



Reti cellulari GSM

Reti Avanzate, a.a. 2012/2013

Un. of Rome "La Sapienza"

Chiara Petrioli[†]

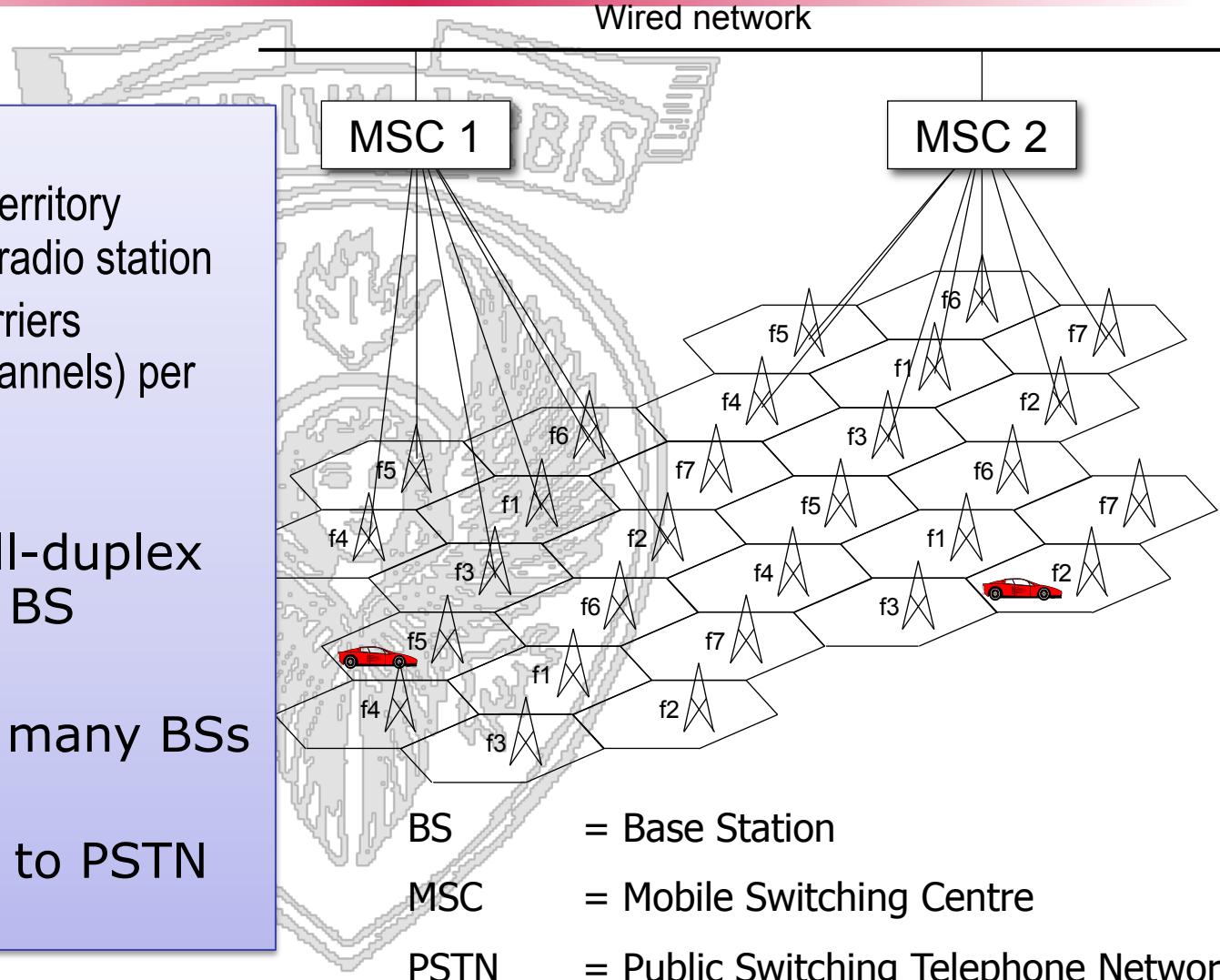
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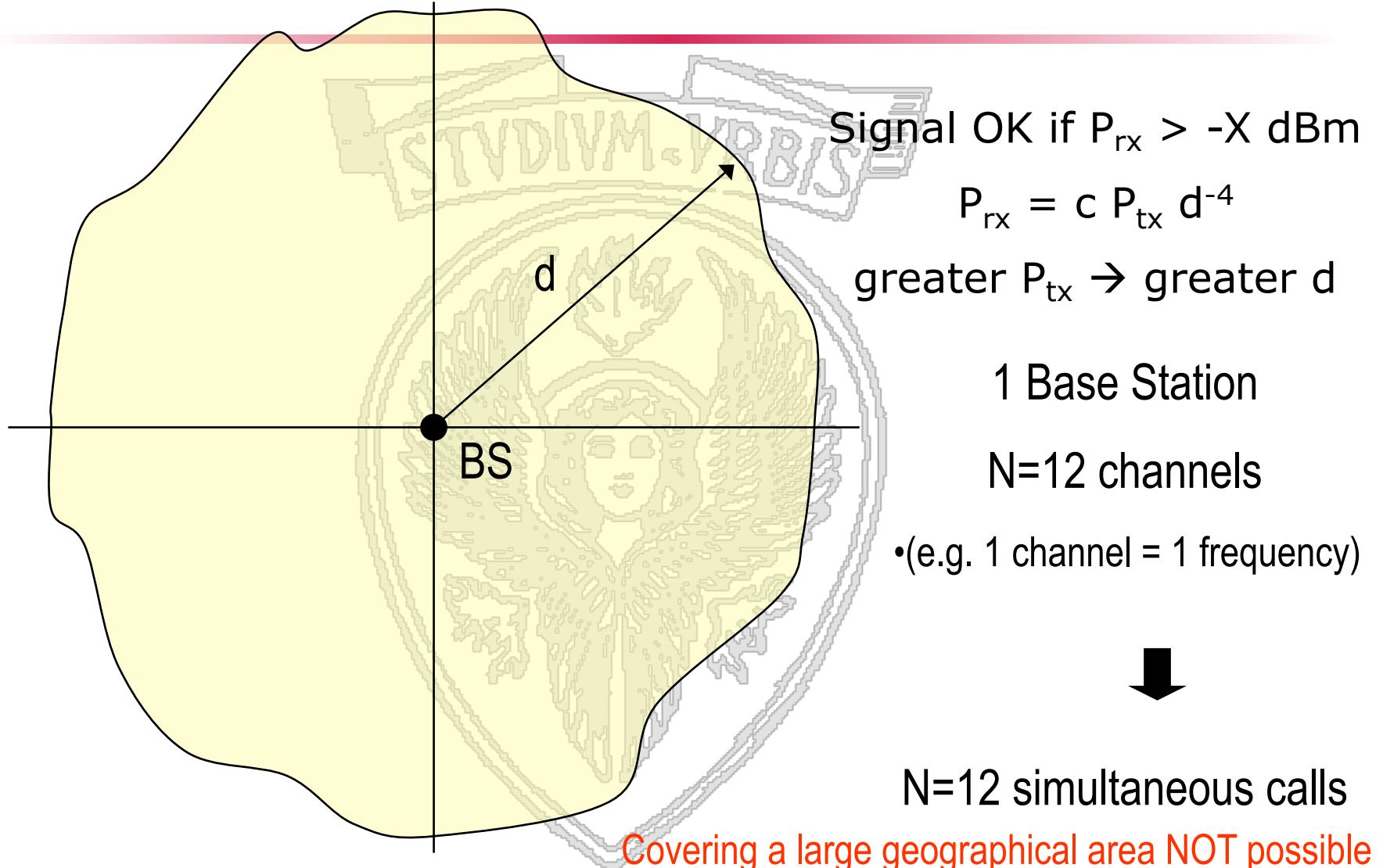


Allocazione delle frequenze: organizzazione a celle



- 1 BS per cell
 - ⇒ Cell: Portion of territory covered by one radio station
 - ⇒ One or more carriers (frequencies; channels) per cell
- Mobile users full-duplex connected with BS
- 1 MSC controls many BSs
- MSC connected to PSTN

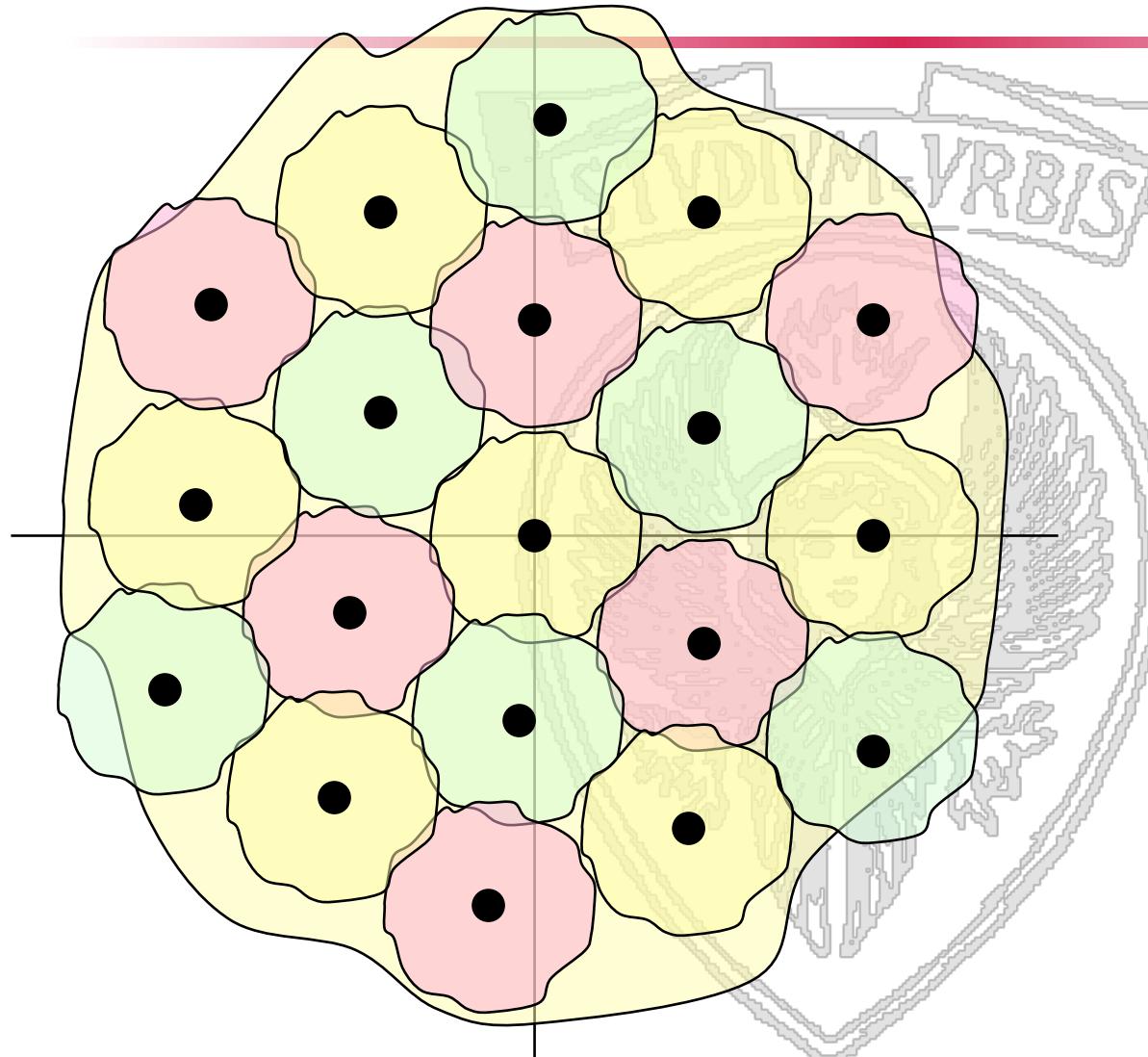






Cellular coverage

target: cover the same area with a larger number of BSs



19 Base Station

12 frequencies

4 frequencies/cell



Worst case:

4 calls (all users in same cell)

Best case:

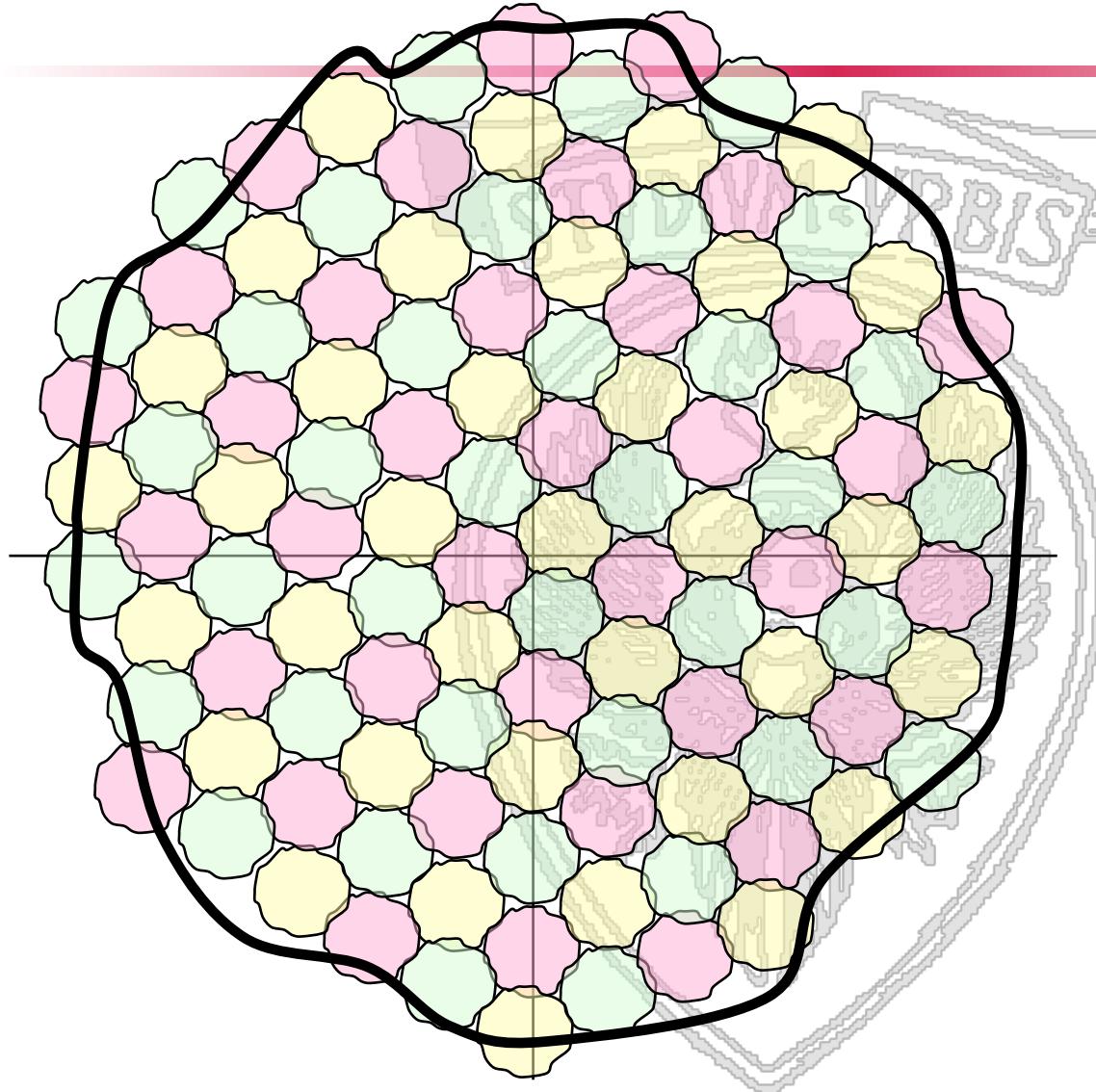
76 calls (4 users per cell, 19 cells)

Average case $\gg 12$

Low transmit power

Key advantages:

- Increased capacity (freq. reuse)
- Decreased tx power



many BS

Very low power!!

Unlimited capacity!!

