ITTI

Computer Graphics
and Visualisation

Lighting and Shading

Overhead Projection (OHP) Overviews

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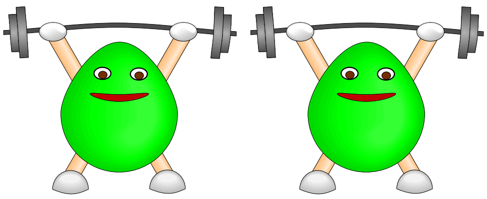

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Lighting and Shading

Introduction

- Generation of **realistic images** given description of 3D scene...
  - object shape
  - object location and orientation
  - surface properties
  - light sources
- ...and viewing information
  - find out which points are visible
  - find out how these points are lit
- Here, we are only concerned with the second of these problems

Introduction

Having chosen a view of our scene, need to establish how the visible points are illuminated

- Each point has two sources of illumination
  - **direct illumination**
    - light which arrives straight from the light sources
  - **indirect illumination**
    - light which arrives after interacting with the rest of the scene
- Can group algorithms according to how they handle these two components
  - **global illumination** algorithms
    - care is taken to evaluate both
  - **local illumination** algorithms
    - only direct light is accounted for

Introduction

Typically, when generating realistic images, one of two approaches is adopted

- **Empirically-based rendering**
  - image is found using enough fudge-factors to ensure an accurate image
- **Physically-based rendering**
  - image is found by modelling the processes which determine how light is transported around a real scene
- Physically-based algorithms are more computationally expensive, but give better realism

Introduction

We can further classify rendering algorithms

- **Image-space algorithms**
  - first find out visible points and then how these points are illuminated
  - e.g. ray-tracing
- **Object-space algorithms**
  - first find out how the whole scene is illumination and then which points are visible
  - e.g. radiosity

Introduction

This course will cover the following topics

- Local illumination modelling
  - points light sources
  - ambient lighting
  - diffuse and specular reflection
- Shading
  - flat, Gouraud and Phong
- Texture mapping and transparency
  - pattern mapping
  - bump mapping
  - environment mapping

Introduction

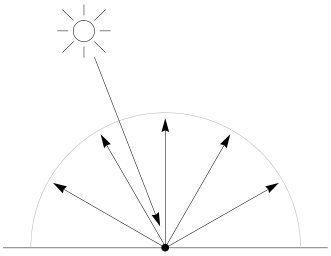
- Ray-tracing
  - image-space algorithm
  - sharp shadows
  - object intersections
  - acceleration techniques
  - image aliasing
- Radiosity
  - object-space algorithm
  - form-factors
  - hemi-cube
  - progressive refinement

Local Illumination Models

Types of Reflection

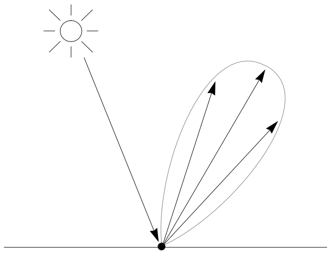
There are two ways in which reflection can occur

- **Diffuse reflection**
  - light is scattered uniformly, making the surface look *matte*



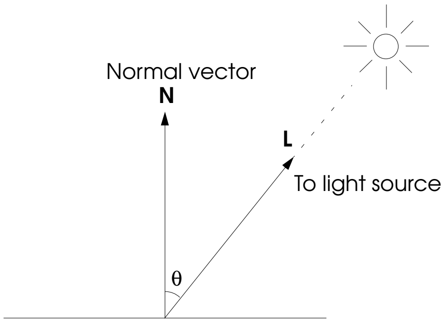
Types of Reflection

- **Specular reflection**
  - a large proportion of the incident light is reflected over a narrow range of angles, making the surface look *shiny*



- **Combined reflection**
  - in practice, many materials exhibit both of these effects to different degrees

Surface Orientation



- **N** is **surface normal** and
- **L** is direction *to* light source
- Vectors **N** and **L** are *unit vectors*
- θ is **angle of incidence**

Illumination Model 1

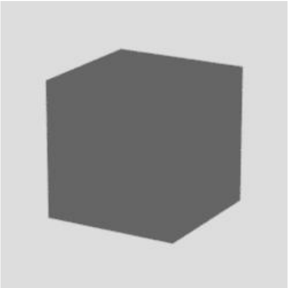
We only consider illumination from ambient lighting

- **Ambient lighting**
  - uniform illumination from all directions
  - arises from multiple reflections
  - I_a = intensity of ambient light
  - $I = k_d I_a$
- **Diffuse reflection coefficient**, k_d
  - measures reflectivity of suface for diffuse light
  - values in the range 0 - 1

Illumination Model 1

There is a problem...

