MULTI_RADIO and its Impact on Network Connectivity and More

Stefano Basagni
Northeastern University

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Ad Hoc Networks

- Networks with NO fixed infrastructure
  - Every node is a router
- Every node can be mobile
  - According to different mobility models
    - Random waypoint, Brownian, Manhattan, etc.
  - Different speed or pause times
- Very different from cellular telephony

Ad Hoc vs. Cellular

- Centralized vs. decentralized
- Asymmetric vs. symmetric
- With vs. without infrastructure
- “Master/slave” vs. peer-to-peer
- Overall, a different architecture
  - Different handling of mobility
  - Different applications

Ad Hoc Challenges

- At all levels of the protocol stack
  - MAC: hidden terminal, exposed terminal
  - Routing: How to update table, how to find routes
  - Transport: Flow and congestion control non-trivial
  - Applications: Need for new services

Ad Hoc Applications (Some)

- Disaster recovery and law enforcement
  - Public safety networks
- Vehicular networks (VANETs)
- Mesh networking
  - Multi-hop serving the mobile users
- Wireless sensor networks (WSNs)
- ALL: One radio

Multi-radio: Why?

- Multiple radio interfaces in a single node --- technically feasible today (802.11, Bluetooth…)
- Can be viewed as a network with ***multiple physical layers***
- Even a single interface can logically act as multi-radio (several channels)
- Future systems may combine physical layers in even more varieties (radio + infrared + laser…)
- Generates many new research issues
- ***Question:*** Does the added complexity pay off in network performance?
  - *Can we somehow quantify the gain?*

Possible Use for Multiple Radios

- Devices with multiple interfaces could use them for common tasks to achieve:
  - Reliability: One interface stops working and another substitutes the first one
  - Efficiency: Closer nodes are contacted via the “best” interface (e.g., the one that minimized power consumption)
  - Bandwidth increase: Splitting a single communication on multiple channel

Specifying the Question

- Gain in general network performance - too ambitious as a first step
- We focus on the gain in *network connectivity*
- The degree of connectivity is important for performance: related to fault tolerance, routing, load balancing etc.

Model

- Network topology with multiple physical layers → ***multigraph***
- Parallel edges between nodes represent various physical layers
- Basic merging operation: multigraph sum

$$G = G_1 \uplus G_2$$

- For more graphs:

$$\uplus_{i=1}^{N} G_i = G_1 \uplus G_2 + \cdots + \uplus G_N$$

A Curious Property: The "Multigraph Advantage"

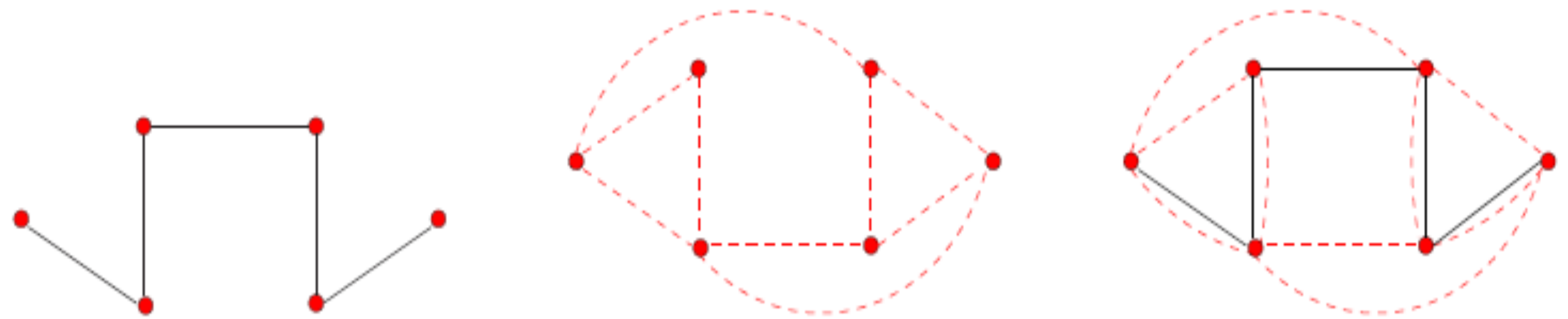
- We explain it through the property of edge-connectivity
- Edge connectivity *λ(G):* min number of edges that need to be deleted to disconnect the graph – important parameter
- First insight: we always have

$$\lambda\left(\biguplus_{i=1}^{N} G_i\right) \geq \sum_{i=1}^{N} \lambda(G_i)$$

- Why? Simply because any cut in the multigraph is the disjoint union of the corresponding cuts in the components