Usage of same spectrum

(12 frequencies)

(4 freq/cell)

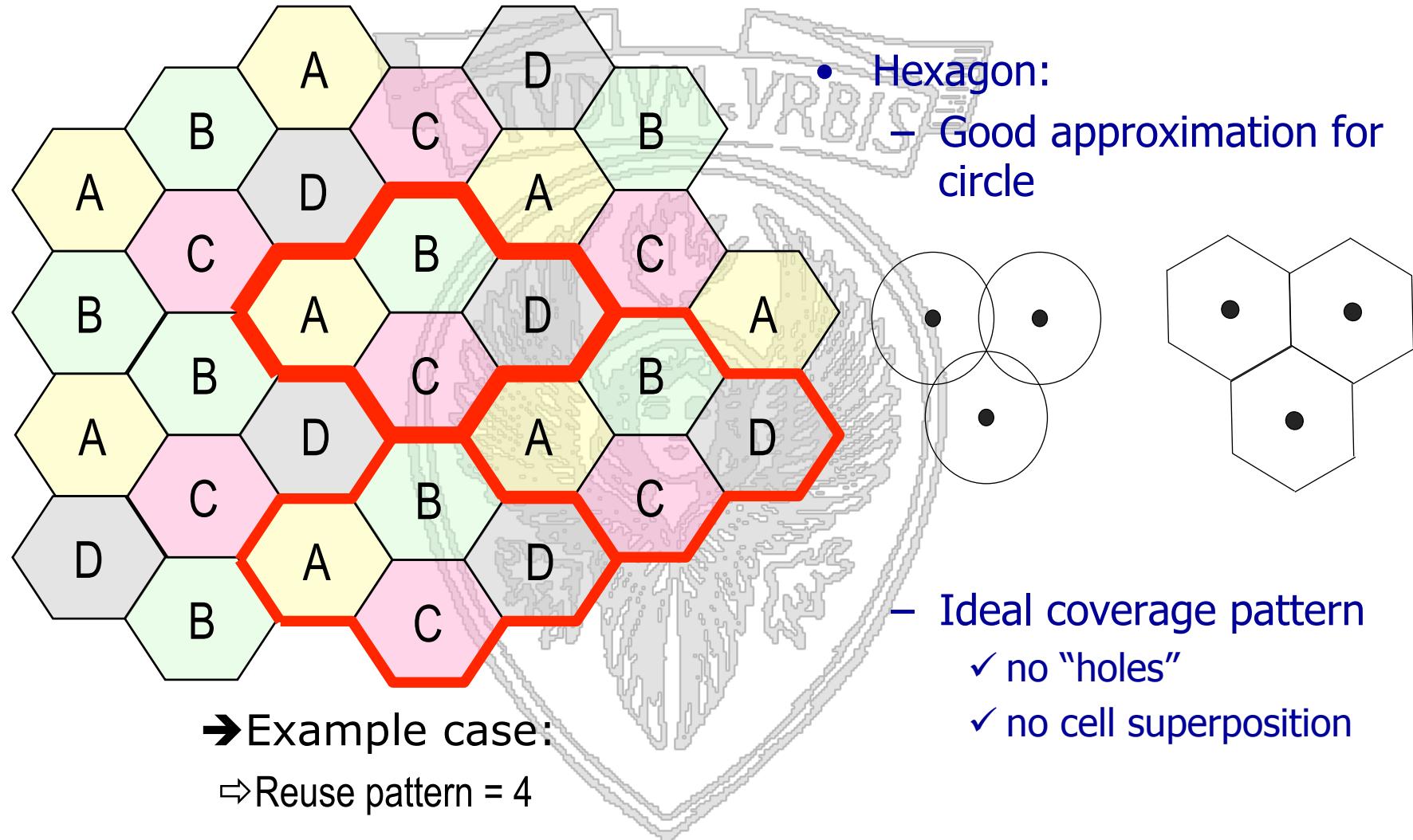
Disadvantage:

mobility management

additional infrastructure costs



- Increased via frequency reuse
 - Frequency reuse depends on interference
 - need to sufficiently separate cells
 - ✓ reuse pattern = cluster size ($7 \rightarrow 4 \rightarrow 3$): discussed later
- Cellular system capacity: depends on
 - overall number of frequencies
 - ✓ Larger spectrum occupation
 - frequency reuse pattern
 - Cell size
 - ✓ Smaller cell (cell → microcell → picocell → femtocell) = greater capacity
 - ✓ Smaller cell = lower transmission power
 - ✓ Smaller cell = increased handover management burden





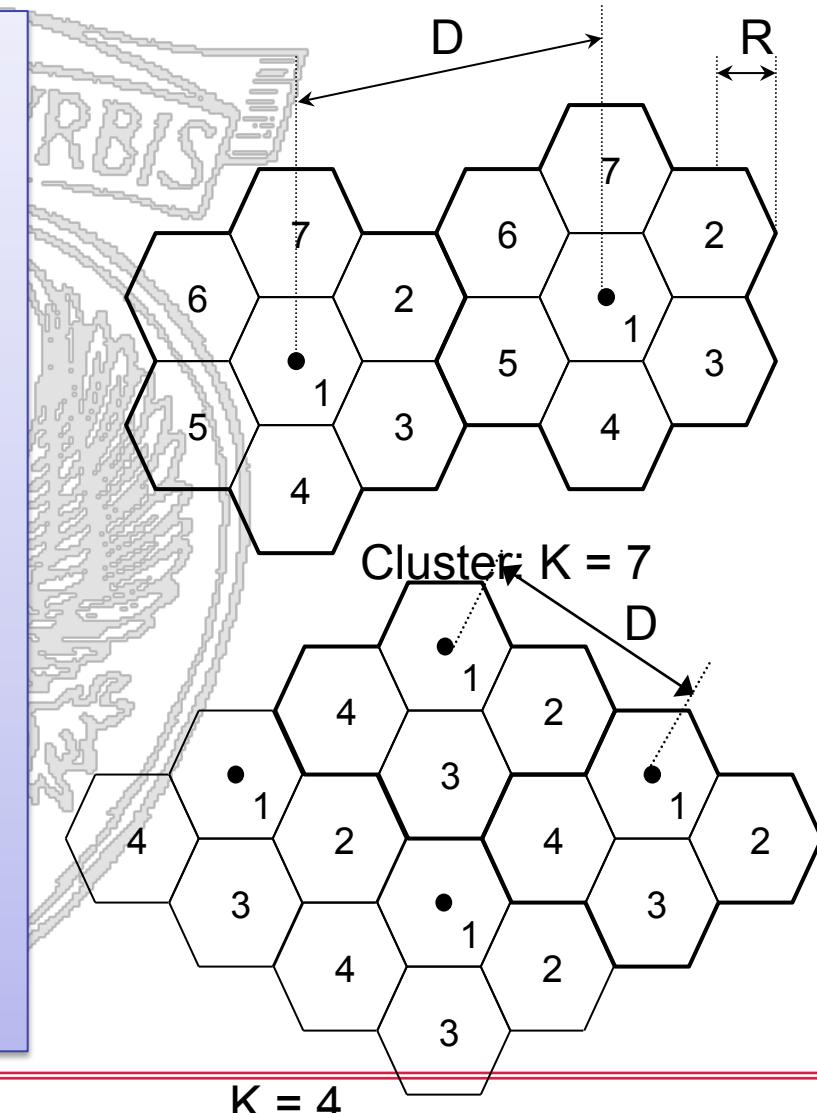
PART 2

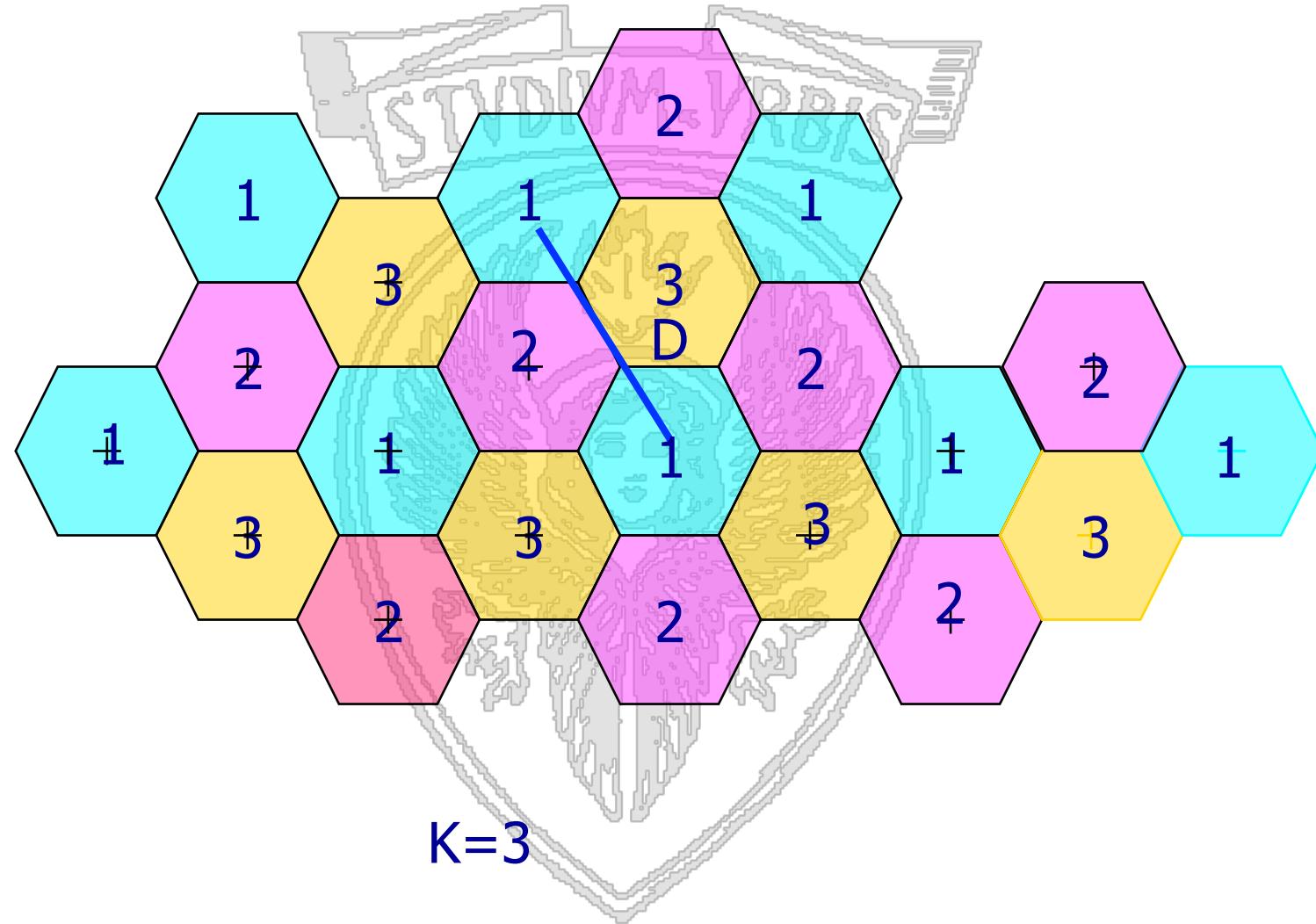
Cellular Coverage Concepts (piu' in dettaglio sull'organizzazione di un sistema cellulare)

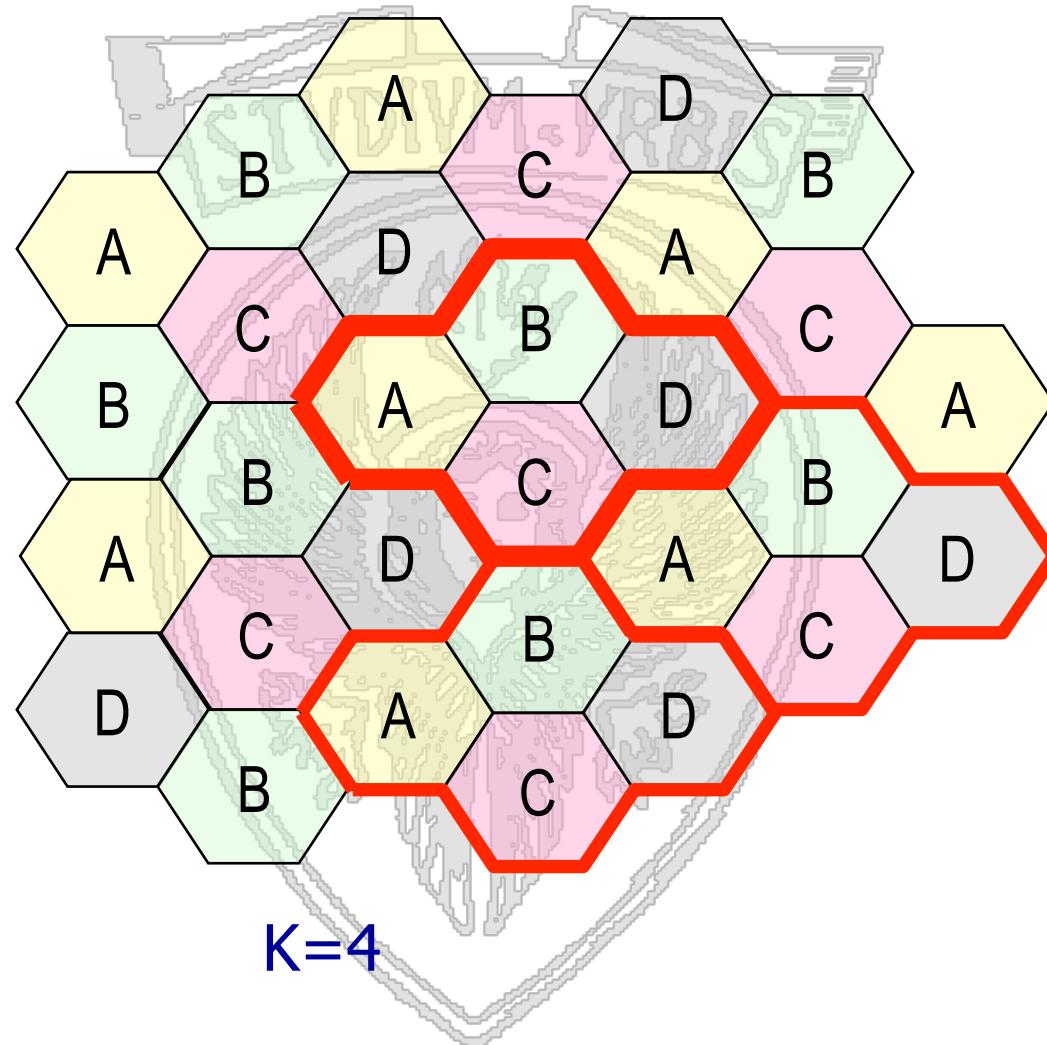
Lecture 2.2
Clusters and CCI

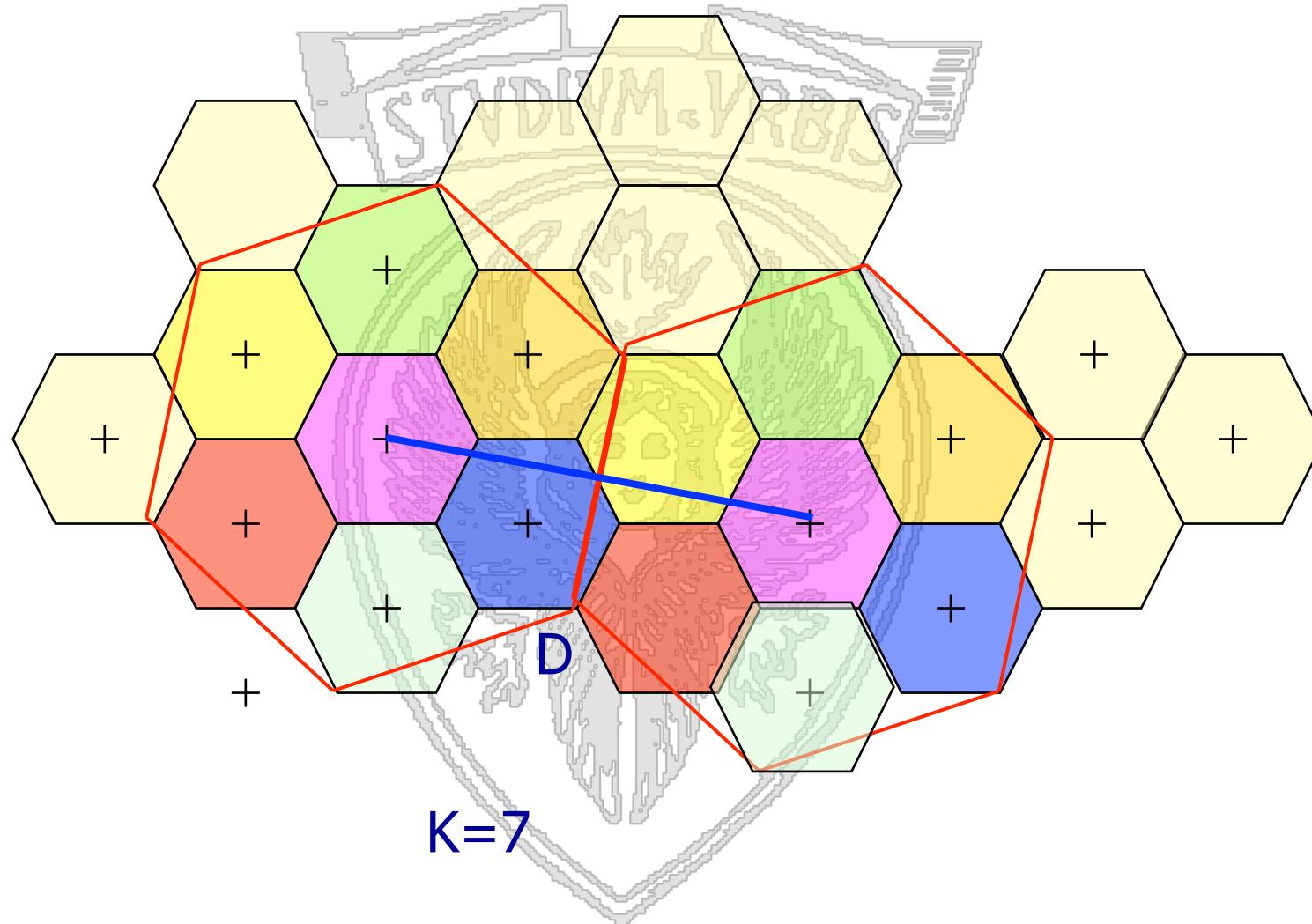


- Reuse distance:
 - Key concept
 - In the real world depends on
 - ✓ Territorial patterns (hills, etc)
 - ✓ Transmitted power
 - and other propagation issues such as antenna directivity, height of transmission antenna, etc
- Simplified hexagonal cells model:
 - reuse distance depends on reuse pattern (cluster size)
 - Possible clusters:
 - ✓ 3,4,7,9,12,13,16,19,...





