Computer Network Performance Projects. Project topic: task assignment and path planning for swarms of aerial drones

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***Abstract*—This is a short summary of some discussions we had in class, when I introduced some of the topics for the project of the course of Computer Network Performance. We hereby consider the problem of assigning tasks to multiple cooperating drones, under several application contexts, including complete exploration of an area of interest, and maximization of number of critical targets being visited within a given time.**

I. Introduction

While nowadays single remotely controlled drones are commonly used for monitoring regions of interest in multiple applications, we outline situations in which monitoring target points in a sequence is not sufficient for the purpose of reconstructing a sufficiently representative view of the ongoing events, and the application requires monitoring from multiple drones working concurrently.

For instance, in critical scenarios where drones are looking for survivors, or aim at distributing medicines or water to trapped or disabled humans or animals, it is of uttermost urgency for them to deliver service within strict time constraints.

We consider also completely different application contexts requiring simultaneous monitoring of multiple points in a region. In agriculture scenarios, for example, especially in developing countries, drones may be deployed with the objective of measuring the spread of viruses in crop fields. In these contexts it is important that multiple points of the region of interest be monitored simultaneously or in a short time window, to be able to correlate simultaneous measurements of geographically distant points, and reconstruct the front-line of a spreading disease.

In these applications, we need a monitoring network formed by multiple aerial devices capable of sampling multiple targets in parallel, with the objective to minimize the overall sampling time or maximize the monitoring jobs done within a given time span.

In this work we formulate the problem of assigning monitoring tasks and planning paths for multiple drones forming a swarm with the purpose of achieving a common goal.

We contribute an analytic formulation of the mentioned problems in terms of MILPs.

II. Problem formulation

We assume that the ultimate objective of the monitoring swarm is to monitor a given set of target points Ψ in the region of interest. Each drone flying above a surface can monitor a finite area with a certain precision, which depends on the type of sensing devices mounted on-board, on the height of the drone, on the presence of obstacles on the ground and on several other factors. It follows that even for applications requiring a continuous and complete coverage of the area of a region of interest, we can reasonably approximate this requirement to the problem of monitoring a given set of target points.

As a simplifying example, consider a smooth and flat region of interest. With a rough approximation, we can assume that a drone is able to monitor a circular area of radius R (a function of its height with respect to the ground). In this case, a potential representation of Ψ could be the set of central points of the tiles of a squared grid, with side of length $\sqrt{2}R$, such that by monitoring the points of Ψ we ensure the inspection of all the tiles and consequently of the entire region of interest.

Let $\mathcal{U}$ be the set of aerial vehicles forming the swarm, and let d_u be the home depot of drone $u \in \mathcal{U}$, i.e. the point of the region of interest from which the drone departs and to which it must go back for recharging and device recollection.

We introduce the following binary decision variables $x^u_{ij} \in \{0,1\}$, with $i,j \in \Psi \cup \{d_u\}$ and $u \in \mathcal{U}$ to represent the decision to let the vehicle u traverse the region from the region point i to the region point j, exploring them in a sequence ($x^u_{ij} = 1$) or not ($x^u_{ij} = 0$).

A. Coverage of target points

Depending on the formulation of the optimization problem, we may want to tackle coverage by means of an explicit constraint, by imposing coverage of all the target points (or a given percentage of them), or we may prefer to have coverage as an objective to be maximized.

1) Coverage as a constraint: In order to impose coverage completeness, we want every target point to be covered by at least one vehicle. This implies that for every target point $i \in \Psi$ there must be an edge adjacent to i which is traversed at least once, by at least a vehicle $u \in \mathcal{U}$. This translates into the following set of constraints:

$$\sum_{u\in\mathcal{U},j\in\Psi\cup\{d_u\}} x^u_{ij} \geq 1, \forall i \in \Psi. \tag{1}$$

The use of the equality sign in this constraint, would instead imply that each point will be covered exactly once by only one vehicle. Notice that, depending on energy availability, or time requirements, and number of available drones, it may be impossible for the given monitoring network to cover all the target points. Observe that the introduction of this constraint may affect feasibility of the problem solution, especially if used jointly with constraints on battery life or target exploration time. Depending on the application, it may be helpful to tackle coverage as an objective of the problem[1], as we show in the following paragraph.