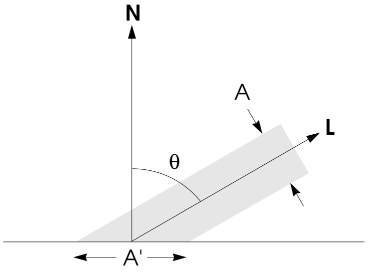
- Lose 3D information because an object is illuminated uniformly
- An example



- Therefore, need to consider reflection of light from a *localised* source

Lambert's Cosine Law

- Amount of light received from light source depends on orientation of surface



- ❑ light intensity I_p of area A
- ❑ light spread over area A'
- ❑ A' = A/cosθ and A = A' cosθ, so...
- ❑ effective intensity I_e at surface is

$$I_e = I_p \cos\theta$$

Diffuse Reflection

We can calculate intensity of diffusely reflected light as

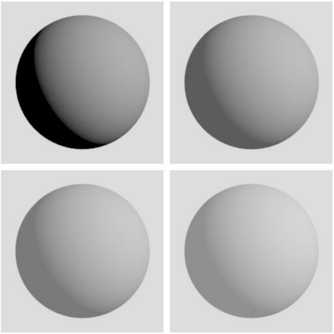
- $I = k_d I_p \cos\theta$
- But $\cos\theta = \mathbf{N} \bullet \mathbf{L}$ since **N** and **L** are unit vectors
- Amount of light diffusely reflected is

$$I = k_d I_p (\mathbf{N} \bullet \mathbf{L})$$

Illumination Model 2

We consider illumination from ambient and a point light source

- I = *ambient* + *diffuse*
- $I = k_d I_a + k_d I_p (\mathbf{N} \bullet \mathbf{L})$

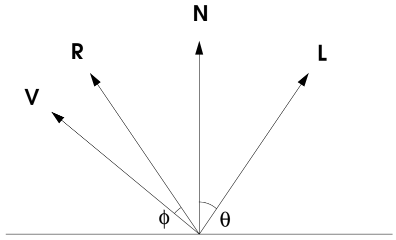


Specular Reflection

Not all surfaces exhibit diffuse reflection

- Surfaces that only show diffuse reflection are dull and **matte**
- In reality, many surfaces are *shiny*
  - ❏ at certain viewing angles, shiny surfaces produce *specular highlights*
  - ❏ highlights occur over a narrow range of angles
  - ❏ colour of highlight usually same as the illuminating light
- Mirrors are examples of ideal specular reflection
  - ❏ angle of incidence equals angle of reflection

More Notation!

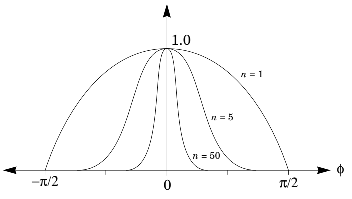


- **N** is surface normal
- **L** is direction *to* light source
- **V** is direction *towards* view point
- **R** is direction of ideal specular reflection
- The intensity of specular reflection depends on the angle ϕ such that $I_s \propto f(\phi)$

Phong Model

Phong's empirical model provides a simple way of controlling the size of the specular highlight

- $f(\phi) = \cos^n\phi$
- *n* depends on surface properties
  - ❏ for perfect reflector $n = \infty$
  - ❏ for very poor reflector $n = 1$
  - ❏ in practice use $1 \le n \le 200$



Phong Model

We can evaluate the cosine term solely in terms of vectors

- With **R** and **L** normalised

$$\cos\phi = \mathbf{R} \bullet \mathbf{V}$$

- Hence

$$I_s \propto (\mathbf{R} \bullet \mathbf{V})^n$$

- Specular reflection also depends on θ, so that

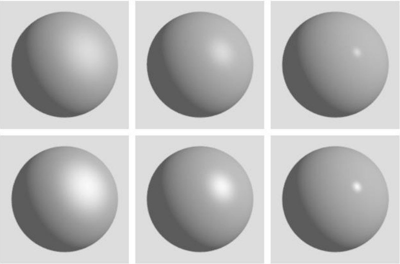
$$I_s \propto W(\theta)\ (\mathbf{R} \bullet \mathbf{V})^n$$

