left(\sum_{i=0}^{m-1} (2p)^i + \frac{(2p)^m}{1-p} \right) + \frac{1}{1-p} \right]$$

from which:

$$b_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)}$$

Transmission Probability τ

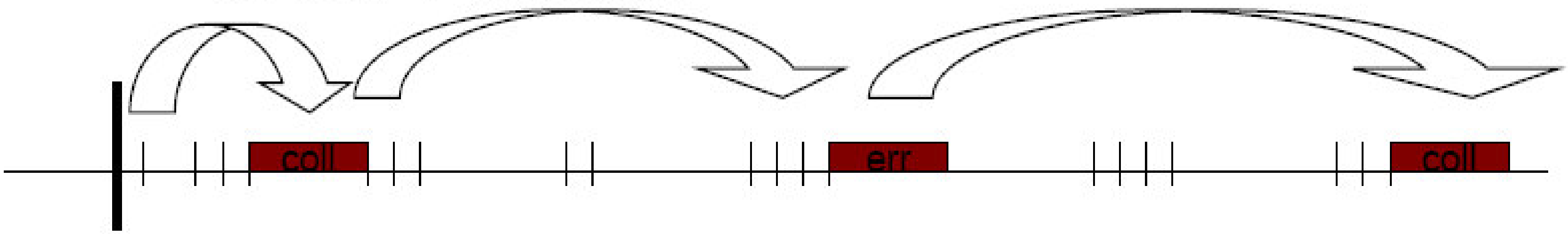
$$\tau = \sum_{i=0}^{m} b_{i,0} = \frac{b_{0,0}}{1-p} = \frac{2(1-2p)}{(1-2p)(W+1) + pW(1-(2p)^m)}$$

➔**Result:**

- ⇨We have expressed the transmission probability τ versus the conditional collision probability p
- ⇨To solve the problem we need to find an explicit value for p **as seen before!**

Error-prone channel

- ➔ **802.11 does NOT distinguish collision from wireless error**
  - ⇨ No ACK → Retransmit



- ➔ **Trivial extension if we assume:**
  - ⇨ Uncorrelated losses
  - ⇨ Constant PER value ζ
  - ⇨ Neglect RTS/CTS/ACK errors
    - →Or include all them in ζ

Error prone channel - eqs

Tau(p) expression remains the same:

$$\tau = \frac{1}{\sum_{i=0}^{R} \frac{1-p}{1-p^{R+1}} p^i \left(1 + E[b_i]\right)}$$

However p now shall include channel errors:

$$p = 1 - (1-\zeta)(1-\tau)^{n-1}$$

And the throughput computation will also account for channel errors

$$S = \frac{(1-\zeta) P_{success} E[P]}{P_{idle}\sigma + (1-\zeta) P_{success} T_s + \zeta P_{success} T_e + (1 - P_{idle} - P_{success}) T_c}$$

EDCA model

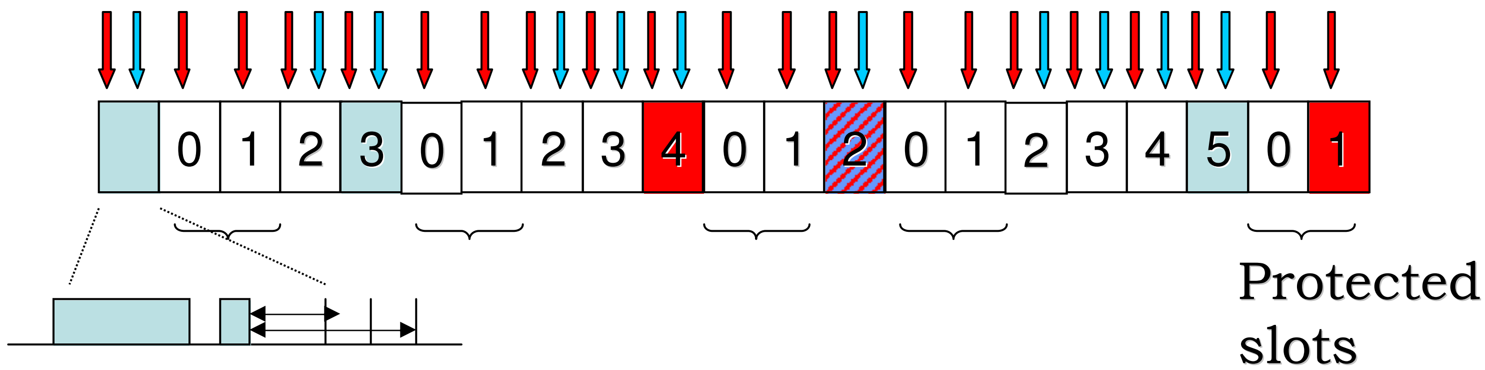
Is EDCA p-persistent?