2) Coverage as an objective of the optimization problem: When the energy availability or the number of vehicles are limited such that it is not granted that the exploration of all the targets is feasible, it may be convenient to formulate the problem as a coverage maximization problem.

Given the variables defined so far, we do not have an explicit representation of this objective. Indeed, while the expression $\sum_{u\in\mathcal{U},j\in\Psi\cup\{d_u\}} x^u_{ij}$ is equal to 0 when the target point i is not covered, this expression may have a value that is larger than 1, when the target is covered.

We may have two situations: (a) each target point is explored no more than once by no more than one vehicle, or (b) each target point is explored potentially multiple times by the same or by multiple vehicles.

In the first scenario (a), the expression $\sum_{u\in\mathcal{U},j\in\Psi\cup\{d_u\}} x^u_{ij}$ is either 0 (point i not covered) or 1 (point i covered exactly once by one drone). Hence if the purpose is to maximize the number of covered points we need to use the following objective function:

$$\max \sum_{i\in\Psi} \sum_{u\in\mathcal{U},j\in\Psi\cup\{d_u\}} x^u_{ij}. \tag{2}$$

In the second scenario (b), we need to introduce a new binary variable $\delta_i \in \{0,1\}$, for $i \in \Psi$, to represent the decision to cover target i a non null number of times, by any number of drones ($\delta_i = 1$), or not covering it ($\delta_i = 0$). We then need to have the following objective function,

$$\max \sum_{i\in\Psi} \delta_i. \tag{3}$$

In scenario (b), to represent a relationship between the values of the variables δ_i and x^u_{ij}, we impose an additional constraint

$$\delta_i \leq \sum_{u\in\mathcal{U},j\in\Psi\cup\{d_u\}} x^u_{ij}, \tag{4}$$

where the inequality bounds the value of the variables $\delta_i = 0$ when the point i is not covered by any vehicle, while all the covered points j will have $\delta_j = 1$ because of the $\max$ operator given by equation 3.

B. Cyclic trajectory constraints

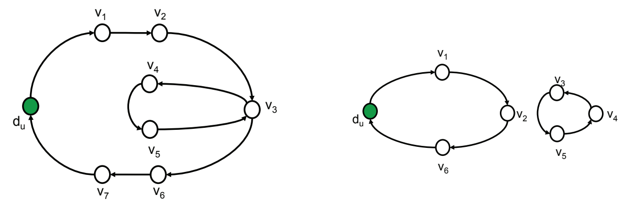
To impose a cyclic trajectory with an explicit constraint acting on those variables, we impose the following equation 5 which implies that any drone that enters the point j should also leave it,

$$\sum_{i\in\Psi\cup\{d_u\}} x^u_{ij} = \sum_{k\in\Psi\cup\{d_u\}} x^u_{jk}, \forall j \in \Psi \cup \{d_u\}, u \in \mathcal{U}, \tag{5}$$

and with the following equation 6, we impose the connectedness of the trajectory with the home depot d_u of drone u,

$$\sum_{j\in\Psi} x^u_{d_u j} = 1, \forall u \in \mathcal{U}. \tag{6}$$

[1]Notice that coverage can never be tackled as both an objective or a constraint simultaneously



(a) (b)

Fig. 1. Path with subcycles(a), path with disconnected cycles (b)

Notice that the constraints of Equation 5 and 6 do not preclude the formation of disconnected cycles or subcycles like those in the Figure 1.

For the purpose of eliminating solutions containing either sub-cycles or disconnected cycles we could write an additional constraint, observing that any cycle, not containing sub-cycles contains a number of nodes which is equal to the number of edges. Therefore, for any given subset of nodes Ψ, i.e. $\forall \Omega \subset \Psi$, s.t. it must hold that

$$\sum_{i,j \in \Omega} x_{ij}^u \leq |\Omega| - 1.$$

In fact, if some nodes form a disconnected cycle or a sub-cycle, by choosing Ω containing the only nodes of the sub-cycle we observe a violation of this constraint. Sub-cycles containing the home depot d_u are instead not captured by this constraint, but are already excluded by constraint 6, which imposes that the depot is traversed only once in each drone trajectory.