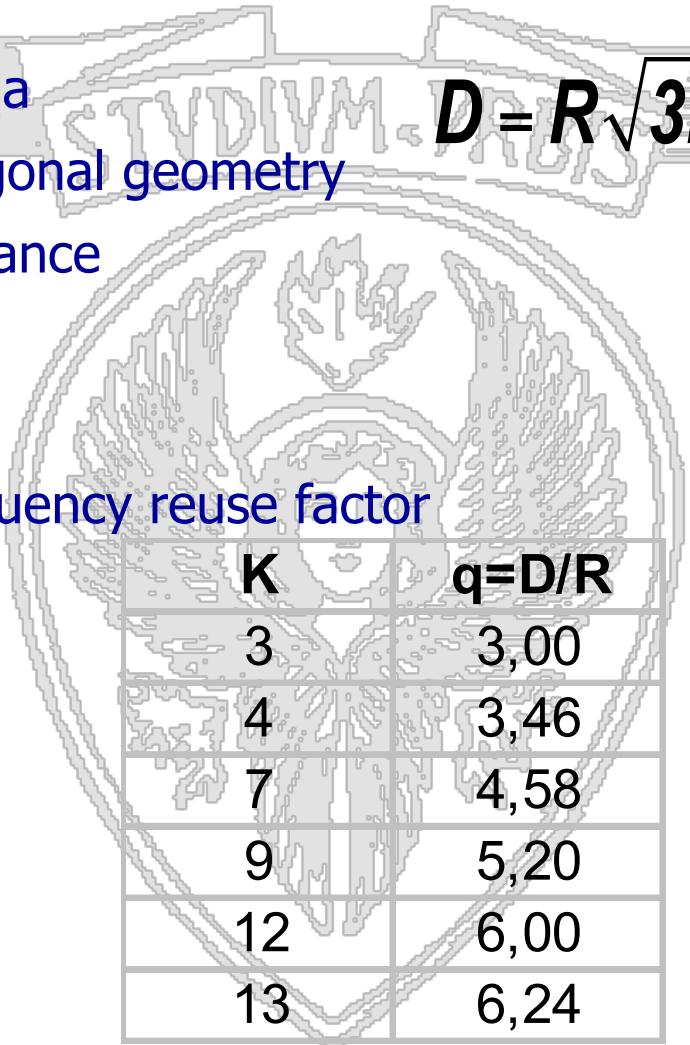




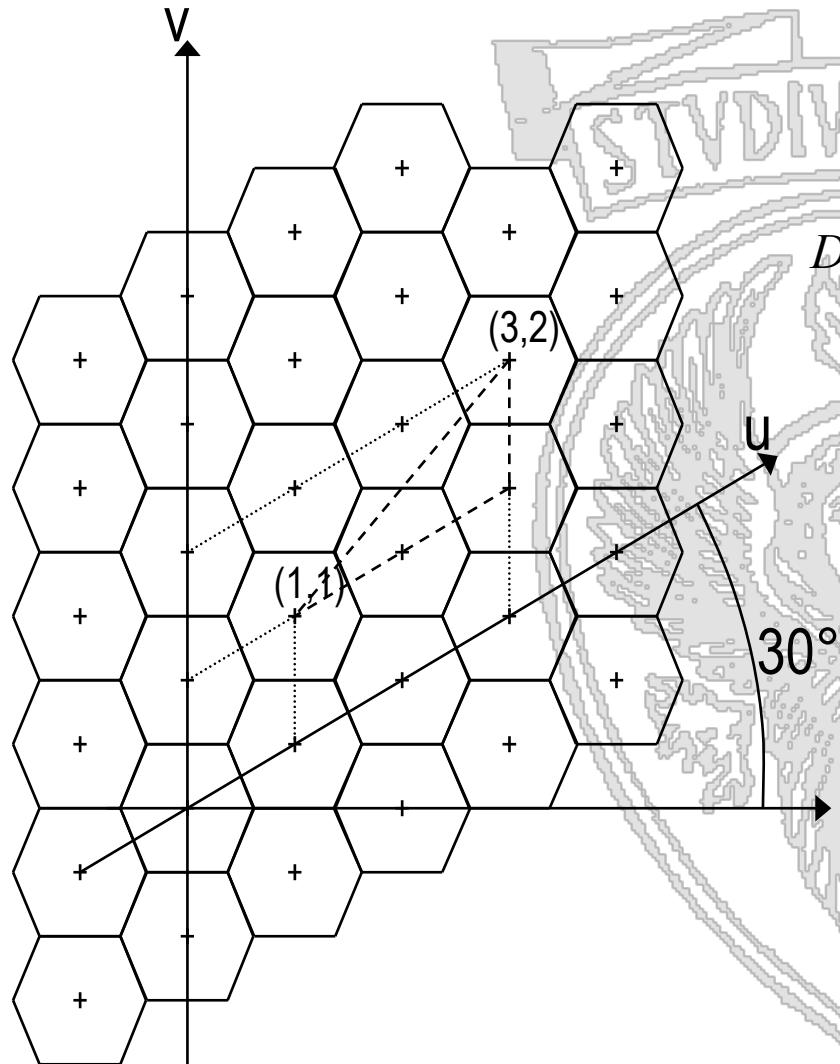


- General formula
- Valid for hexagonal geometry
- $D = \text{reuse distance}$
- $R = \text{cell radius}$
- $K = \text{cluster size}$
- $q = D/R = \text{frequency reuse factor}$

$$D = R\sqrt{3K}$$



K	$q=D/R$
3	3,00
4	3,46
7	4,58
9	5,20
12	6,00
13	6,24



- Distance between two cell centers:
– $(u_1, v_1) \leftrightarrow (u_2, v_2)$

$$D = \sqrt{[(u_2 - u_1) \cos 30^\circ]^2 + [(v_2 - v_1) + (u_2 - u_1) \sin 30^\circ]^2}$$

– Simplifies to:

$$D = \sqrt{(u_2 - u_1)^2 + (v_2 - v_1)^2 + (u_2 - u_1)(v_2 - v_1)}$$

– Distance of cell (i,j) from $(0,0)$:

$$D = \sqrt{i^2 + j^2 + ij} \sqrt{3R}$$

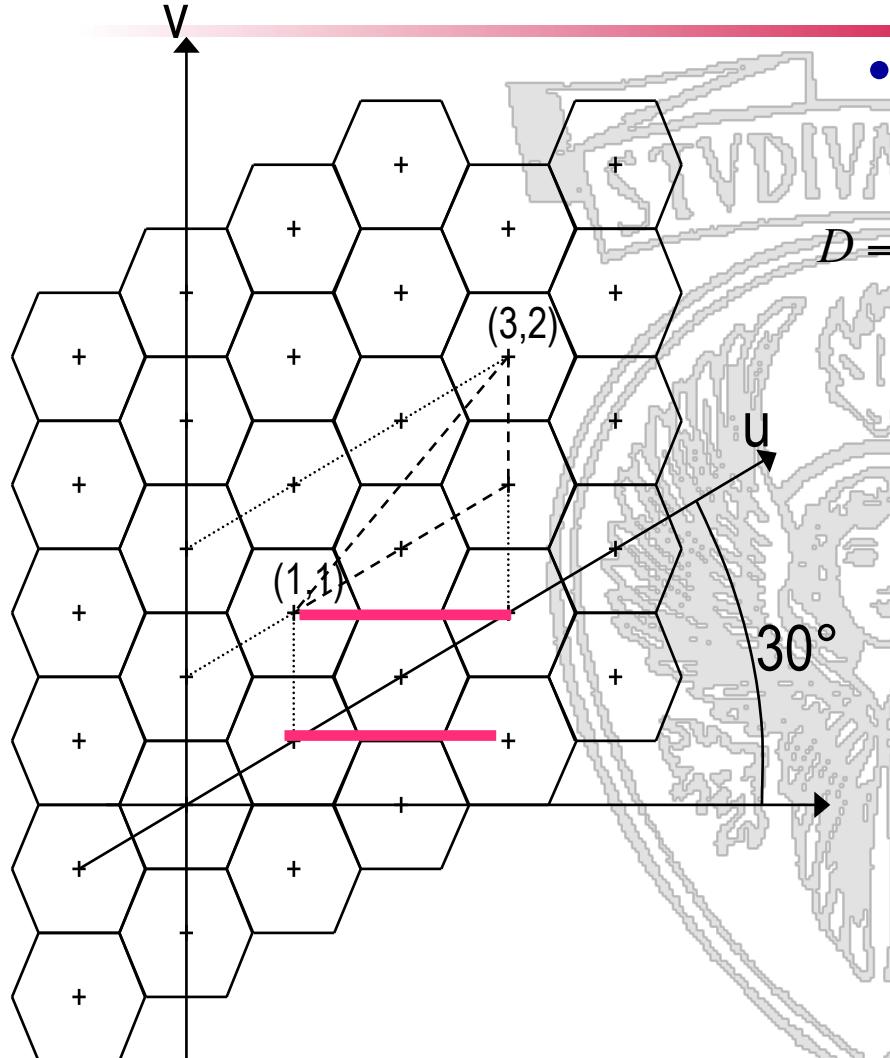
$$D_R = \sqrt{i^2 + j^2 + ij}$$

– Cluster: easy to see that

$$K = D_R^2 = i^2 + j^2 + ij$$

– hence:

$$D = R\sqrt{3K}$$



- Distance between two cell centers:

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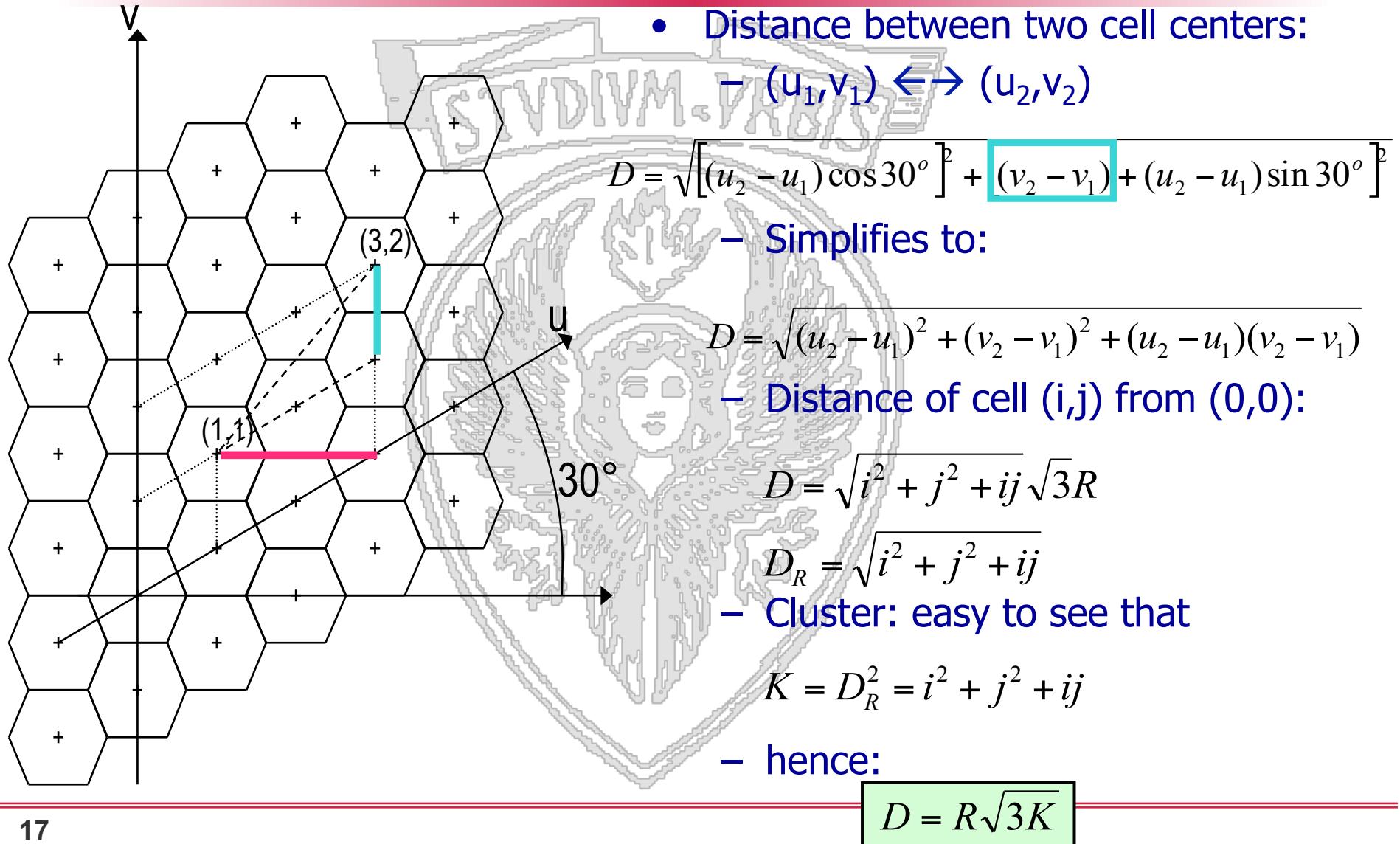
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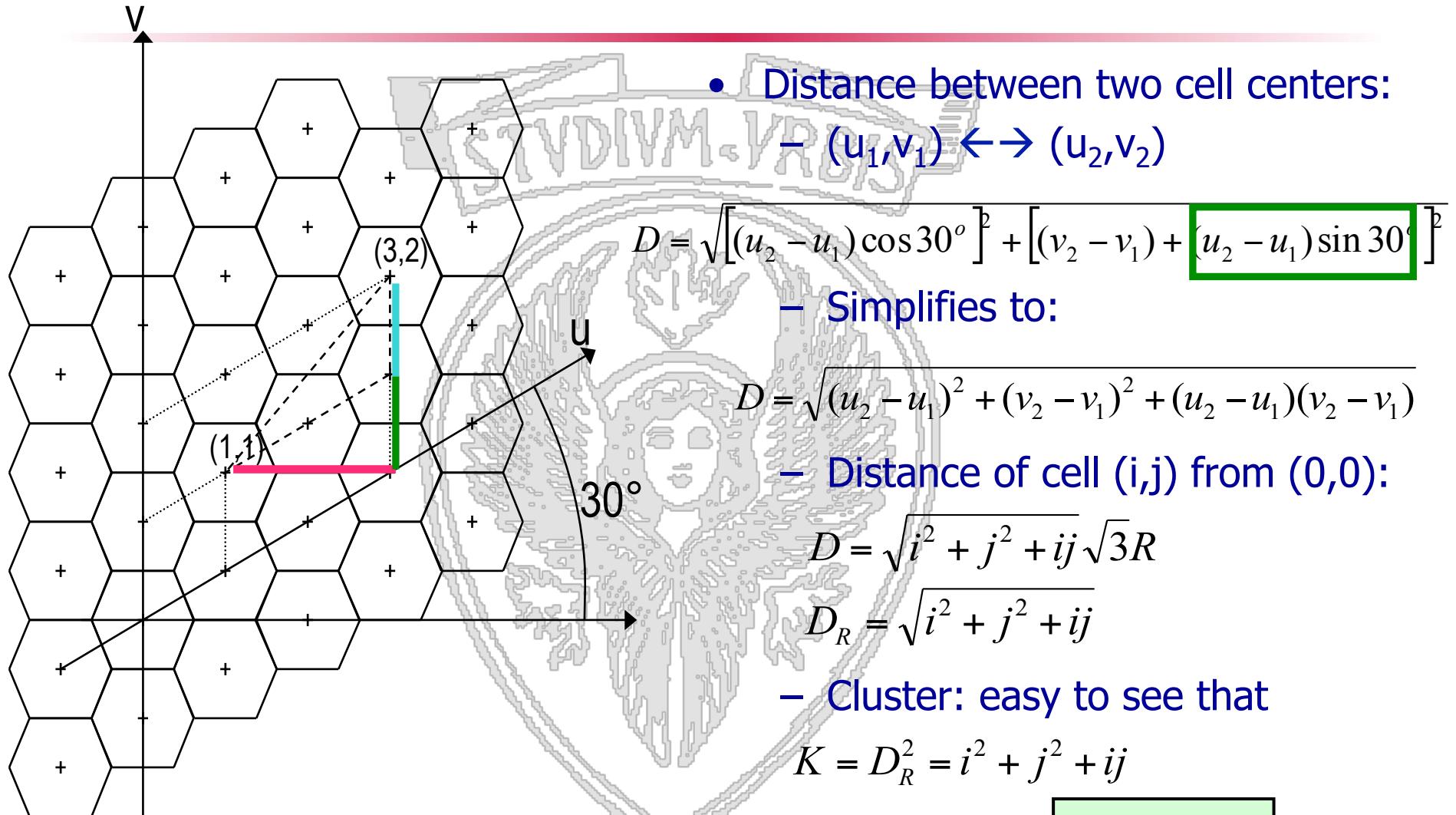
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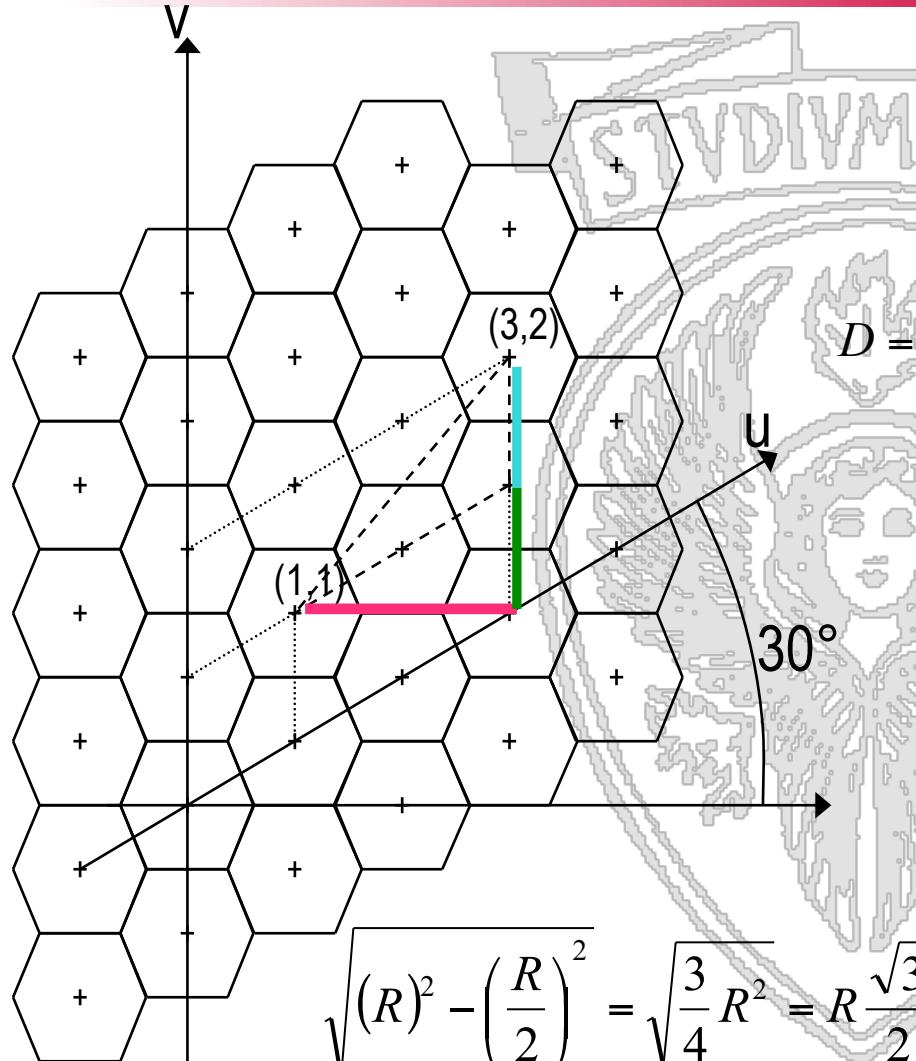
$$K = D_R^2 = i^2 + j^2 + ij$$