-Proposal: differentiating τ_1 and τ_2 for different service classes

-Conceptually wrong: some slots can be accessed only by some stations and the collision probabilities p_1 and p_2 are not constant slot by slot

-Slots are numbered according to the time *n* elapsed by the last busy slot

-For each service class, we need the distribution $\tau(n)$ and p(n)



CW differentiation

Each class k, has a different minimum contention window W_k, thus resulting in a different transmission probability τ_k

$$\tau_k = \frac{2(1-2p_k)(1-p_k^{R_k+1})}{(1-2p_k)(1-p_k^{R_k+1}) + W_k(1-p_k)(1-(2p_k)^{R_k+1})}$$

As a consequence, each traffic class experiences a different collision probability p_k

$$p_k = 1 - \frac{\prod_{r=1}^{C}(1-\tau_k)^{n_r}}{(1-\tau_k)}$$

Per-class Throughput

Each class has a different successful access probability:

$$P_{success}(k) = n_k \tau_k \left(1-\tau_k\right)^{n_{k-1}} \prod_{r=1, r\neq k}^{C} \left(1-\tau_r\right)^{n_r} = n_k \tau_k (1-p_k)$$

Thus, the per-class throughput results:

$$S_k = \frac{P_{success}(k) E[P]}{\prod_{r=1}^{C}(1-\tau_r)^{n_r}\sigma + \sum_{r=1}^{C} P_{success}(k) T_s + \left(1 - \prod_{r=1}^{C}(1-\tau_r)^{n_r} - \sum_{r=1}^{C} P_{success}(k)\right) T_c}$$