The Surplus

- First we might guess that equality holds, that is, *λ(G)* is *additive*
- But check out this example:



(a) G_1: $\lambda(G_1) = 1$ (b) G_2: $\lambda(G_2) = 2$ (c) $G = G_1 \uplus G_2$: $\lambda(G) = 4$

- What we find is that *λ(G)* is *superadditive!*

Analysis

- Is this gain *typical* or it shows up only in a few specially chosen examples?
- Will it occur in a *large, random* network topology?
- We analyze the multigraph advantage asymptotically in a *random graph* model

The random graph model

- There are many random graph models → which one to choose?
- As a first step, we choose the Erdos-Renyi model, in which edges are picked independently at random
- In some situations it better captures the radio network than the distance based geometric random graph model:
  - When random obstacles are the main reasons for missing links, not distance
  - Realistic propagation models tend to decrease link correlations → becomes similar to Erdos-Renyi model
  - When power control counterbalances the effect of distance

Some background on random graphs

- $G_{n,p}$: a random graph on n nodes with edge probability $p=p(n)$
- Asymptotically almost surely connected if and only if

$$p(n) = \frac{\log n + \omega(n)}{n}$$

where $\omega(n)$ tends to infinity

- Moderately dense regime:

$$p(n) = \frac{c \log n}{n}$$

with $c>1$.

→ Only constant time more dense than the min needed for connectivity

Main result

Theorem 1 *Let G_{n,p_1} and G_{n,p_2} be independently drawn random graphs in the moderately dense regime, on the same set of nodes. Let their edge probabilities be*

$$p_1(n) = \frac{a \log n}{n} \quad \text{and} \quad p_2(n) = \frac{b \log n}{n}$$