Nevertheless, in order to impose this condition for any drone u and for any choice of Ω not containing the depot d_u we would have $|U| \cdot 2^{|\Psi|}$ additional constraints, which would cause the problem formulation to be exponential in the input size.

By contrast, by adding new auxiliary variables, we can formulate the condition of avoidance of disconnected cycles and sub-cycles, with a number of variables and constraints which is polynomial in the input size of the problem.

We therefore introduce the auxiliary variables $z_i^u \in \{1, \ldots, |\Psi|\}$ to represent the ordinal position of the target point $i \in \Psi$ in the trajectory of drone u. These variables should respect the following constraint:

$$z_j^u - z_i^u \geq x_{ij}^u + |\Psi| \cdot (x_{ij}^u - 1), \forall u \in \mathcal{U}, \forall i, j \in \Psi, \quad (7)$$

with also

$$z_i \in \{1, \ldots, |\Psi|\}, \forall i \in \Psi. \quad (8)$$

Fig. 2. DJI Phantom 4 PRO

In summary, in the final problem formulation, the equations 5, 6, 7 and 8 impose that each drone traverses a path starting and ending in the home depot, including a subset of the target points, without traversing the same target more than once.

C. Energy consumption of a drone

It must be noted that, without a limitation on execution time or battery life, the entire problem could be solved by using a single drone, inspecting all the target points in a sequence.

Nevertheless, the battery life of a drone imposes a strict limitation on the number of target points that can be inspected in a unique flight, before going back to the depot point for recharging.

Just to make an example, the DJI Phantom 4 Pro [1] shown in Figure 2, on sale for about €1200, with no additional payload, can fly for about 30 minutes and needs about 1hr to recharge its battery from 15% to 100%.

If we do not consider recharging and demand drones to complete their tasks and go back to the depot for recollection before the battery is completely depleted, we should give a model of energy consumption for movement and for target point inspection.

As a first approximation we can consider an energy consumption for movement which is proportional to the length of the traversed distance. Under this approximation if edge x_{ij} is l_{ij} meters long, the battery consumption for movement is proportional to l_{ij}. Likewise, if ϕ_i is the necessary time to inspect the target point i, we can consider a contribution to the energy consumption related to target inspection proportional to ϕ_i. It follows that, as a first approximation we can model the energy consumption of a drone along its path as the summation of terms related to both inspected target points and traversed edges. We incorporate these two energy consumption components into an edge-based measure, so we model the energy ω_{ij} that a drone spends for exploring the region point i and moving to the region point j, with $i, j \in \Psi \cup \{d_u : u \in \mathcal{U}\}$ as follows: $\omega_{ij} \triangleq a \cdot \phi_i + b \cdot l_{ij}$ where a and b are dimensional coefficients which reflect the energy consumption for a unit time of inspection of a target and unit length of movement, respectively, and where we consider $\phi_i = 0$ if i is a depot of a drone, i.e. $i \in \{d_u : u \in \mathcal{U}\}$[2].

Similarly to coverage, the energy consumption of a drone can be tackled either as an objective to be minimized or as a constraint. We explain the two approaches in the following paragraphs.