Throughput Ratio

$$\frac{S_j}{S_k} = \frac{P_{success}(j)}{P_{success}(k)}$$

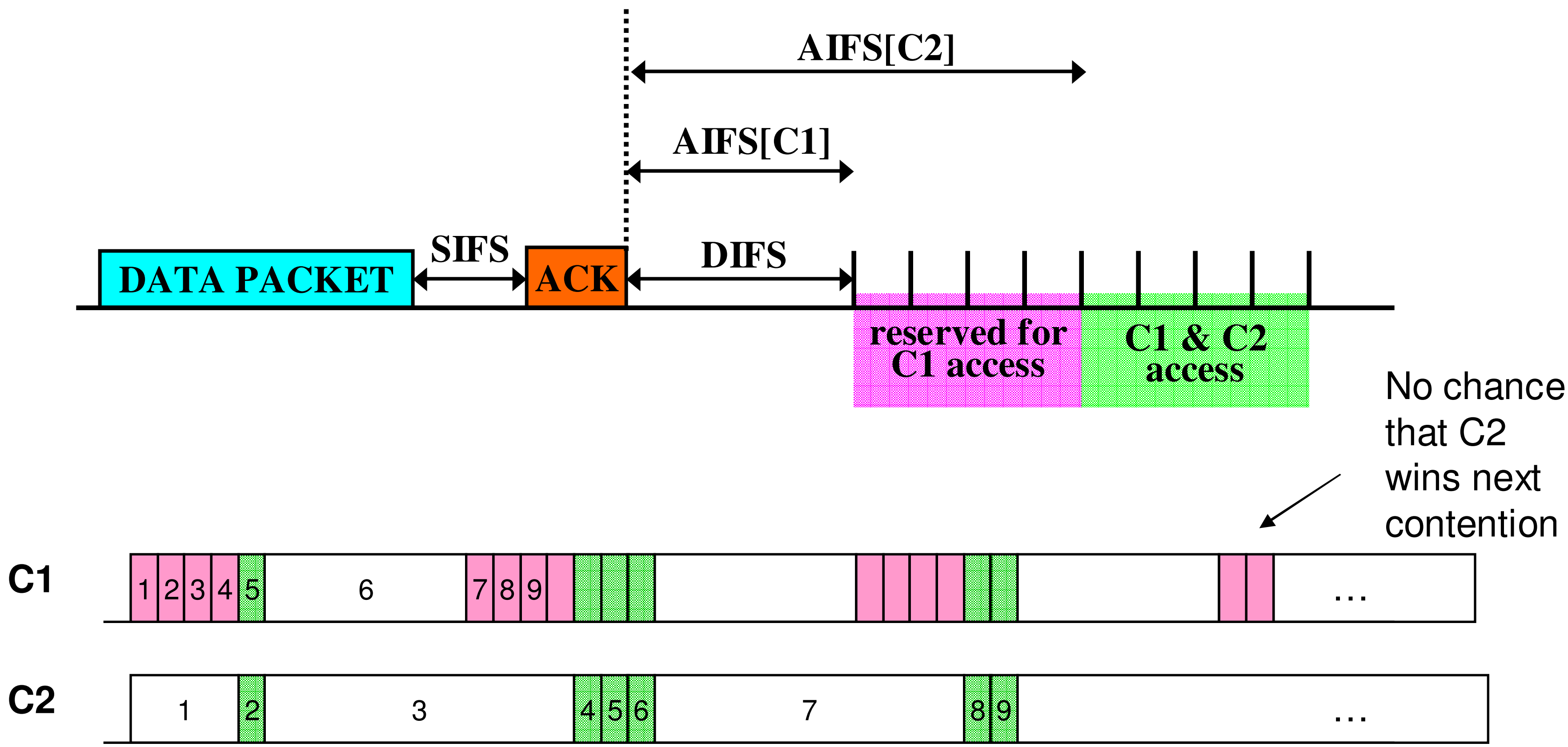
In low loaded conditions.. $P_{success}(k)=n_k\tau_k$.

$$\tau_k \approx \frac{2}{1+W_k}$$

$$\frac{S_j}{S_k} \approx \frac{n_j\tau_j}{n_k\tau_k} = \frac{n_j(1+W_k)}{n_k(1+W_j)} \approx \frac{n_j}{W_j} \cdot \frac{W_k}{n_k}$$

AIFS differentiation

In low-loaded conditions, same τ, but different available slots!

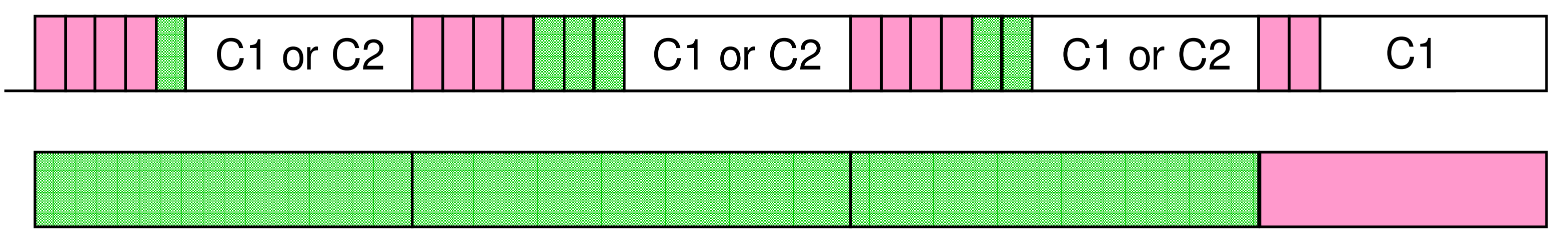


Simplified Analysis (1)

$$\tau_k \approx \tau_j \approx \frac{2}{1+W} = \tau$$

Let δ_j the AIFS[k]-AIFS[j] difference in terms of slots..