- hence:

$$D = R\sqrt{3K}$$







- Distance between two cell centers:

- $$(u_1, v_1) \leftrightarrow (u_2, v_2)$$

$$D = \sqrt{[(u_2 - u_1) \cos 30^\circ]^2 + [(v_2 - v_1) + (u_2 - u_1) \sin 30^\circ]^2}$$

- Simplifies to:

$$D = \sqrt{(u_2 - u_1)^2 + (v_2 - v_1)^2 + (u_2 - u_1)(v_2 - v_1)}$$

- Distance of cell (i,j) from (0,0):

$$D = \sqrt{i^2 + j^2 + ij\sqrt{3}R}$$

Distanza tra
i centri di celle
adiacenti

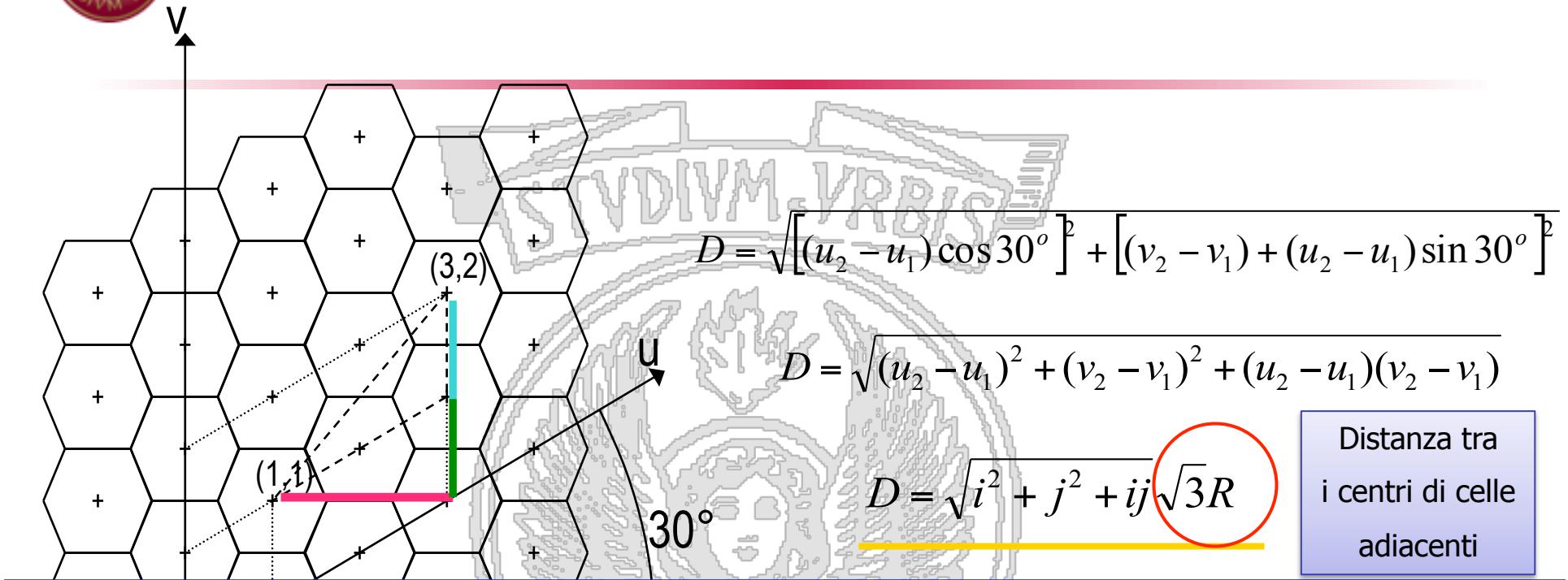
$$D_R = \sqrt{i^2 + j^2 + ij}$$

- Cluster: easy to see that

$$\sqrt{(R)^2 - \left(\frac{R}{2}\right)^2} = \sqrt{\frac{3}{4}R^2} = R\frac{\sqrt{3}}{2}$$

$$K = D_R^2 = i^2 + j^2 + ij$$

$$D = R\sqrt{3K}$$



Se R è il raggio di un esagono la metà della distanza tra due celle adiacenti è

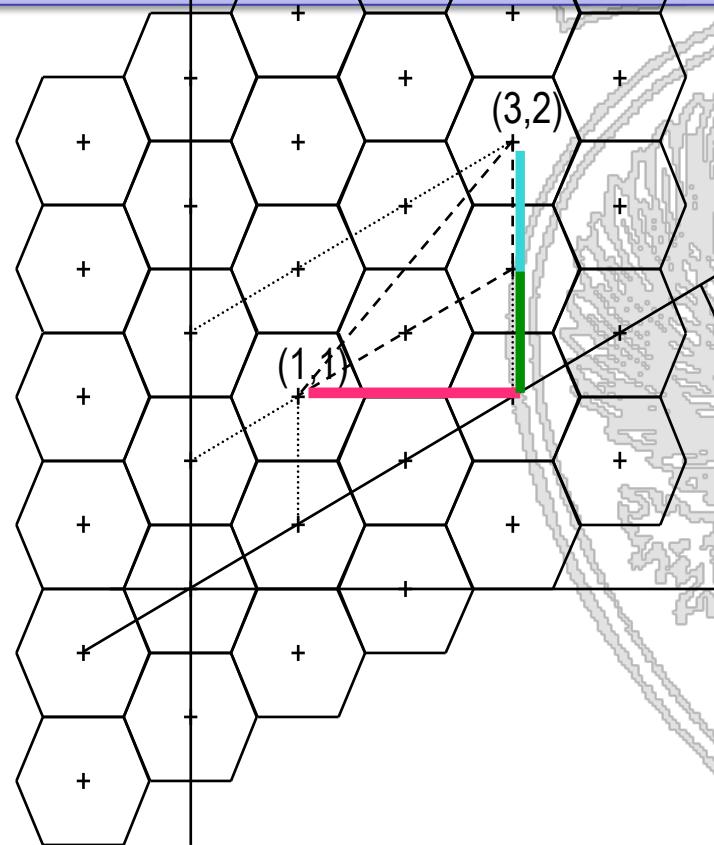
$$\sqrt{(R)^2 - \left(\frac{R}{2}\right)^2} = \sqrt{\frac{3}{4}R^2} = R\frac{\sqrt{3}}{2}$$

Quindi la distanza tra due celle adiacenti è 2 volte questa quantità!



- Distance between two cell centers:
 v

$$- (u_1, v_1) \leftrightarrow (u_2, v_2)$$



$$D = \sqrt{[(u_2 - u_1) \cos 30^\circ]^2 + [(v_2 - v_1) + (u_2 - u_1) \sin 30^\circ]^2}$$

$$D = \sqrt{(u_2 - u_1)^2 + (v_2 - v_1)^2 + (u_2 - u_1)(v_2 - v_1)}$$

$$D = \sqrt{i^2 + j^2 + ij\sqrt{3}R}$$

$$D_R = \sqrt{i^2 + j^2 + ij} \quad (\text{definizione di } D_R)$$

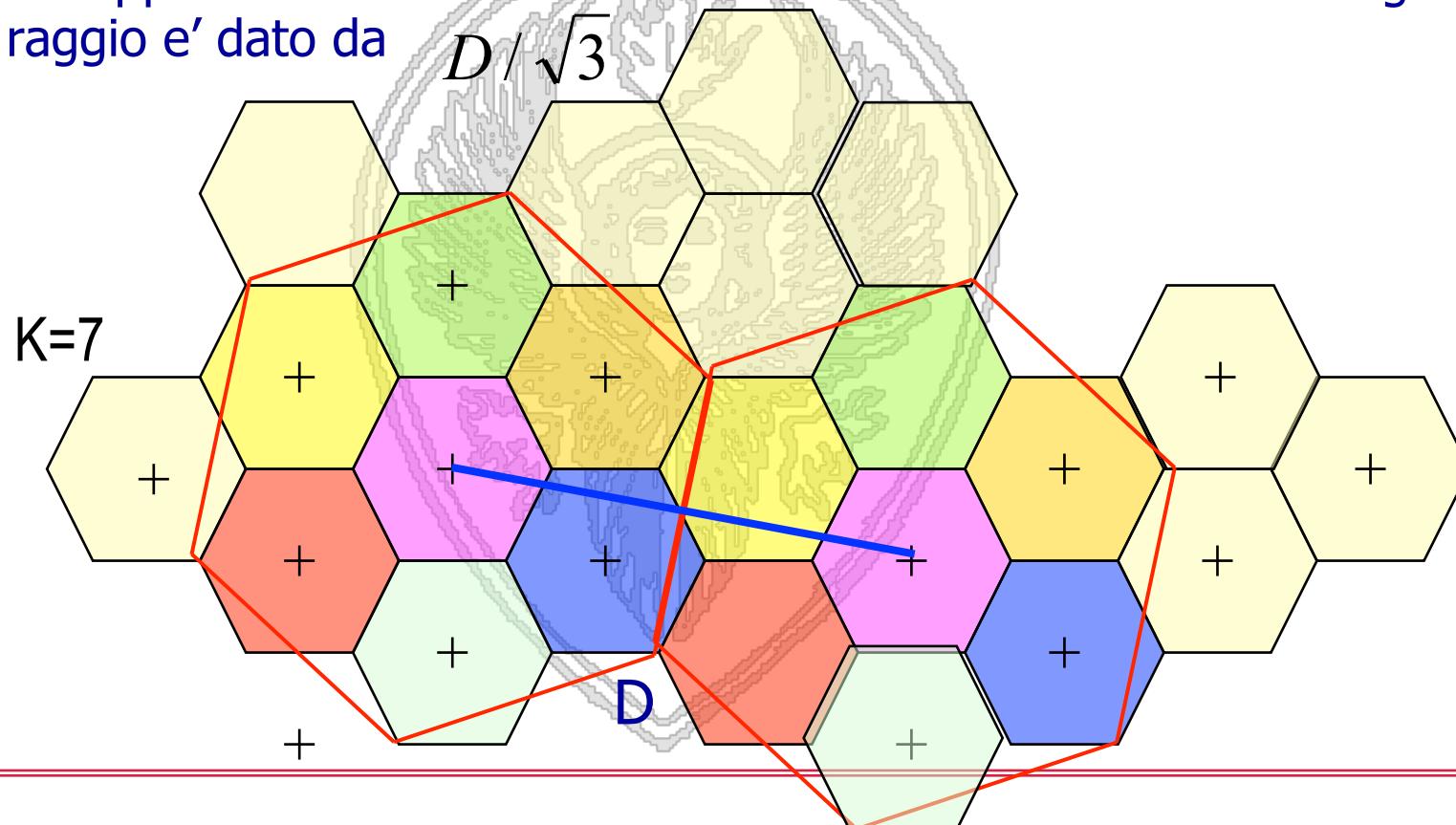
$$K = D_R^2 = i^2 + j^2 + ij$$

$$K = D^2 / 3R^2$$

$$D = R\sqrt{3K}$$



- Se considero una cella che usa un determinato gruppo di frequenze A dato che voglio ricoprire l'area con cluster i centri delle celle interferenti saranno a distanza D
- Posso approssimare l'area di ciascun cluster con l'area di un esagono il cui raggio e' dato da