1) Constraint on battery life: Given that the term ω_{ij} represents the energy that a drone u spends on point i and on the route from i to j, the *total energy spent by a drone* along its trajectory, including inspections on targets, is

$$\sum_{ij \in \Psi \cup \{d_u\}} \omega_{ij} \cdot x_{ij}^u,$$

for $u \in \mathcal{U}$. Assuming that the energy available for the drones is heterogeneously distributed, i.e. a drone u has b_u available energy (in energy units), with $b_u \gtreqless b_w$, for any $u, w \in \mathcal{U}$ and $u \neq w$, a constraint that imposes that drone u does not consume more than its available energy in its route is the following:

$$\sum_{ij \in \Psi \cup \{d_u\}} \omega_{ij} \cdot x_{ij}^u \leq b_u, \forall u \in \mathcal{U}. \tag{9}$$

2) Energy saving as an objective of the optimization problem: Depending on the application context, it may be beneficial to minimize the total energy consumption of the drones. Let ς_u be the energy consumed by a drone in its route, it holds

$$\varsigma_u = \sum_{ij \in \Psi \cup \{d_u\}} \omega_{ij} \cdot x_{ij}^u.$$

It is easy to see that *a solution that minimizes the total energy spent by the drones, also minimizes the average energy spent*. This is because, if a route assignment solution minimizes $\sum_{u \in \mathcal{U}} \varsigma_u$, it also minimizes the value of $\sum_{u \in \mathcal{U}} \varsigma_u / |\mathcal{U}|$.

[2]We are implicitly assuming that the depot of a drone does not coincide with any of the target points

Hence, for the purpose of minimizing either the *total or the mean energy spent by the drones*, the objective function of the optimization problem should be the following:

$$\min \sum_{u \in \mathcal{U}} \sum_{ij \in \Psi \cup \{d_u\}} \omega_{ij} \cdot x_{ij}^u. \tag{10}$$

Nevertheless, it must be noted that this objective may result in an uneven distribution of the energy requirements of the drones.

By contrast, it may be beneficial to minimize the maximum energy spent by any drone, which implies some load balancing among the drones. In order to do so, we can introduce a new decision variable (this time a continuous one) γ to represent the maximum energy spent by any drone, and minimize the value of γ:

$$\begin{array}{ll} \min \gamma & (a) \\ \gamma \geq \sum_{u \in \mathcal{U}} \sum_{ij \in \Psi \cup \{d_u\}} \omega_{ij} \cdot x_{ij}^u. & (b) \end{array} \tag{11}$$

Notice that without an explicit constraint, such as the one represented by Equation 9, there is no guarantee that the solution does not exceed the energy availability of a drone.

D. Time to visit a target

For safety critical applications the waiting time spent between the launch of the swarm and the inspection of a target, is a very important performance metric which needs to be accounted either as a constraint or in the objective function. In order to model visit time in our problem we modify the subcycle elimination constraints of Equations 7 and 8. We replace the integer variables z_i^u with continuous variables $t_i^u \in \mathbb{R}_0^+$ representing the time at which drone u visits the target point $i \in \Psi$ or 0 if drone u does not visit point i at all. We modify the constraint given in Equation 7 in the following, where L is a large upper bound on t_i^u, $\forall i, u$:

$$\begin{aligned} t_j^u - t_i^u \geq \omega_{ij} x_{ij}^u + L(x_{ij}^u - 1), & \\ \forall i \in \Psi \cup \{d_u\}, \quad j \in \Psi, \quad u \in \mathcal{U} & \end{aligned} \tag{12}$$

$$t_{d_u} = 0, \quad u \in \mathcal{U} \tag{13}$$

$$t_i^u \leq \sum_{j \in \Psi \cup \{d_u\}} L \cdot x_{ij}^u, \quad u \in \mathcal{U}. \tag{14}$$

With Equation 12 we calculate the time to explore any target in the route of drone u and at the same time we rule out potential sub-cycles or disconnected cycles from the solution. Equation 13 sets the initial exploration time to 0, while Equation 14 sets to zero the exploration time of all the targets not explored by drone u.

With this setting, the time to visit target i is $\tau_i = \sum_{u\in\mathcal{U}} t_i^u$.

As a consequence, **under the assumption that all targets are inspected**, the *average inspection time* is

$$(1/|\Psi|) \cdot \sum_{i\in\Psi} \tau_i. \tag{15}$$

Notice that another way to optimize the inspection time is to minimize the time to complete the target inspection.