The probability that C2 stations are enabled is the probability that no C1 station transmits in δj slots



Simplified Analysis (2)

Probability to win the next contention:

C1	1	$n_1/(n_1+n_2)$
C2	0	$n_2/(n_1+n_2)$
Occurrence probability ⇨	1- $(1-\tau)^{n_k \delta_j}$	$(1-\tau)^{n_k \delta_j}$

Throughput Ratio

Neglecting collisions, throughput repartition only depends on the ratio of the channel access probabilities:

$$\frac{S_j}{S_k} \approx \frac{\dfrac{n_j}{n_j + n_k}(1-\tau)^{n_k \cdot \delta_j}}{1 - \dfrac{n_j}{n_j + n_k}(1-\tau)^{n_k \cdot \delta_j}}$$

Performance Optimizations

Maximum Saturation Throughput

$$S = \frac{P_{success} E[P]}{E[slot]} = \frac{P_{success} E[P]}{P_{idle}\sigma + P_{success} T_s + (1 - P_{idle} - P_{success}) T_c} =$$

$$= \frac{E[P]}{T_s + \sigma \frac{P_{idle} + T_c^* (1 - P_{idle} - P_{success})}{P_{success}}} =$$

For τ value that maximizes the above expression

$$(1 - \tau_{\max})^N - T_c^* \left\{ N \tau_{\max} - \left(1 - (1 - \tau_{\max})^N\right) \right\} = 0$$

Optimal τ (approx)

$$\tau_{\max} = \frac{\sqrt{1 + 2\left(T_c^* - 1\right)\frac{(N-1)}{N}} - 1}{(N-1)\left(T_c^* - 1\right)} \approx \frac{1}{N\sqrt{T_c^* / 2}}$$

$$\tau_{\max} = \frac{1}{1 + CW_{opt} / 2}$$

$$CW_{opt} \approx 2N\sqrt{T_c^* / 2} - 2 \approx N\sqrt{2T_c^*}$$

And in actual networks?

- ➔ CW_{opt}=f(load, AIFSN), **but..**
  - ⇨ Load estimation
  - ⇨ Function f evaluation
    - → f depends on the traffic sources, which need to be estimated and modeled !
    - → complex f expressions for not-saturated traffic sources
    - → no close expression for every traffic conditions..

How to compute CWopt?

➔ Our solution: not fixed CWmin, but adaptive corrections on the basis of the channel monitoring status, to force idle slots and collision equalization **[Gallagher]**