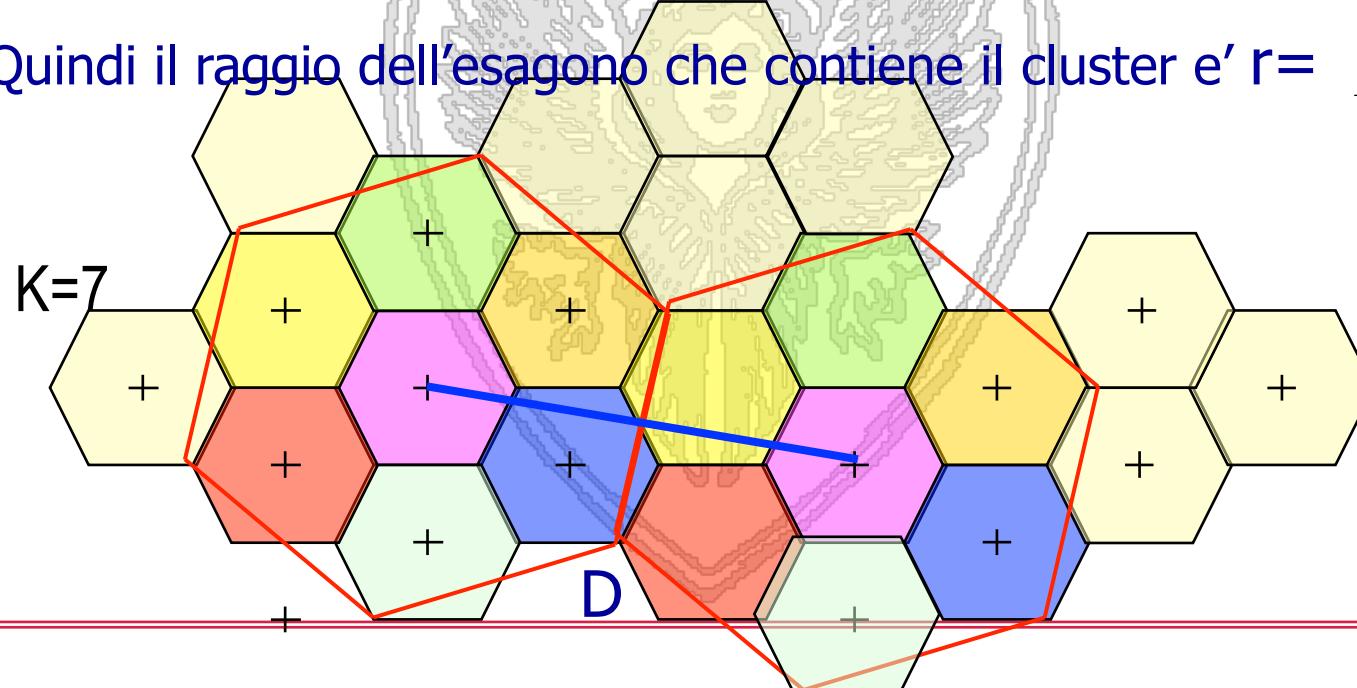


- Posso approssimare l'area di ciascun cluster con l'area di un esagono il cui raggio e' dato da $D/\sqrt{3}$

→ Se il raggio di una cella esagonale e' r la distanza tra i centri di due esagoni adiacenti e' $d = \sqrt{3}r$

→ Nel caso di cluster adiacenti la distanza tra i loro centri e' D

Quindi il raggio dell'esagono che contiene il cluster e' $r = D/\sqrt{3}$





- L'area occupata da un cluster A_{cluster} e' quindi data da:

$$\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}$$

- Quanti esagoni di area $\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}$ possono stare

in un'area pari a

$$\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}$$

?

- Risposta:

$$K = \frac{A_{\text{cluster}}}{A_{\text{cella}}} = \frac{\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}}{\frac{3}{2} \left(R \right)^2 \sqrt{3}} = \left(\frac{D}{R\sqrt{3}} \right)^2 = (D_R)^2$$

$$K = \left(\frac{D}{R\sqrt{3}} \right)^2 = \frac{D^2}{3R^2}$$



$$D = \sqrt{3KR^2} = R\sqrt{3K}$$



- L'area occupata da un cluster $A_{cluster}$ è quindi data da:

$$\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}$$

- Quanti esagoni di area $\frac{3}{2} (R)^2 \sqrt{3}$ possono stare

in un'area pari a $\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}$?

- Risposta:

dato che:

$$D = \sqrt{i^2 + j^2 + ij} \sqrt{3} R$$
$$D_R = \sqrt{i^2 + j^2 + ij}$$

$$K = \frac{A_{cluster}}{A_{cella}} = \frac{\frac{3}{2} \left(\frac{D}{\sqrt{3}} \right)^2 \sqrt{3}}{\frac{3}{2} (R)^2 \sqrt{3}} = \left(\frac{D}{R\sqrt{3}} \right)^2 = (D_R)^2$$

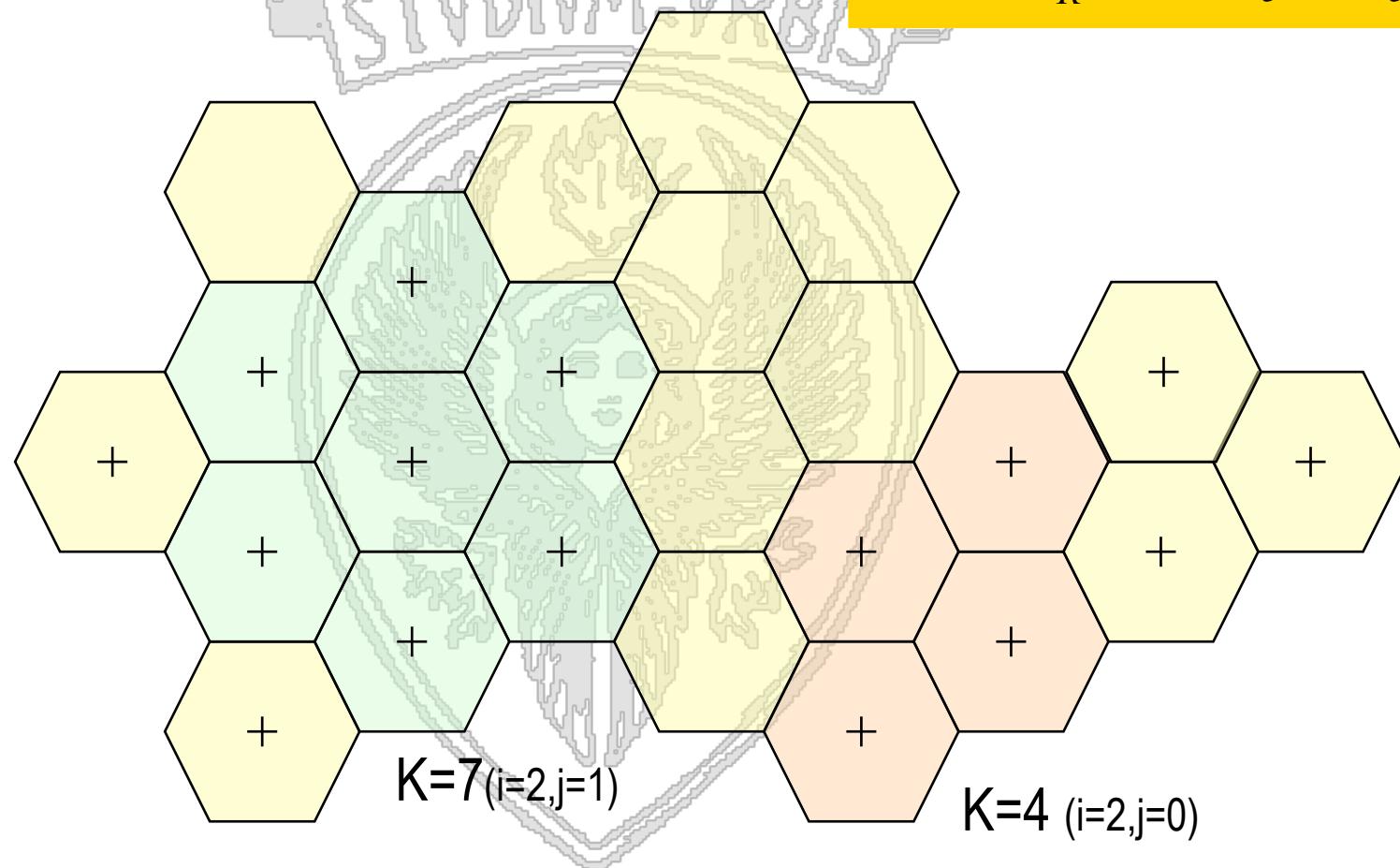
$$K = \left(\frac{D}{R\sqrt{3}} \right)^2 = \frac{D^2}{3R^2}$$



$$D = \sqrt{3KR^2} = R\sqrt{3K}$$



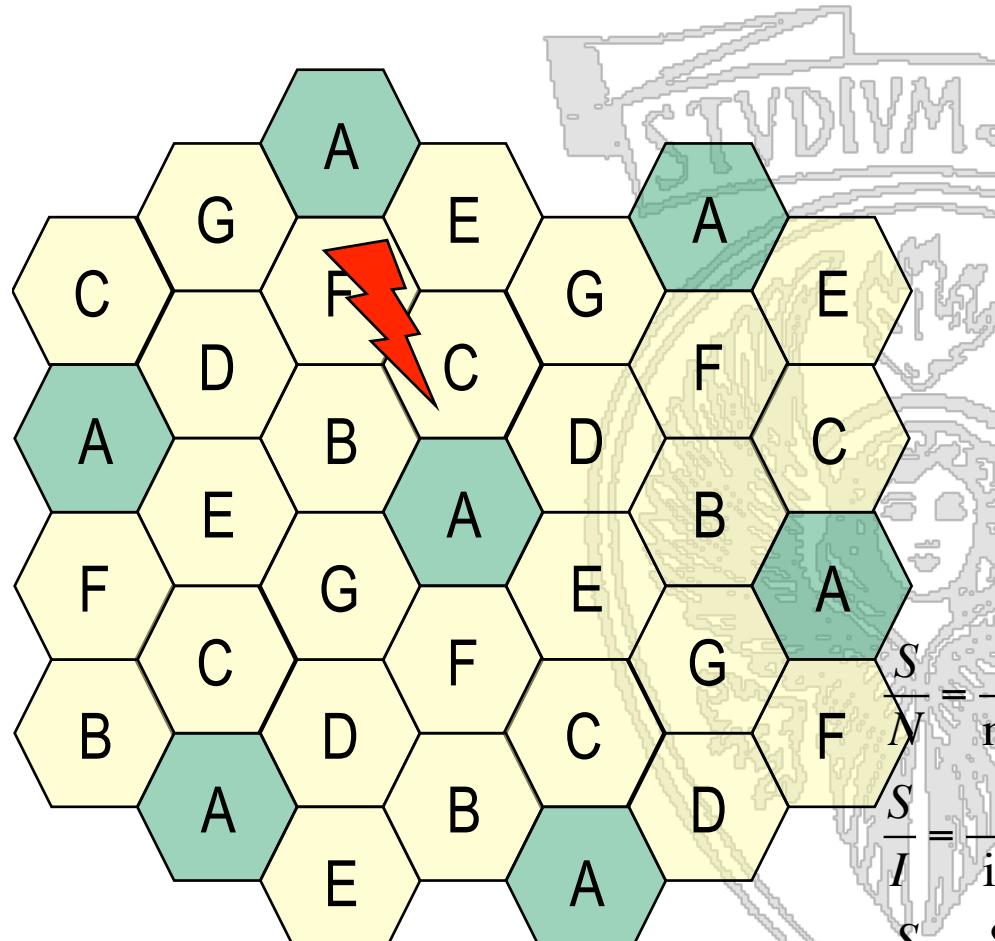
$$K = D_R^2 = i^2 + j^2 + ij$$





i	j	K=ii+jj+ij	q=D/R
1	0	1	1,73
1	1	3	3,00
2	0	4	3,46
2	1	7	4,58
2	2	12	6,00
3	0	9	5,20
3	1	13	6,24
3	2	19	7,55
3	3	27	9,00
4	0	16	6,93
4	1	21	7,94
4	2	28	9,17
4	3	37	10,54
4	4	48	12,00
5	0	25	8,66
5	1	31	9,64

Dim.ammissibili dei cluster 1,3,4,7,9,12,13,16,...

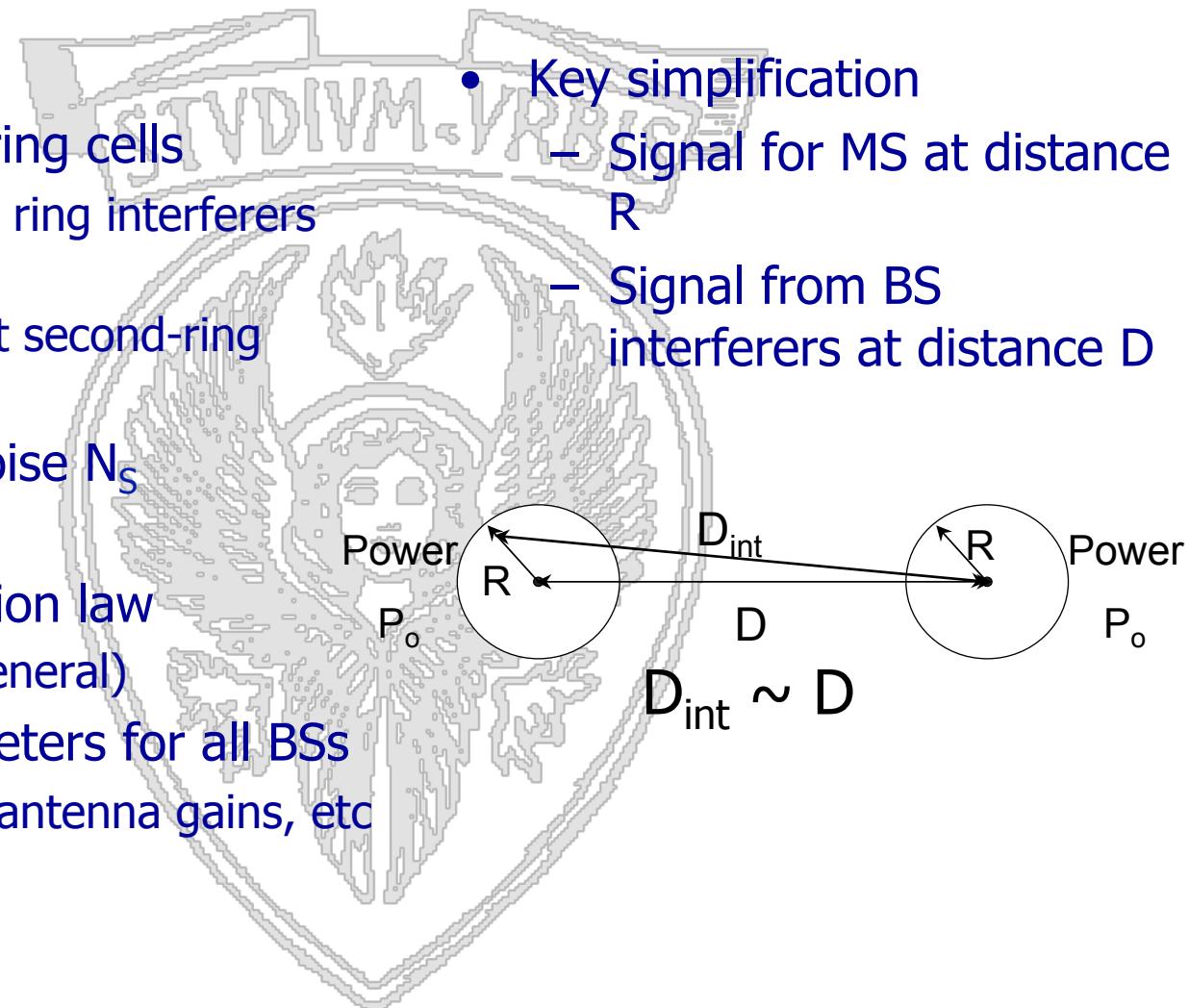


- Frequency reuse implies that remote cells interfere with tagged one
- Co-Channel Interference (CCI)
 - sum of interference from remote cells

$$\frac{S}{N} = \frac{\text{signal power (S)}}{\text{noise power (N}_s\text{) + interfering signal power (I)}}$$
$$\frac{S}{I} = \frac{\text{signal power (S)}}{\text{interfering signal power (I)}}$$
$$\frac{S}{N} \approx \frac{S}{I} \quad \text{as } N_s \text{ small}$$



- Assumptions
 - $N_I=6$ interfering cells
 - ✓ $N_I=6$: first ring interferers only
 - ✓ we neglect second-ring interferers
 - Negligible Noise N_S
 - ✓ $S/N \sim S/I$
 - $d^{-\eta}$ propagation law
 - ✓ $\eta=4$ (in general)
 - Same parameters for all BSs
 - ✓ Same P_{tx} , antenna gains, etc
- Key simplification
 - Signal for MS at distance R
 - Signal from BS interferers at distance D





$$\frac{S}{N} \approx \frac{S}{I} = \frac{\text{cost} \cdot R^{-\eta}}{\sum_{k=1}^{N_I} \text{cost} \cdot D^{-\eta}}$$

$$= \frac{1}{N_I} \left(\frac{R}{D} \right)^{-\eta} = \frac{1}{N_I} \left(\frac{D}{R} \right)^\eta = \frac{1}{N_I} q^\eta$$