  - ❏ in practice, we set $W(\theta) = k_s$
  - ❏ k_s is the **coefficient of specular reflection**
  - ❏ k_s has values in the range 0 - 1

Illumination Model 3

Reflection from a surface with diffuse and specular properties

- *I = ambient + diffuse + specular*
- $I = k_d I_a + I_p\left[k_d(\mathbf{N} \bullet \mathbf{L}) + k_s(\mathbf{R} \bullet \mathbf{V})^n\right]$
- Examples



Light Source Distance

The intensity of light from a point light source depends on how far away it is

- Physically, intensity falls off as the **square** of the distance
  - the effective intensity of a light source I_p after travelling a distance d is

$$I_e = \frac{I_p}{4\pi d^2}$$

  - in practice, this does not works very well
- Instead, experiment shows that a better formula is

$$I_e = \frac{I_p}{d + d_0}$$

Illumination Model 4

Reflection from a surface when the attenuation of light is included

- *I = ambient + dist-factor(diffuse + specular)*

$$I = k_d I_a + \frac{I_p}{d + d_0}\left[k_d(\mathbf{N} \bullet \mathbf{L}) + k_s(\mathbf{R} \bullet \mathbf{V})^n\right]$$

  - this represents a linear fall-off of intensity
  - d_0 is a constant, used to prevent infinite intensity when $d = 0$
- Multiple light sources
  - use linear superposition

$$I = ambient + \sum_{i=1}^{n} diffuse_i + \sum_{i=1}^{n} specular_i$$

Colour

So far our models make no mention of colour, only light intensities

- Choose a **colour model** and apply the illumination model to each colour component
  - simple colour model is monitor RGB
    - surface defined by k_{dR}, k_{dG} and k_{dB}
    - similarly for the light source
  - an example for the Red component

$$I_R = k_{dR} I_{aR} + \frac{I_{pR}}{d + d_0}\left[k_{dR}(\mathbf{N} \bullet \mathbf{L}) + k_s(\mathbf{R} \bullet \mathbf{V})^n\right]$$

  - assumes specular highlight is the same colour as the light source
- More sophisticated, spectrally-based colour models are available

Applying the Illumination Model

Polygon Shading

Typically, objects are represented by meshes of *polygons*

■ Our illumination model computes the intensity at a *single point* on a surface

■ How can we compute the intensity across the polygon?

- ❏ compute the shade at the centre and use this to represent the whole polygon
  - **flat shading**
- ❏ compute the shade at all points
  - unnecessary and impractical
- ❏ compute shade at key points and interpolate for the rest
  - **Gouraud** and **Phong shading**

Flat Shading

This shading method is the simplest and most computationally efficient

■ Not realistic

- ❏ polygon structure is still evident
  - *faceted* appearance
- ❏ but we can refine the structue to reduce the visual effect

■ Problem with **Mach banding**

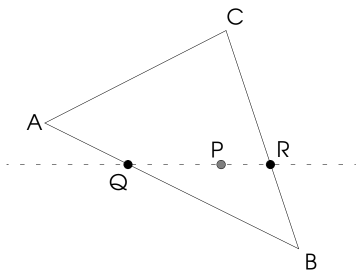
- ❏ the human visual system is extremely good at detecting edges - even when they are not there!
- ❏ abrupt changes in the shading of two adjacent polygons are perceived to be even greater

■ We need to smooth out these changes

Intensity Interpolation

This smooth shading method is also known as *Gouraud shading*

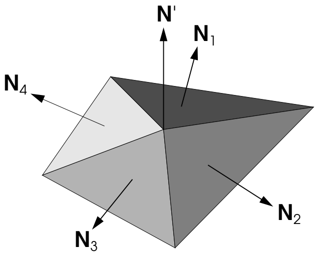
■ Given a polygon and a scan-line, the problem is to determine the intensity at an interior point, such as *P*



- ❏ for this we need the intensity values at the vertices *A*, *B* and *C*

Intensity Interpolation

- First compute the intensity values at each polygon vertex
  - ❏ need vertex normals
  - ❏ compute vertex normal approximately by averaging the surface normals of surrounding polygons