By introducing the decision variable $\tau_{\texttt{final}}$ as objective function to be minimized, and adding the following constraint to the formulation, we obtain a representation of the time until the target inspection is completed:

$$\tau_{\texttt{final}} \geq \tau_i + \phi_i, \quad \forall i \in \Psi \tag{16}$$

III. Optimization problems

A. Maximum Coverage under Battery Constraints

When energy available is constrained, it may happen that the number of targets is too high to be inspected by the monitoring network. In such a case it makes sense to formulate the problem of maximizing the number of targets being covered under the limitation of energy available per drone.

The problem can be formulated as an ILP, as follows:

$$\begin{array}{lr}
\max \sum_{i\in\Psi} \sum_{u\in\mathcal{U}, j\in\Psi\cup\{d_u\}} x_{ij}^u & (a)\\
s.t. & \\
\sum_{i\in\Psi\cup\{d_u\}} x_{ij}^u = \sum_{k\in\Psi\cup\{d_u\}} x_{jk}^u, \forall j \in \Psi\cup\{d_u\}, u\in\mathcal{U} & (b)\\
\sum_{j\in\Psi} x_{d_u j}^u = 1, \forall u \in \mathcal{U} & (c)\\
z_j^u - z_i^u \geq x_{ij}^u + |\Psi|\cdot(x_{ij}^u - 1), \forall u\in\mathcal{U}, \forall i,j\in\Psi & (d)\\
z_i \in \{1,\ldots,|\Psi|\}, \forall i \in \Psi & (e)\\
\sum_{ij\in\Psi\cup\{d_u\}} \omega_{ij}\cdot x_{ij}^u \leq b_u, \forall u\in\mathcal{U} & (f)\\
x_{ij}^u \in \{0,1\} & (g)
\end{array}$$

Problem 1. Max Coverage under Limited Battery

In Problem 1, the optimization function expressed by equation (a) is the one already discussed in Equation 2 of Section II-A2. Constraints (b-e) ensure that the trajectory of a drone u is a Hamiltonian cycle on a subset of the target points of Ψ, including the depot d_u and no sub-cycles, nor separate cycles, as we discussed in Section II-B. Finally, constraint (f) of Problem 1 ensures that every drone does not consume more energy than the initially available b_u.

B. MinMaxEnergy

The problem of ensuring complete coverage of all the target points in the area of interest, when minimizing the energy spent by the drone which consumes the most, is formulated as follows, in agreement with Equations 11(a-b).

$$\begin{array}{lr}
\min \gamma & (a)\\
s.t. & \\
\gamma \geq \sum_{u\in\mathcal{U}} \sum_{ij\in\Psi\cup\{d_u\}} \omega_{ij}\cdot x_{ij}^u & (b)\\
\sum_{u\in\mathcal{U}, j\in\Psi\cup d_u} x_{ij}^u \geq 1, \forall i\in\Psi & (c)\\
\sum_{i\in\Psi\cup\{d_u\}} x_{ij}^u = \sum_{k\in\Psi\cup\{d_u\}} x_{jk}^u, \forall j \in \Psi\cup\{d_u\}, u\in\mathcal{U} & (d)\\
\sum_{j\in\Psi} x_{d_u j}^u = 1, \forall u \in \mathcal{U} & (e)\\
z_j^u - z_i^u \geq x_{ij}^u + |\Psi|\cdot(x_{ij}^u - 1), \forall u\in\mathcal{U}, \forall i,j\in\Psi & (f)\\
z_i \in \{1,\ldots,|\Psi|\}, \forall i \in \Psi & (g)\\
x_{ij}^u \in \{0,1\} & (h)
\end{array}$$

Problem 2. Minimization of the Maximum Energy

C. Minimization of the Average Inspection Time

Notice that, any solution which minimizes the average inspection time of the targets, also minimizes the sum of these inspection times. Hence, in our minimization we can get rid of the constant factor $(1/|\Psi|)$ from the expression of the average inspection time of Equation 15.