By using the assumptions of same cost and same D:

Results depend

on ratio $q=D/R$
(q =frequency reuse factor)

Alternative expression: recalling that

$$\frac{S}{N} \approx \frac{S}{I} = \frac{1}{N_I} \left(\frac{R}{R\sqrt{3K}} \right)^{-\eta} = \frac{1}{N_I} (3K)^{\eta/2} = \frac{(3K)^{\eta/2}}{6}$$

$$N_I=6, \eta=4 \rightarrow \frac{S}{I} = \frac{(3K)^2}{6} = \frac{3}{2} K^2$$

USAGE: Given an S/I target, cluster size K is obtained



- target conditions:
 - $S/I = 9 \text{ dB}$
 - $\eta = 4$
- Solution:

$$\frac{S}{I} = 10^{0.9} = 7.94 \approx 8$$

$$\left. \frac{S}{I} = \frac{(3K)^{\eta/2}}{6} \right|_{\eta=4} \Rightarrow K = \sqrt{\frac{2 \cdot S}{3 \cdot I}}$$

$$K \geq 2.3 \Rightarrow K = 3$$

- target conditions:
 - $S/I = 18 \text{ dB}$
 - $\eta = 4.2$
- Solution:

$$\frac{S}{I} [\text{dB}] = 5\eta \log(3K) - 10 \log 6$$

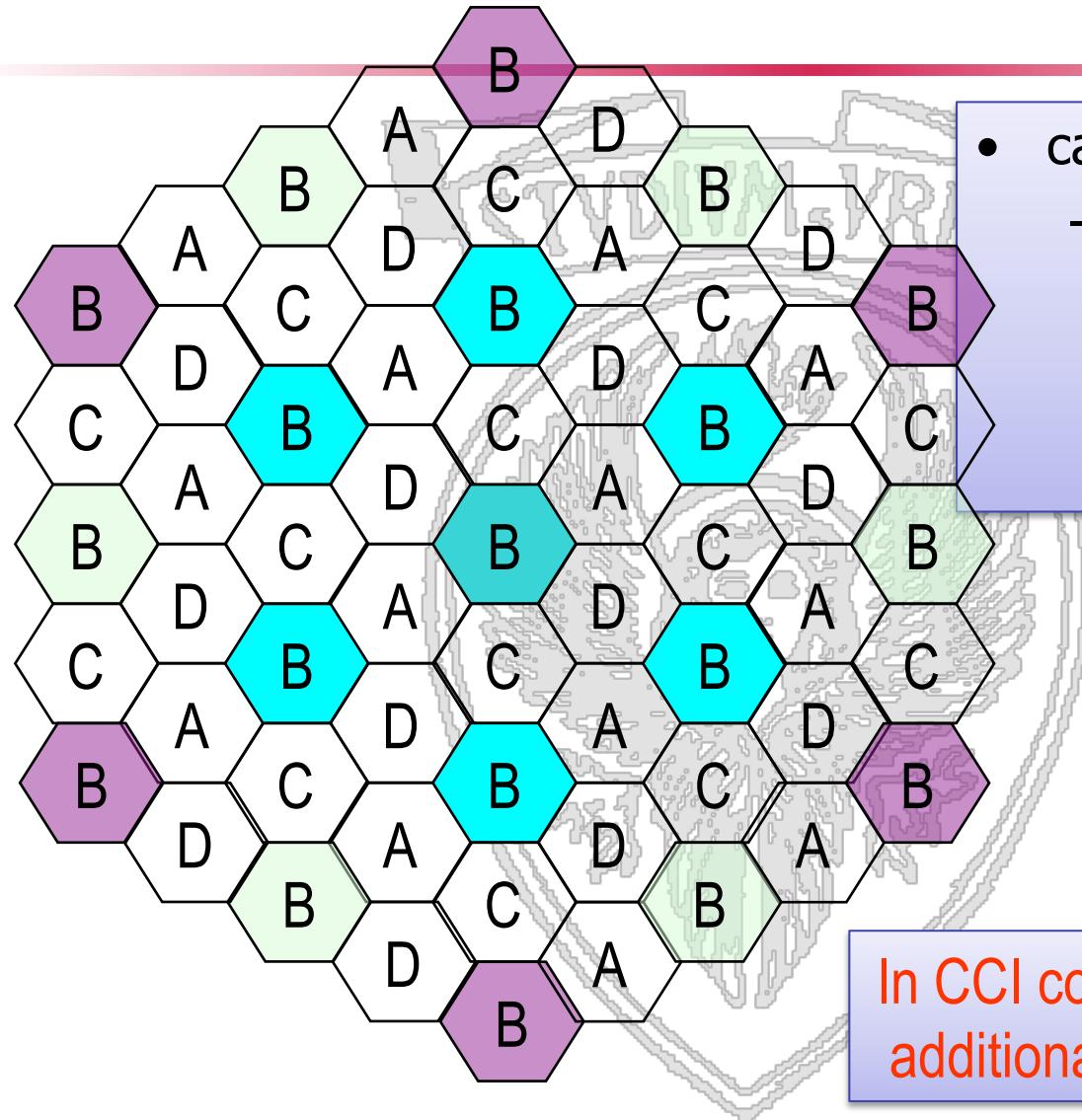
$$\log(3K) = \frac{18 + 7.78}{21} = 1.23$$

$$K \geq \frac{10^{1.23}}{3} = 5.63 \Rightarrow K = 7$$

$$\frac{S}{N} \approx \frac{S}{I} = \frac{(3K)^{\eta/2}}{6}$$



K	q=D/R	S/I	S/I dB
3	3,00	13,5	11,3
4	3,46	24,0	13,8
7	4,58	73,5	18,7
9	5,20	121,5	20,8
12	6,00	216,0	23,3
13	6,24	253,5	24,0
16	6,93	384,0	25,8
19	7,55	541,5	27,3
21	7,94	661,5	28,2
25	8,66	937,5	29,7



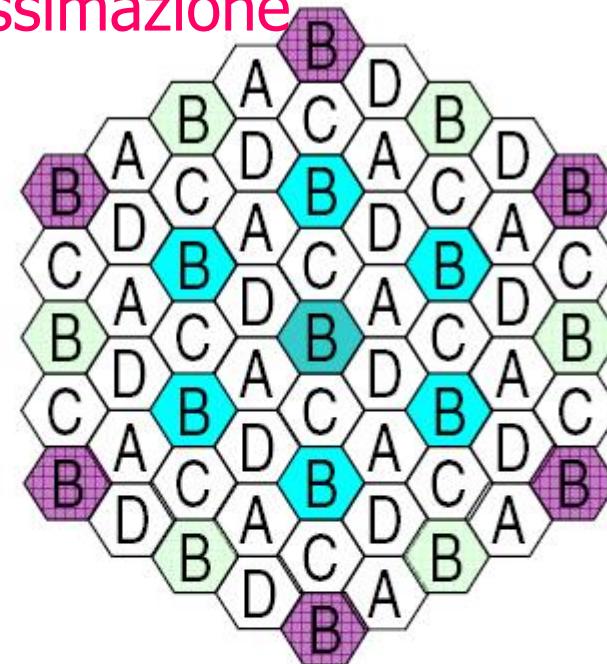
- case K=4
 - note that for each cluster there are always $N_I=6$ first-ring interferers

In CCI computation, contribute of additional interferers is marginal



- Primo livello di interferenti a distanza D , secondo a distanza $2D$, terzo a distanza $3D$ etc.
- Spesso gli interferenti oltre il primo livello hanno impatto non significativo
- Formula generale

approssimazione



$$SIR = \frac{R^{-\eta}}{N_{I1}(D)^{-\eta} + N_{I2}(2D)^{-\eta} + N_{I3}(3D)^{-\eta}}$$

N_{Ii} = numero di celle interferenti a livello i



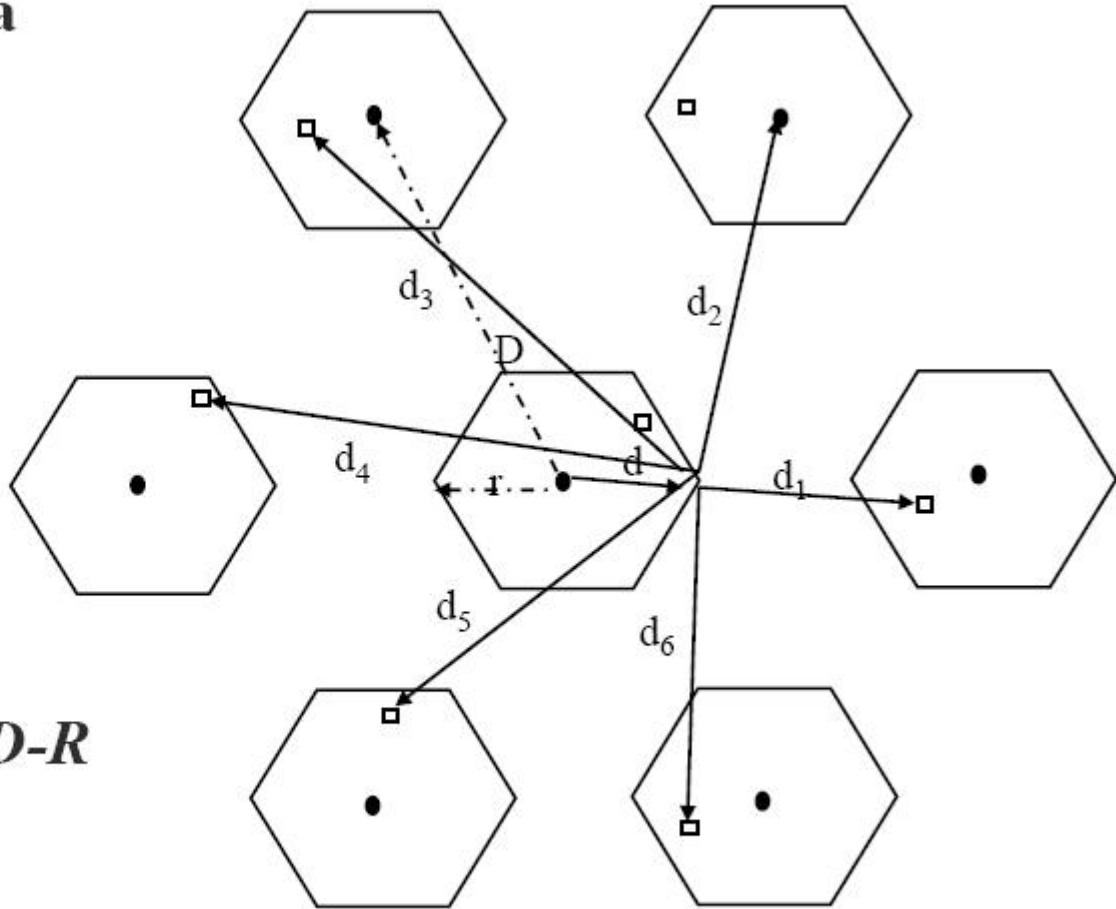
- stesse antenne e stessa potenza

$$SIR = \frac{P_t \cdot G \cdot d^{-\eta}}{\sum_{i=1}^6 P_t \cdot G \cdot d_i^{-\eta}} =$$

$$= \frac{d^{-\eta}}{\sum_{i=1}^6 d_i^{-\eta}}$$

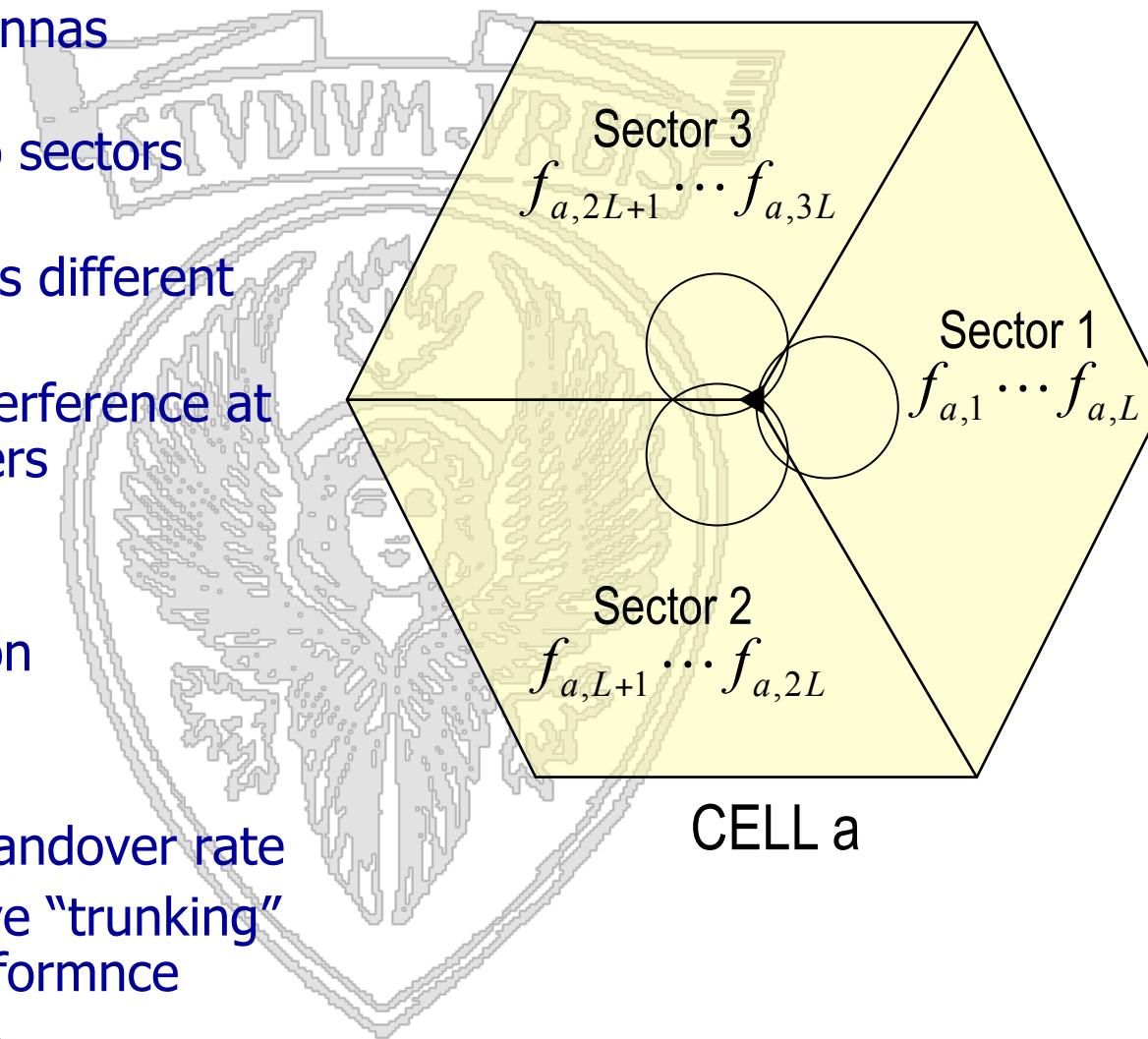
- caso peggiore $d = r$
- approssimazione $d_i = D - R$

$$SIR = \frac{R^{-\eta}}{6(D-R)^{-\eta}}$$





- Directional antennas
- Cell divided into sectors
- Each sector uses different frequencies
 - To avoid interference at sector borders
- PROS:
 - CCI reduction
- CONS:
 - Increased handover rate
 - Less effective “trunking” leads to performance impairments