~~**CWmin $\overset{?}{=}$ CWopt**~~

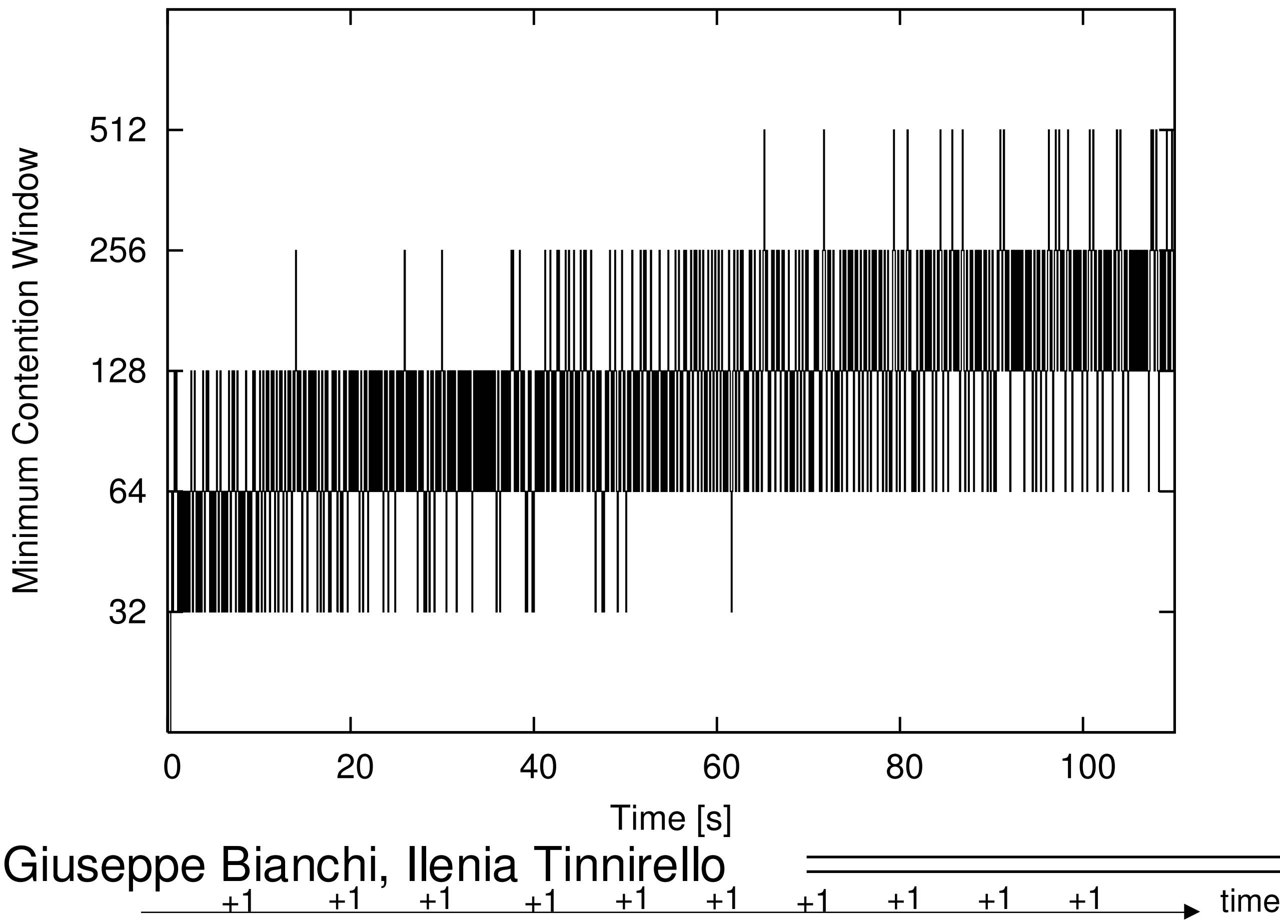
CWmin(t) = CWmin(t-1) + ΔCW

- ⇨ CW has an opposite effects on the two different events of channel wastes of the access protocol: BACKOFF, COLLISIONS
  - →Large CW -> too long backoff expirations
  - →Small CW -> too high collision probability
- ⇨ Optimal CW as a tradeoff between these channel wastes
- ⇨ different tuning algorithms are possible based on:
  - If (COLLISIONS > BACKOFF) -> increase the CWmin
  - If (BACKOFF > COLLISIONS) -> decrease the CWmin

No estimation of the system status, but simple channel monitoring of BACKOFF and COLLISIONS
Intrinsically suitable for dynamic network conditions

An example

- Assume that every 10 seconds a new data station joins the network..
- At each beacon: double the contention window at each beacon in which the collisions overcome the backoff times; half the contention window at each beacon in which the backoff overcomes the collision times.