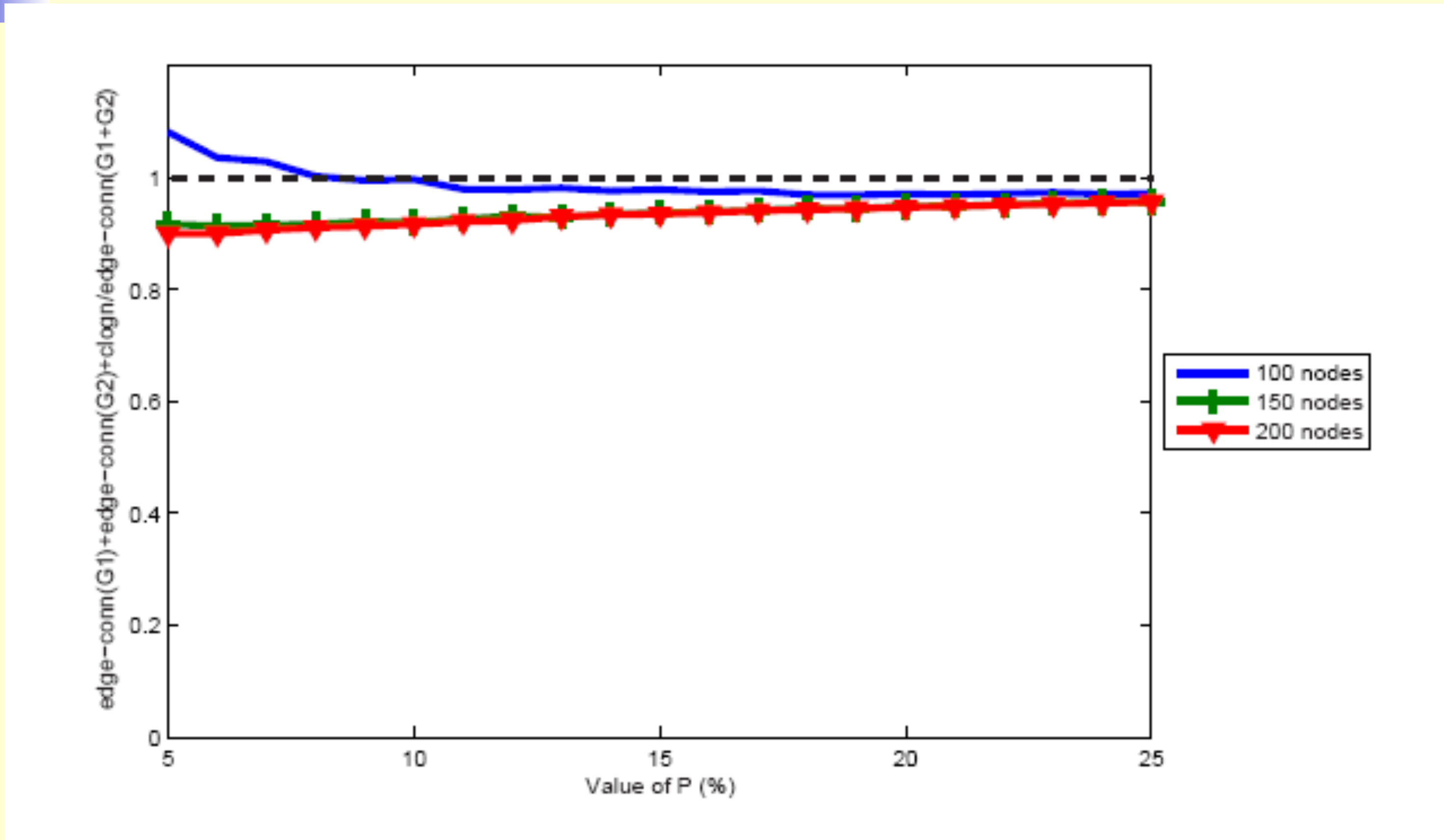
with constants $a, b > 1$. Then there exists a constant $c = c(a, b) > 0$, such that the asymptotic multigraph advantage regarding connectivity is at least $c \log n$. That is,

$$\lim_{n \to \infty} \Pr\left(\frac{\lambda(G_{n,p_1} \uplus G_{n,p_2})}{\lambda(G_{n,p_1}) + \lambda(G_{n,p_2}) + c \log n} \geq 1 \right) = 1$$

Interpretation

- The gain is comparable to the original connectivities, as they are known to be $O(\log n)$ in this regime
- There is a non-vanishing relative gain: a constant percentage gain in connectivity
- The value of $c=c(a,b)$ in the $c \log n$ gain can be computed via solving a nonlinear equation