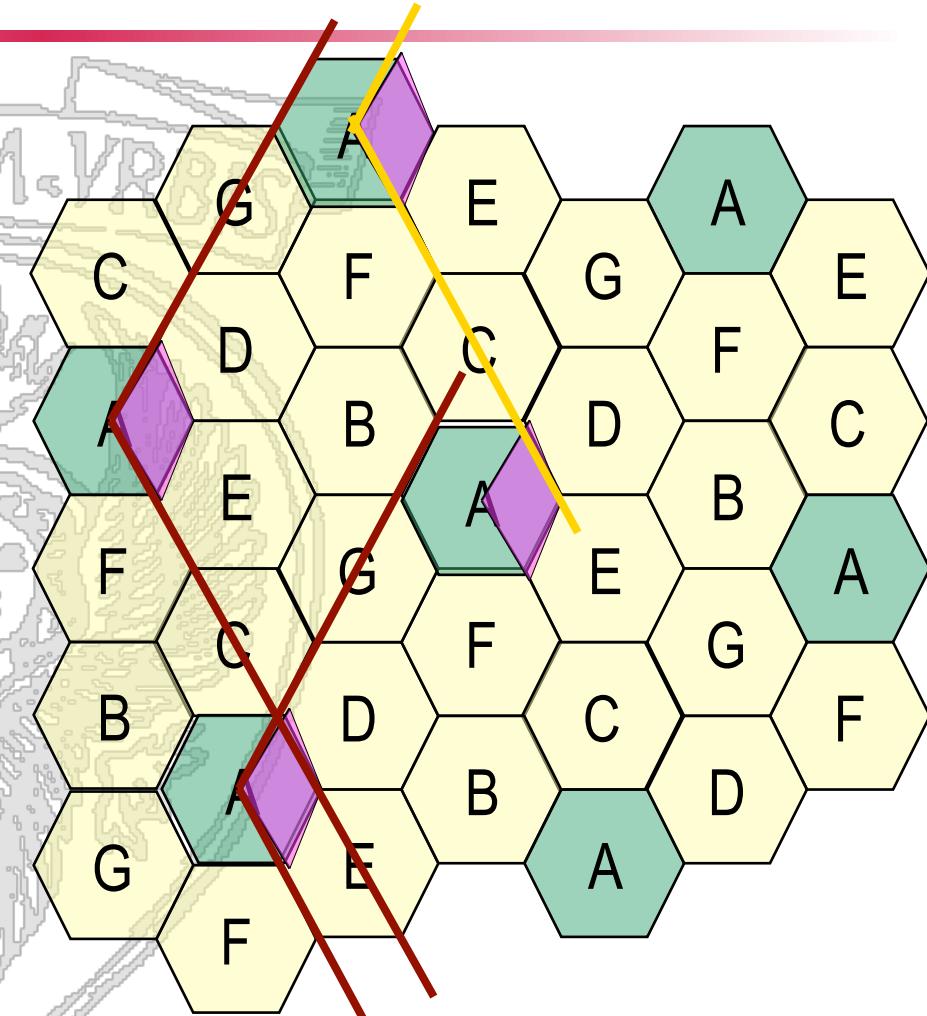
- Inferference from 2 cells,
only
 - Instead of 6 cells

With usual approxs

(specifically, $D_{int} \sim D$)

$$\left[\frac{S}{I} \right]_{120^\circ} = \frac{R^{-\eta}}{2D^{-\eta}} = 3 \cdot \left[\frac{S}{I} \right]_{omni}$$

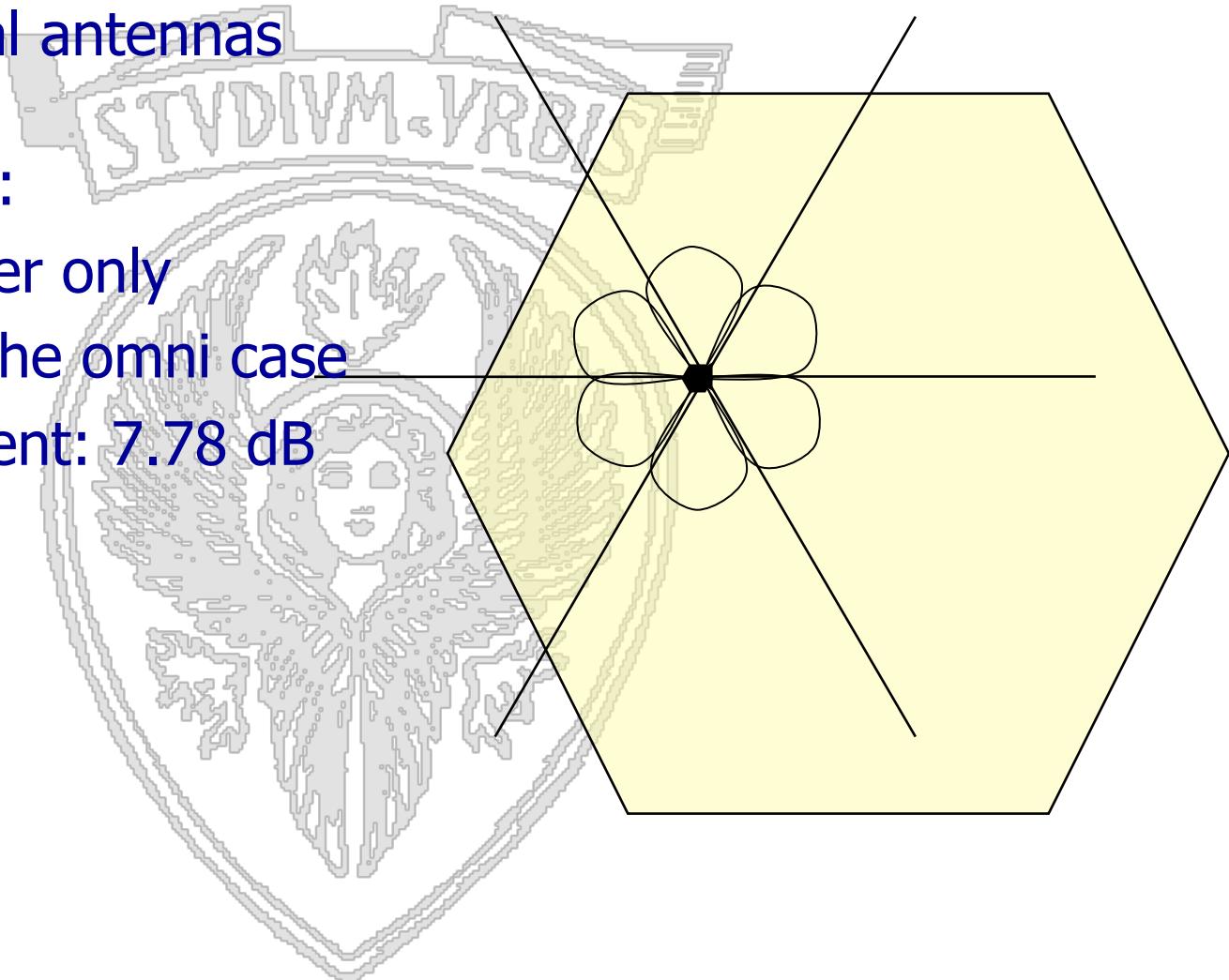
$$\left[\frac{S}{I} \right]_{120^\circ} dB = \left[\frac{S}{I} \right]_{omni} dB + 4.77$$



Conclusion: 3 sectors = 4.77 dB improvement



- 60° Directional antennas

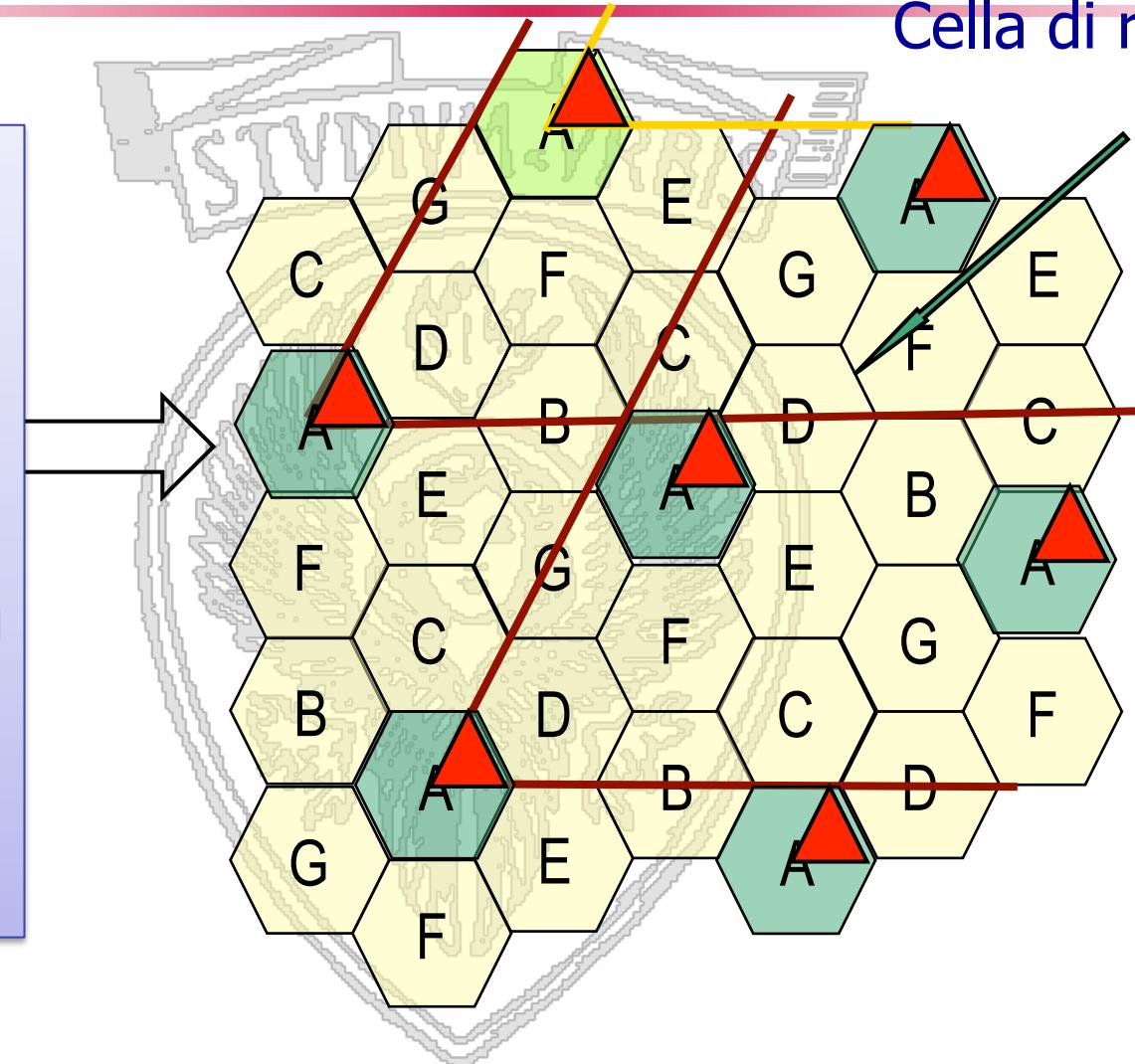


- CCI reduction:
 - 1 interferer only
 - 6 x S/I in the omni case
 - Improvement: 7.78 dB



Cella di riferimento

Unica BS
che disturba
le ricezioni/
trasmissioni
verso/dalle MU
nella cella di
riferimento





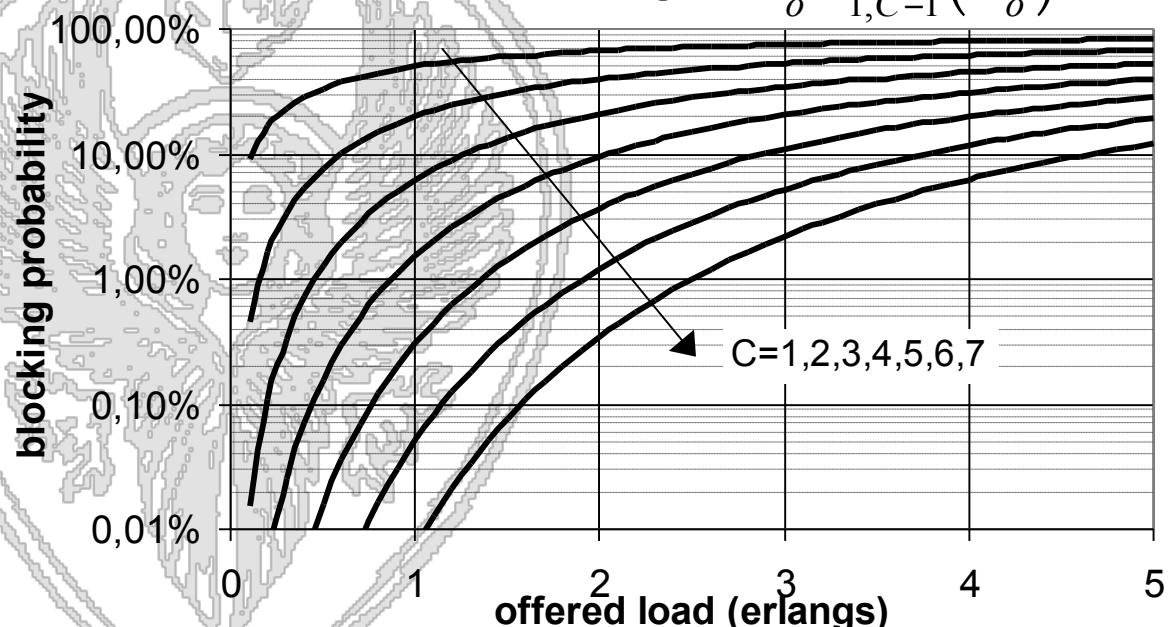
Pianificazione di sistemi cellulari



- Fundamental formula for telephone networks planning
 - A_o =offered traffic in Erlangs

$$E_{1,C}(A_o) = \frac{A_o E_{1,C-1}(A_o)}{C + A_o E_{1,C-1}(A_o)}$$

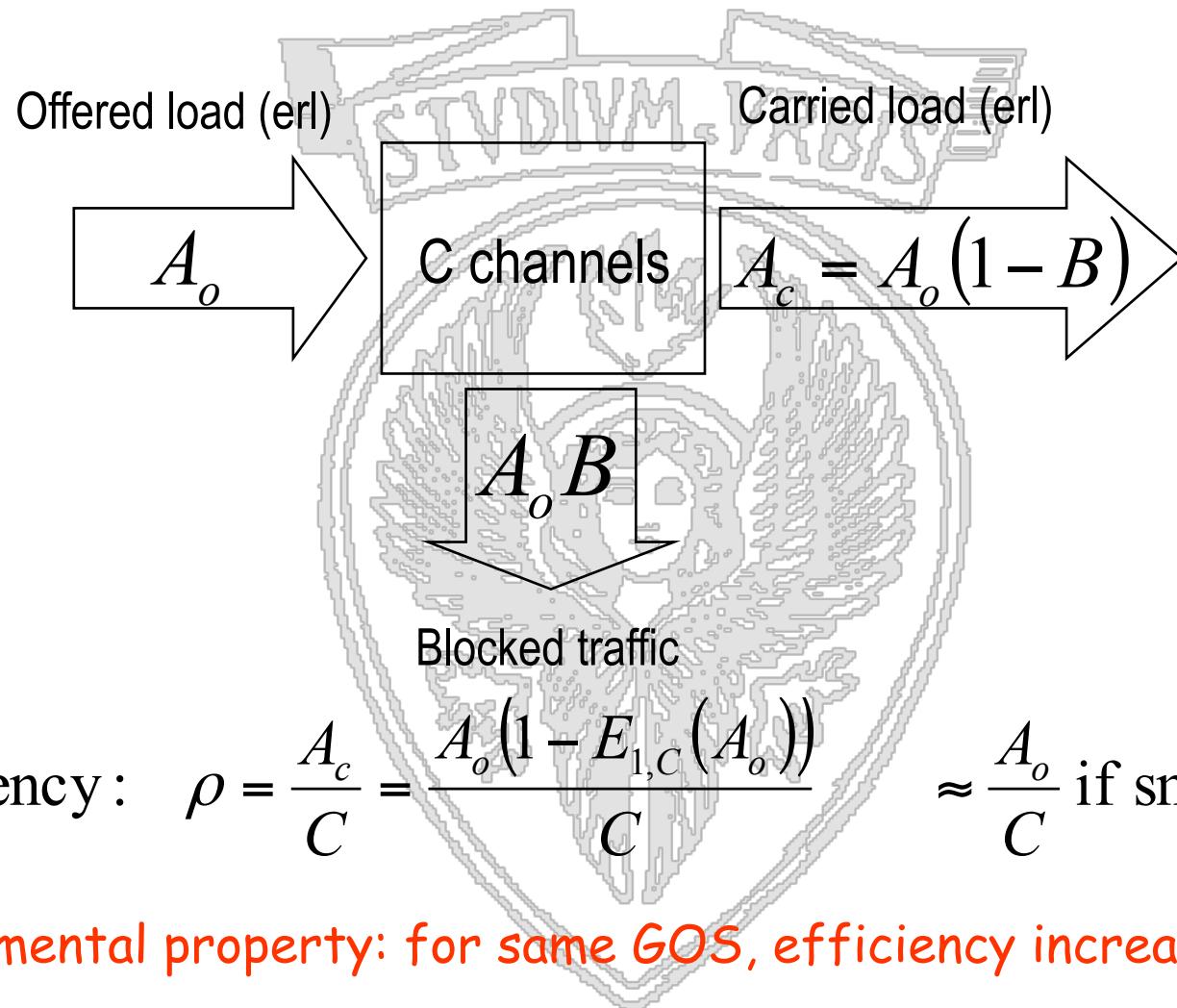
$$\Pi_{block} = \frac{\frac{A_o^C}{C!}}{\sum_{j=0}^C \frac{A_o^j}{j!}} = E_{1,C}(A_o)$$