$$\begin{array}{lr}
\min \sum_{i\in\Psi} \tau_i & (a)\\
s.t. & \\
\sum_{u\in\mathcal{U}, j\in\Psi\cup d_u} x_{ij}^u \geq 1, \forall i\in\Psi & (b)\\
\sum_{i\in\Psi\cup\{d_u\}} x_{ij}^u = \sum_{k\in\Psi\cup\{d_u\}} x_{jk}^u, \forall j \in \Psi\cup\{d_u\}, u\in\mathcal{U} & (c)\\
\sum_{j\in\Psi} x_{d_u j}^u = 1, \forall u \in \mathcal{U} & (d)\\
t_j^u - t_i^u \geq \omega_{ij} x_{ij}^u + L(x_{ij}^u - 1), & \\
\qquad \forall i \in \Psi\cup\{d_u\}, \quad j\in\Psi, \quad u\in\mathcal{U} & (e)\\
t_{d_u} = 0, \quad u\in\mathcal{U} & (f)\\
t_i^u \leq \sum_{j\in\Psi\cup\{d_u\}} x_{ij}^u, \quad u\in\mathcal{U} & (g)\\
\tau_i = \sum_{u\in\mathcal{U}} t_i^u & (h)\\
x_{ij}^u \in \{0,1\} & (i)\\
t_i^u, \tau_i \in \mathbb{R}_0^+ & (j)
\end{array}$$

Problem 3. Minimization of the Average Inspection Time

The constraint (b) of problem 3 is for target coverage completeness, constraints (c-d) are for having cyclic trajectories for each drone, constraints (e-g) are for making t_i^u assume the value of the inspection time of target i by drone u, or 0 if not inspected. Finally constraint (h) is meant to calculate the inspection time of each target point.

D. MinCompletionTime

In the following we aim at minimizing the completion time of the target points inspections. In order to do so, we consider the definition of $\tau_{\texttt{final}}$ given in Equation 16, which we use to replace the objective function of Problem 3 and its constraint (h).

We obtain the following new problem:

$$
\begin{array}{lr}
\min \tau_{\texttt{final}} & (a) \\
s.t. & \\
\sum_{u\in\mathcal{U},j\in\Psi\cup d_u} x_{ij}^u \geq 1, \forall i \in \Psi & (b) \\
\sum_{i\in\Psi\cup\{d_u\}} x_{ij}^u = \sum_{k\in\Psi\cup\{d_u\}} x_{jk}^u, \forall j \in \Psi\cup\{d_u\}, u \in \mathcal{U} & (c) \\
\sum_{j\in\Psi} x_{d_u j}^u = 1, \forall u \in \mathcal{U} & (d) \\
t_j^u - t_i^u \geq \omega_{ij} x_{ij}^u + L(x_{ij}^u - 1), & \\
\qquad \forall i \in \Psi\cup\{d_u\}, \quad j \in \Psi, \quad u \in \mathcal{U} & (e) \\
t_{d_u} = 0, \quad u \in \mathcal{U} & (f) \\
t_i^u \leq \sum_{j\in\Psi\cup\{d_u\}} x_{ij}^u, \quad u \in \mathcal{U} & (g) \\
\tau_{\texttt{final}} \geq \sum_{u\in\mathcal{U}} t_i^u + \phi_i, \quad \forall i \in \Psi & (h) \\
x_{ij}^u \in \{0,1\} & (i) \\
t_i^u, \tau_{\texttt{final}} \in \mathbb{R}_0^+ & (j)
\end{array}
$$

Problem 4. Minimization of the Completion Time

The constraint (b) of problem 3 is for target coverage completeness, constraints (c-d) are for having cyclic trajectories for each drone, constraints (e-g) are for making t_i^u assume the value of the inspection time of target i by drone u, or 0 if not inspected. Finally constraint (h) is meant to calculate the maximum time needed to complete the target inspection.

IV. CONCLUSION

The student will be required to formulate and/or implement one of the variants of the centralized task assignment problem for swarms of aerial drones, and to perform an experiment where some important performance characteristics will be evaluated under varying workload conditions.

REFERENCES

[1] DJI, "Phantom 4 PRO," 2018. [Online]. Available: https://www.dji.com/phantom-4-pro