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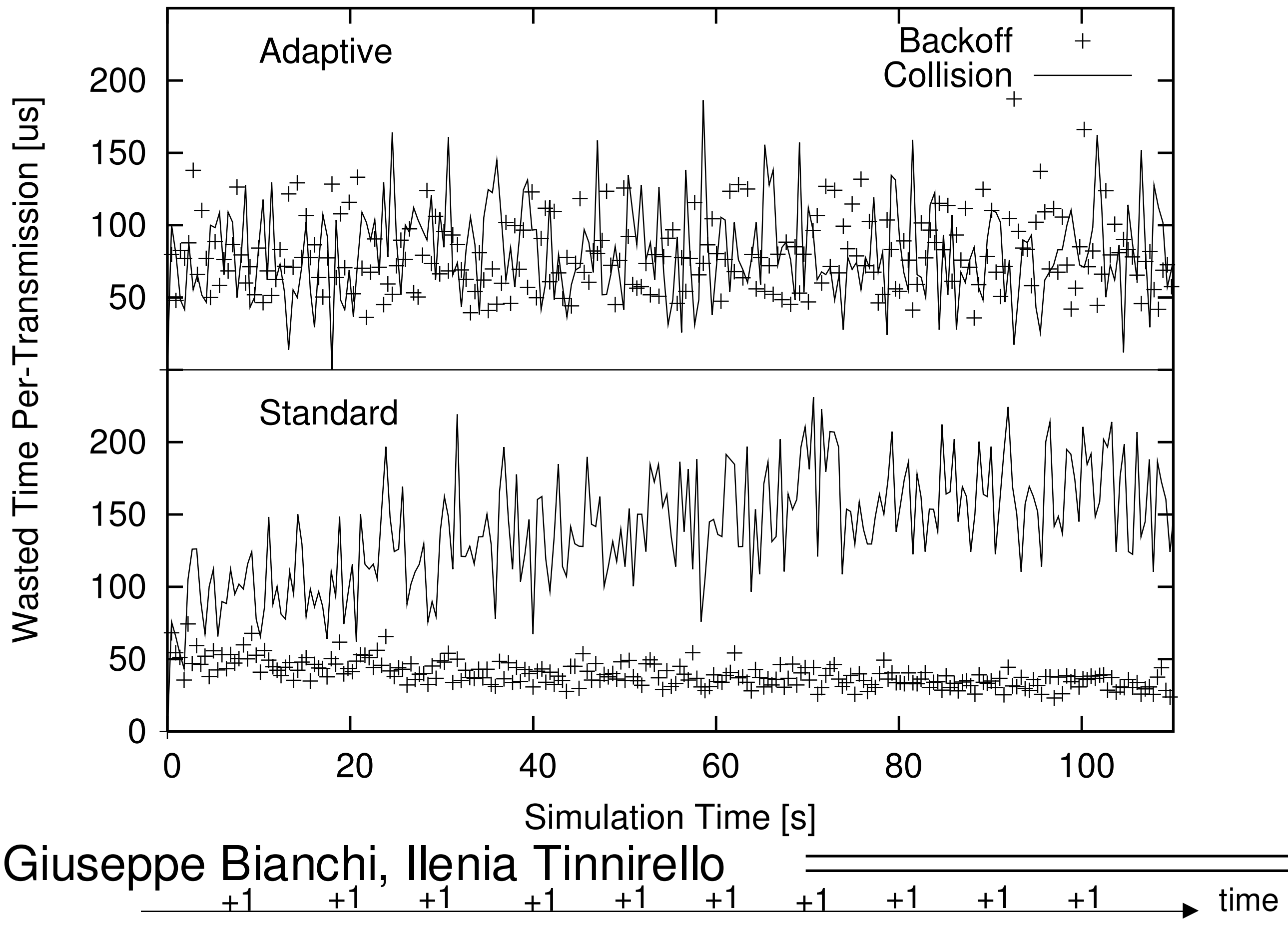
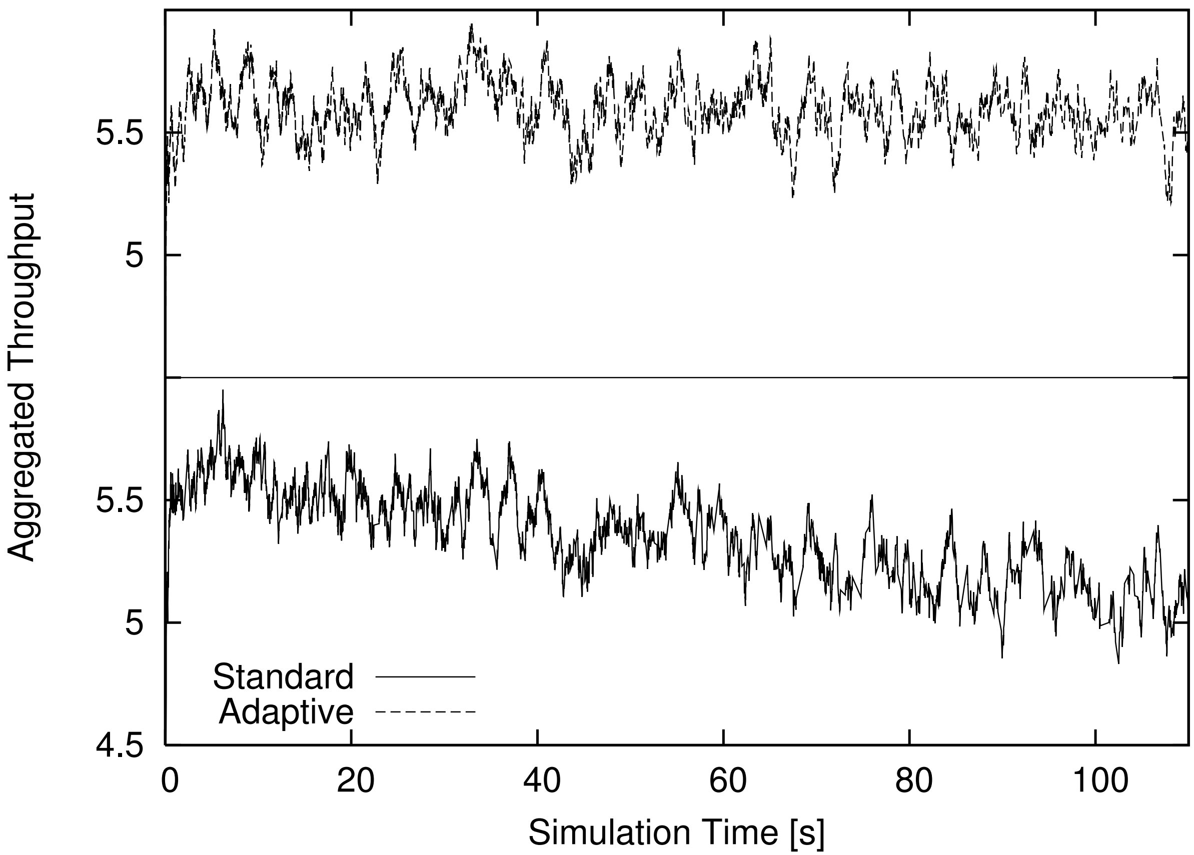
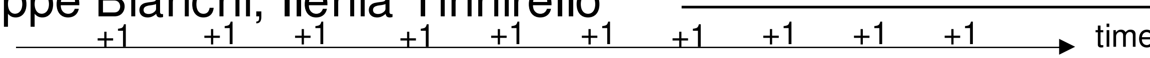


An example

- Assume that every 10 seconds a new data station joins the network..
- At each beacon: double the contention window at each beacon in which the collisions overcome the backoff times; half the contention window at each beacon in which the backoff overcomes the collision times.





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