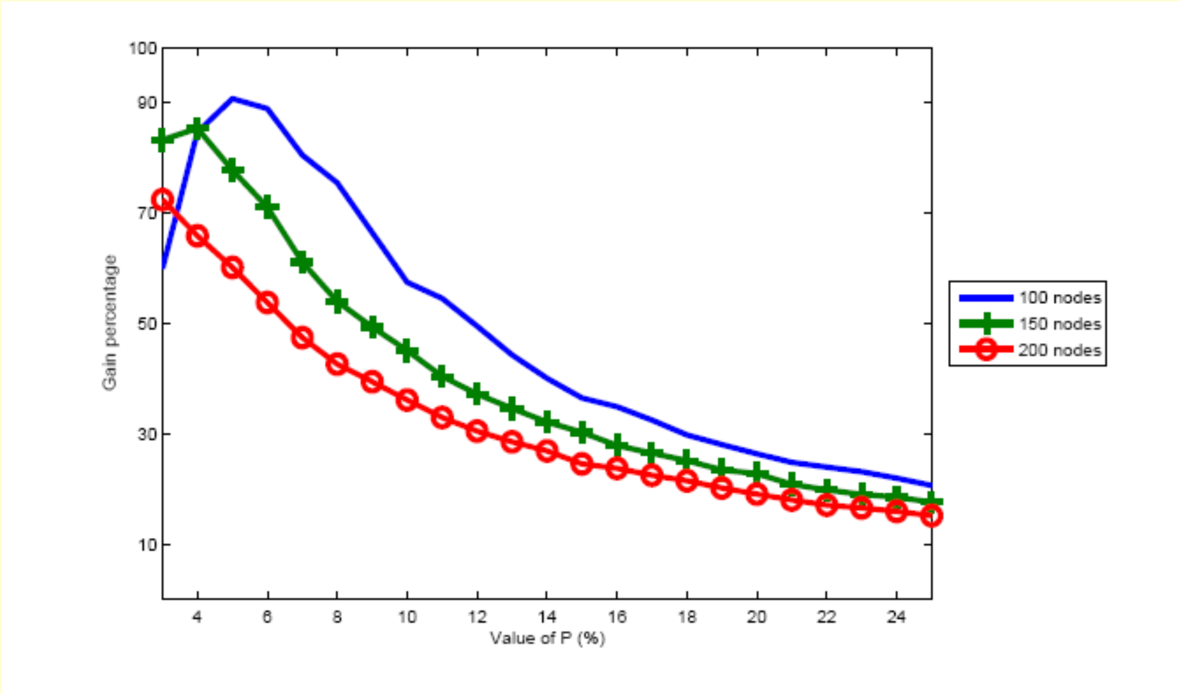
Experimental validation



→Our asymptotic formula well approximates the actual multigraph gain, already for relatively small networks

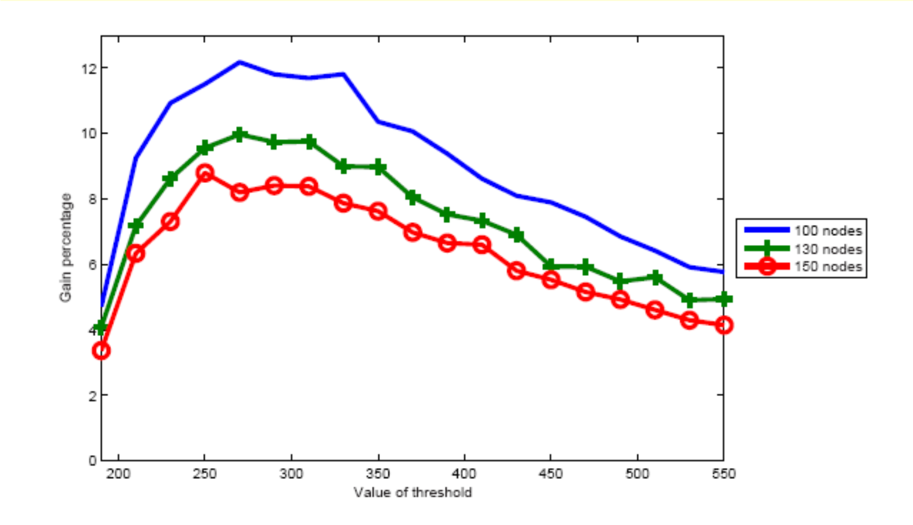
Further experiments

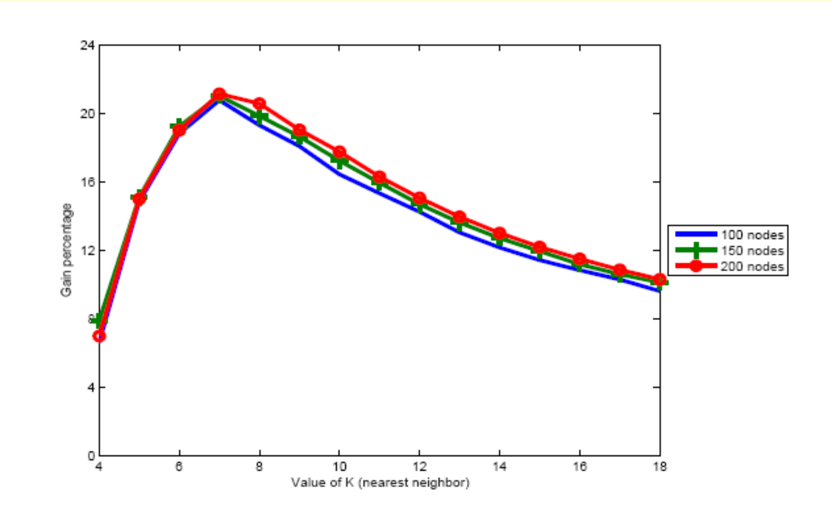
- Dependence of gain on graph densities. Two random graphs: the first with fixed p=0.05, the second with varying p=0.03… 0.25