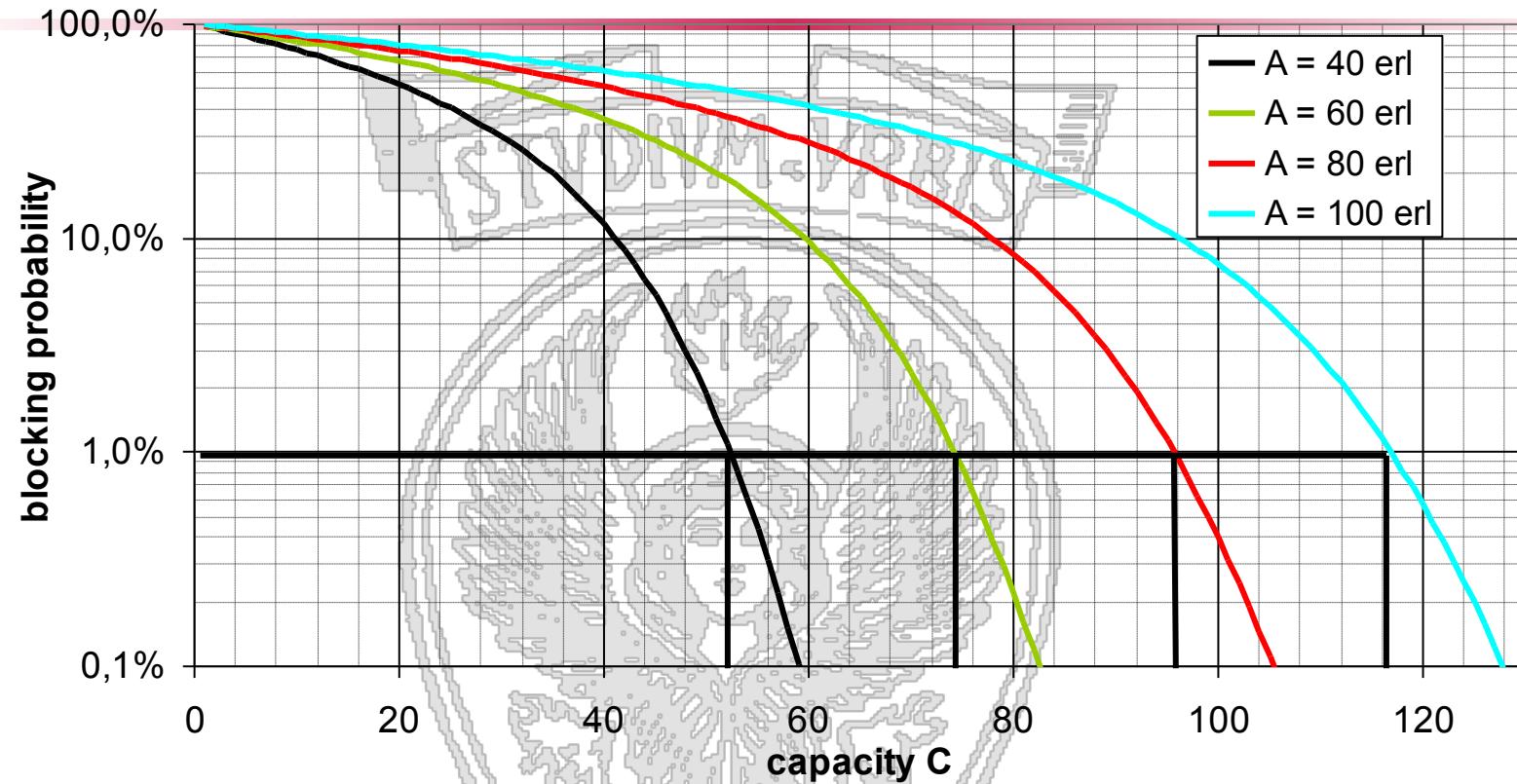




- Target: support users with a given Grade Of Service (GOS)
 - GOS expressed in terms of upper-bound for the blocking probability
 - ✓ GOS example: subscribers should find a line available in the 99% of the cases, i.e. they should be blocked in no more than 1% of the attempts
- Given:
 - ✓ Offered load A_o
 - ✓ Target GOS B_{target}
 - C (number of channels) is obtained from numerical inversion of

$$B_{\text{target}} = E_{1,C}(A_o)$$





GOS = 1% maximum blocking.

Resulting system dimensioning
and efficiency:

40 erl	$C \geq 53$	$\rho = 74.9\%$
60 erl	$C \geq 75$	$\rho = 79.3\%$
80 erl	$C \geq 96$	$\rho = 82.6\%$
100 erl	$C \geq 117$	$\rho = 84.6\%$

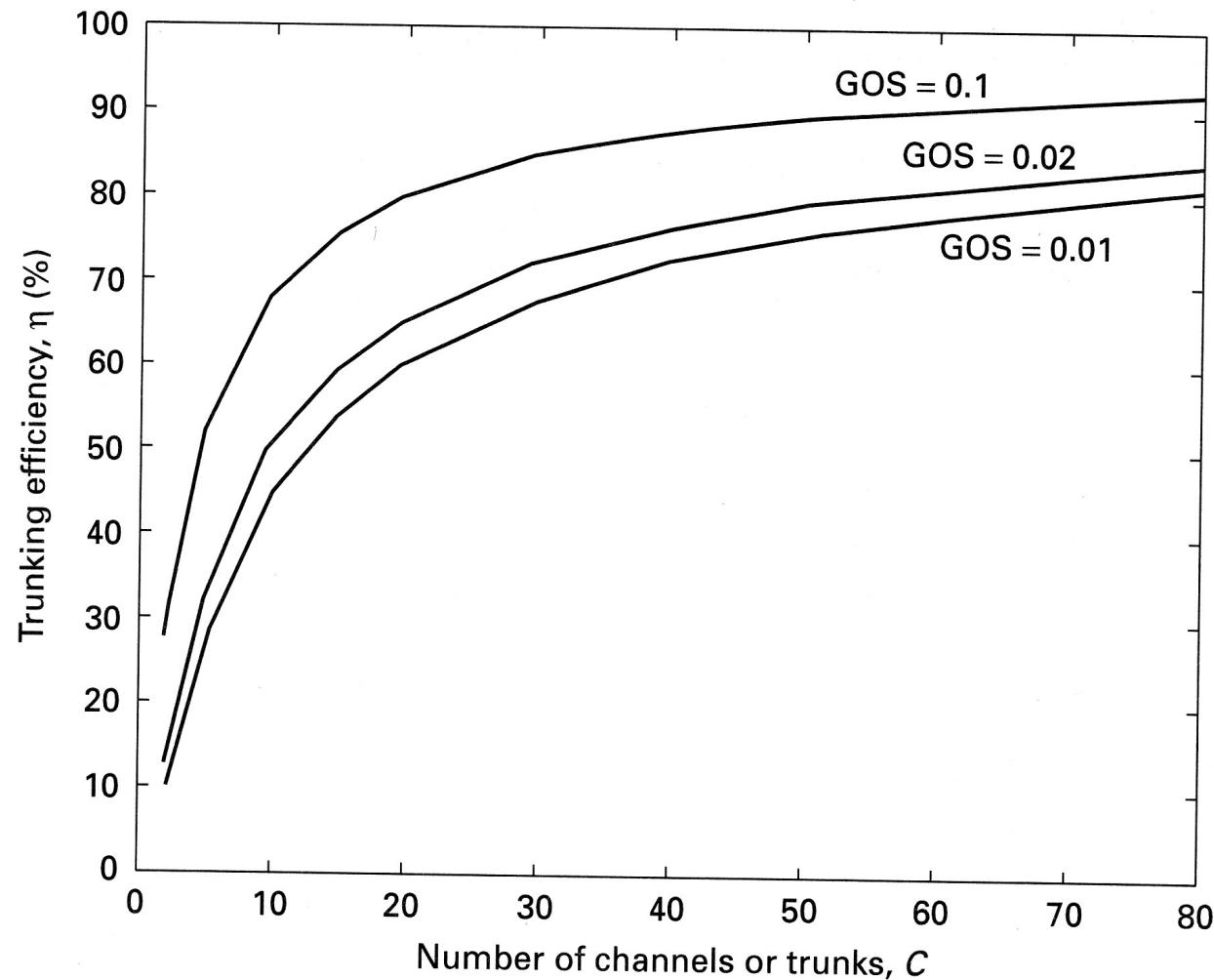


FIGURE 4.21 Trunking efficiency plots.



Example: How many channels are required to support 100 users with a GOS of 2% if the average traffic per user is 30 mE?

$$100 \times 30 \text{ mE} = 3 \text{ Erlangs}$$

$$3 \text{ Erlangs} @ 2\% \text{ GOS} =$$

8 channels

Trunks	0.01	0.015	0.02	0.03
P(B)				
1	0.010	0.015	0.020	0.031
2	0.158	0.190	0.223	0.262
3	0.465	0.586	0.698	0.795
4	0.870	0.992	1.092	1.269
5	1.361	1.624	1.857	2.077
6	1.918	2.114	2.277	2.544
7	2.503	2.743	2.936	3.260
8	3.129	3.405	3.627	3.887
9	3.783	4.096	4.345	4.748
10	4.462	4.808	5.094	5.529

ErlangB Online calculator:

<http://mmc.et.tudelft.nl/~frits/Erlang.htm>



Cell size (radius R) may be determined
on the basis of traffic considerations

Given a provider with 50 channels available, how many users can be supported

If each user makes an average of 4 calls/hour, each call lasting on average 2 minutes?

■ First step:

- Given num channels and GOS
 - C=50 available channels in a cell
 - Blocking probability $\leq 2\%$
- Evaluate maximum cell (offered) load
 - From Erlang-B inversion(tables)
 $A=40.25 \text{ erl}$
- Second step

- Given traffic generated by each user
 - Each user: 4 calls/busy-hour
 - Each call: 2 min on average
 - $A_i = 4 \times 2 / 60 = 0.1333 \text{ erl/user}$
- Evaluate max num of users in cell
 - $M = 40.25 / 0.1333 \sim 302$

Second question: if the user density
Is 500 users/km² how should we
Set the cell radius?

$$\delta = \frac{M}{\pi R^2} \Rightarrow R = \sqrt{\frac{M}{\pi \delta}}$$

→ Third step:

- ⇒ Given density of users
 $\rightarrow \delta = 500 \text{ users/km}^2$
- ⇒ Evaluate cell radius

$$\Rightarrow R \sim 438 \text{ m}$$

Meglio
con
area dell'
esagono !



- Three service providers are planning to provide cellular service for an urban area. The target GOS is 2% blocking. Users make 3 calls/busy-hour, each lasting 3 minutes in average ($A_i=3/20=0.15$)
 - Question: how many users can support each provider?
- Provider A configuration: 20 cells, each with 40 channels
- Provider B configuration: 30 cells, each with 30 channels
- Provider C configuration: 40 cells, each with 20 channels

→ Provider A:

- ⇒ 40 channels/cell
- ⇒ at 2%: $A_o=30.99 \text{ erl}/\text{cell}$
- ⇒ 619.8 erl-total (20 cells)
- ⇒ **M=4132** overall users

→ Provider B:

- ⇒ 30 channels/cell
- ⇒ at 2%: $A_o=21.93 \text{ erl}/\text{cell}$
- ⇒ 657.9 erl-total (30 cells)
- ⇒ **M=4386** overall users

→ Provider C:

- ⇒ 20 channels/cell
- ⇒ at 2%: $A_o=13.18 \text{ erl}/\text{cell}$
- ⇒ 527.2 erl-total (40 cells)
- ⇒ **M=3515** overall users

Compare case A with C! The reason is the lower efficiency of 20 channels versus 40





- Three service providers are planning to provide cellular service for an urban area. The target GOS is 2% blocking. Users make 3 calls/busy-hour, each lasting 3 minutes in average ($A_i=3/20=0.15$)
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- ⇒ 30 channels/cell
- ⇒ at 2%: $A_o=21.93$ erl/cell
- ⇒ 657.9 erl-total (30 cells)
- ⇒ **M=4386** overall users

→ **Provider C:**

il fatto
di avere più celle
con pochi canali
(ad esempio perche'
ho scelto un K
diverso)
non porta
ad un vantaggio
in termini di
capacità del
sistema

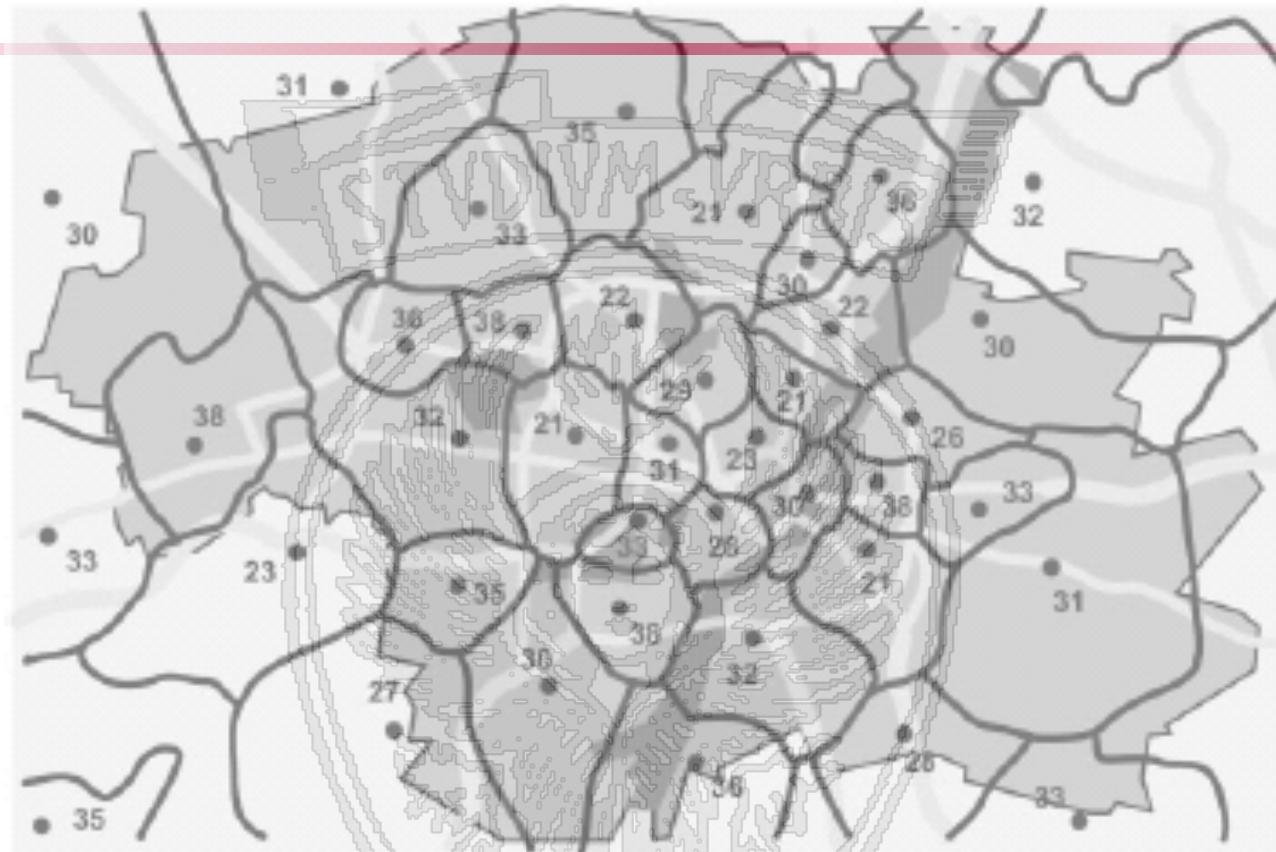
Compare case A with C! The reason is the lower efficiency of 20





- Assume cluster K=7
- Omnidirectional antennas: $S/I = 18.7 \text{ dB}$
- 120° sectors: $S/I = 23.4 \text{ dB}$
- 60° sectors: $S/I = 26.4 \text{ dB}$
- Sectorization yields to better S/I
- BUT: the price to pay is a much lower trunking efficiency!
- With 60 channels/cell, GOS=1%,
 - Omni: 60 channels $A_o = 1 \times 46.95 = 46.95 \text{ erl}$
 $\rho = 77.46\%$
 - 120°: $60/3 = 20$ channels $A_o = 3 \times 12.03 = 36.09 \text{ erl}$
 $\rho = 59.54\%$
 - 60°: $60/6 = 10$ channels $A_o = 6 \times 4.46 = 26.76 \text{ erl}$
 $\rho = 44.15\%$

Erlang supportati per settore



Shaped by terrain, shadowing, etc

Cell border: local threshold, beyond which neighboring BS signal is received stronger than current one