Experiments with geometric random graph models

- Similar phenomena observed in geometric models





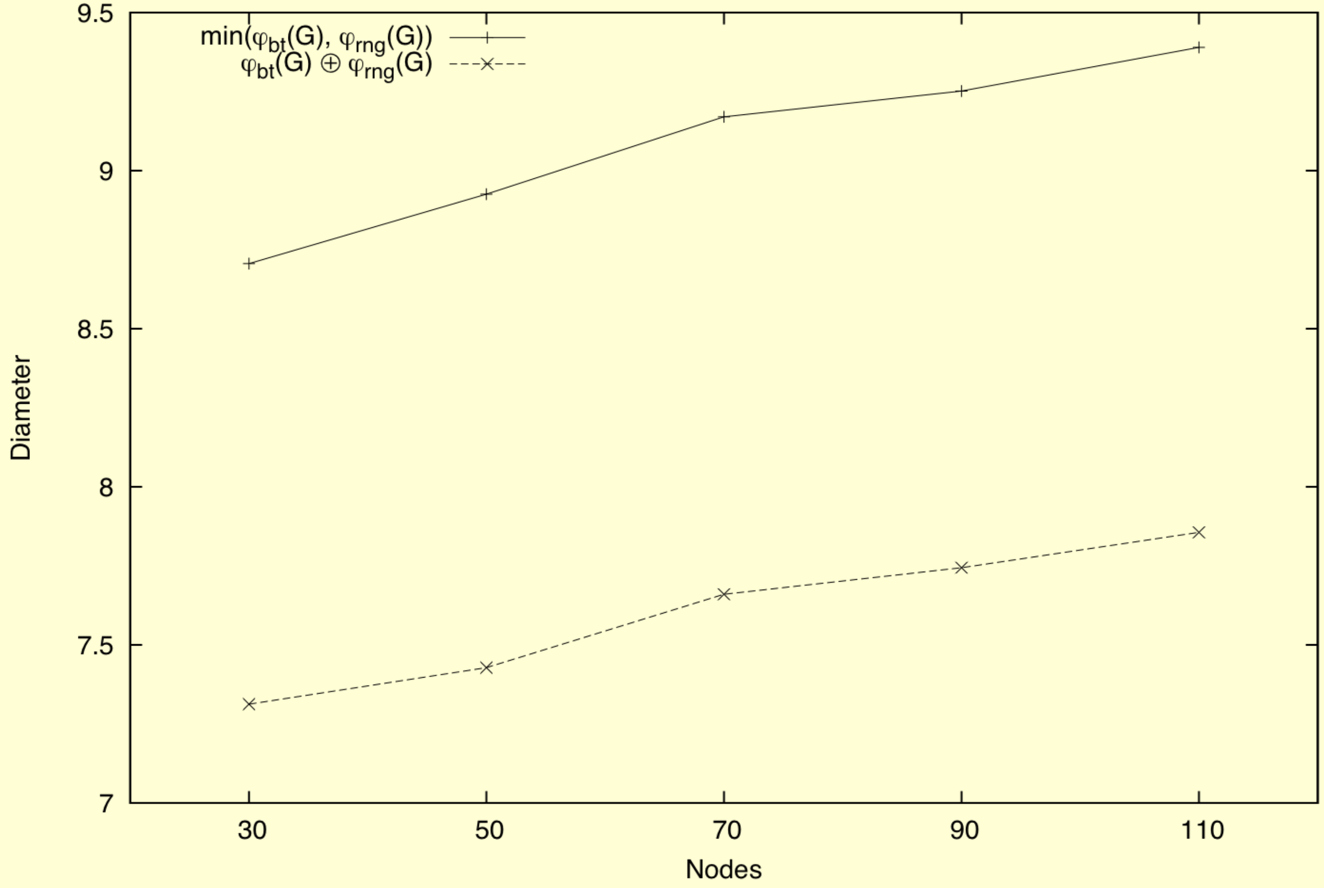
Geometric random graph

k-nearest neighbor graph

→ There is an optimal density difference that maximizes the multigraph gain

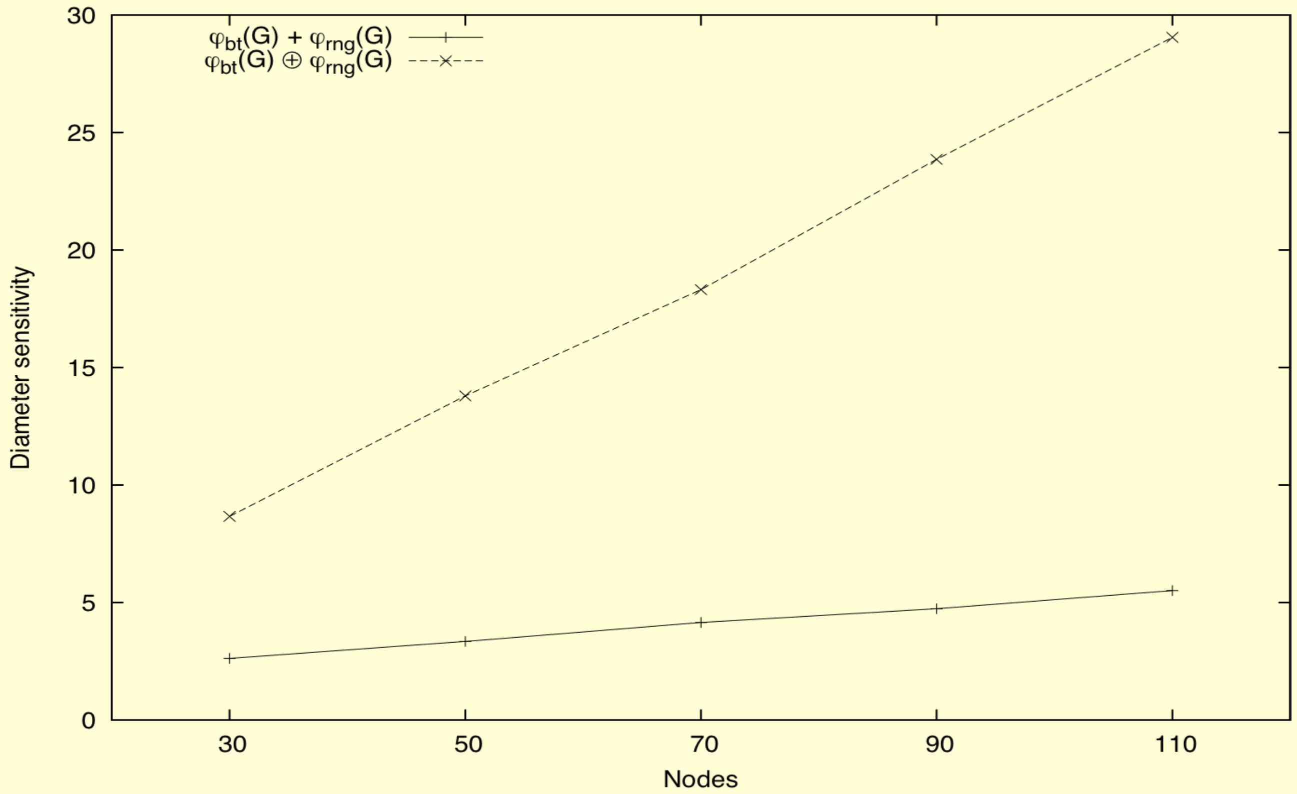
Network diameter

- The longest among the shortest paths



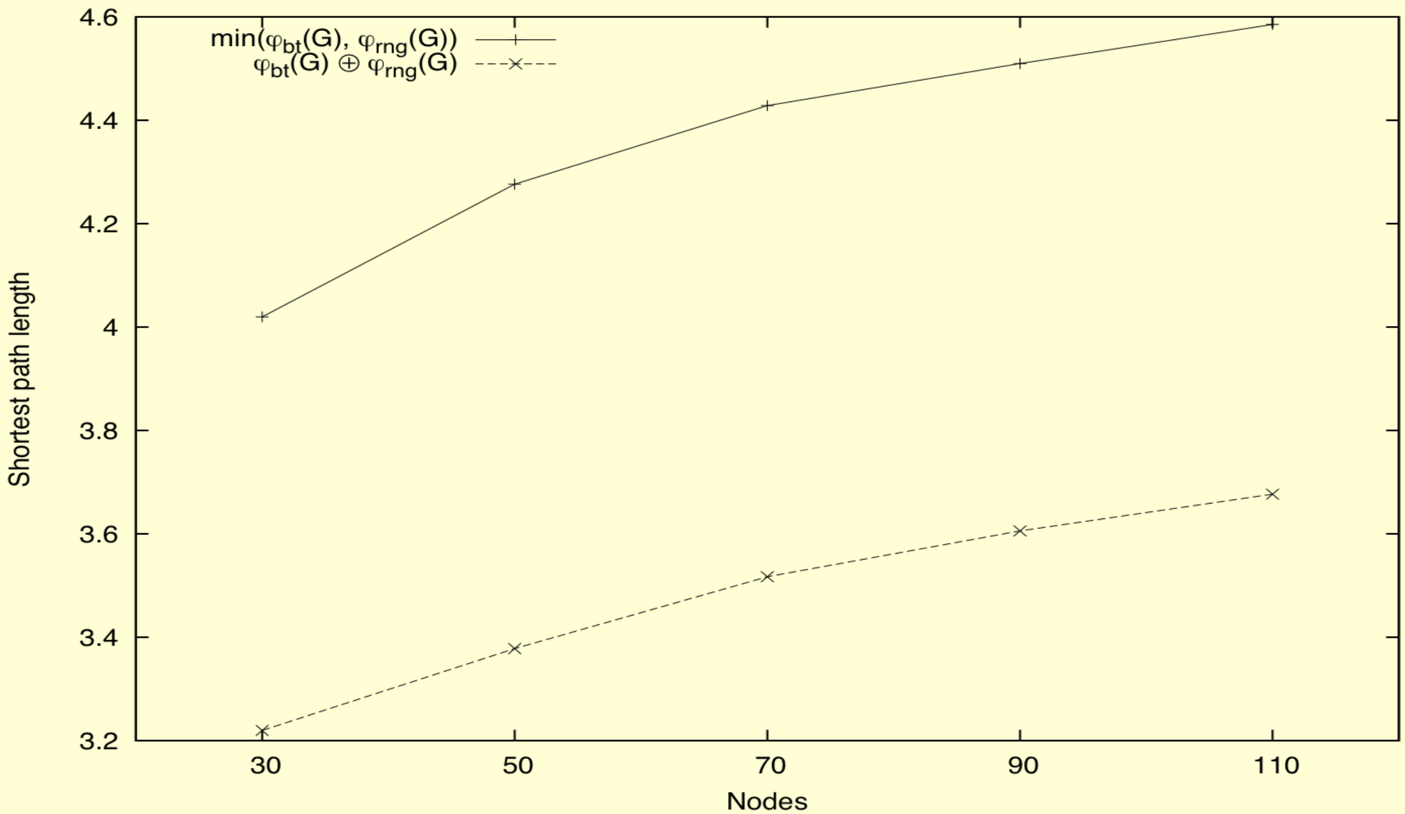
Network diameter sensitivity

- Number of links whose removal makes the network diameter increase



Shortest paths

- Average of the minimum distance between all pairs of nodes



Conclusion

- Analyzed the effect of multiple radio interfaces (multiple physical layers) on connectivity, captured by a multigraph model
- Quantified the multigraph advantage for connectivity
- Experimentally showed that it extends to more complex random graph models, too
- Further work
  - Extend the formal analysis to other random graph models
  - Quantify the gain for other parameters, such as average distance, diameter etc.