Intensity Interpolation

- Next compute the intensities at points *Q* and *R* where the scan-line and polygon intersect
  - ❏ use linear interpolation

$$I_Q = uI_B + (1-u)\,I_A \quad \text{and} \quad u = \frac{AQ}{AB}$$

$$I_R = wI_B + (1-w)\,I_C \quad \text{and} \quad w = \frac{CR}{CB}$$

- Finally, linearly interpolate between I_Q and I_R to get intensity at point *P*

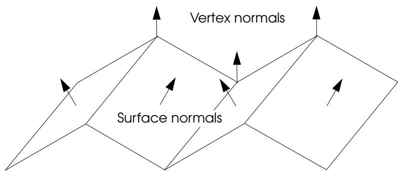
$$I_P = vI_R + (1-v)\,I_Q \quad \text{and} \quad v = \frac{QP}{QR}$$

- This method avoids shading discontinuity for adjacent polygons

Intensity Interpolation

Problems with Gouraud shading

- Smooths out real edges
  - ❏ compute two vertex normals, one for each side of the boundary
- May lose specular reflection if the highlight lies inside a single polygon
- Regular corrugated surfaces will appear to be shaded uniformly



Normal Vector Interpolation

This smooth shading technique is also known as *Phong shading*

- Similar to intensity interpolation, except we linearly interpolate the **surface normal vector** across the polygon
  - ❏ for the example situation described before we have

$$\mathbf{N}_Q = u\mathbf{N}_B + (1-u)\,\mathbf{N}_A$$
$$\mathbf{N}_R = w\mathbf{N}_B + (1-w)\,\mathbf{N}_C$$
$$\mathbf{N}_P = v\mathbf{N}_R + (1-v)\,\mathbf{N}_Q$$

- Now apply illumination model with interpolated normal to find intensity at required point

Normal Vector Interpolation

Pros and cons

- More realistic shading than intensity interpolation
  - specular highlights are preserved
  - Mach banding greatly reduced
- More computationally expensive
  - each interpolation requires complete shading calculation

Texture and Transparency

Texture

We now wish to model surfaces which are *not smooth* or *simply coloured.* There are two kinds of surface texture

- **Patterns** or **colour detail**
  - we superimpose a pattern on a smooth surface but the surface remains smooth
  - also known as *texture mapping*
- **Roughness**
  - we alter the uniformity of the surface using a *perturbation function* that effectively changes the geometry of the surface
  - for example, *bump mapping*
  - but also, *microfacet modelling*

Texture Mapping

Object space mapping

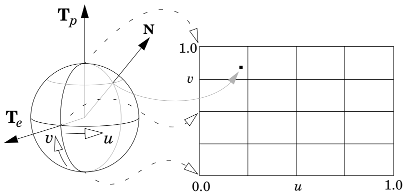
- We map an image onto the surface of an object
  - pattern space specified using (u, v)
  - object space by (θ, ϕ)
  - need to determine the mapping function defined by

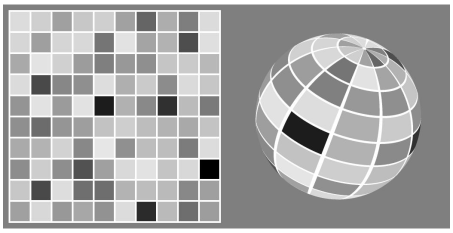
$$u = f(\theta, \phi) \qquad v = g(\theta, \phi)$$

- Hence, given a particular (θ, ϕ)
  - compute the corresponding (u, v)
  - shade the surface with the colour pointed to in the image by (u, v)

Texture Mapping

Mapping function for a sphere





Texture Mapping

There are other ways of mapping patterns

- **Parametric space mapping**
  - ❏ used when surface is defined *parametrically* (*s*, *t*)
  - ❏ similar to object space mapping
  - ❏ map (θ, ϕ) to (*s*, *t*)
- **Environment mapping**
  - ❏ used to simulate mirror-like reflections of the scene surrounding object

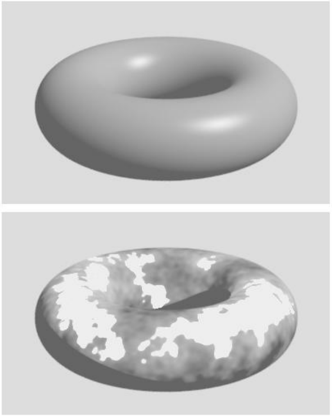
Bump Mapping

This method models features due to large-scale surface roughness

- Alters the uniformity of a surface using a *perturbation function*
  - ❏ for example, use a function which perturbs the normal vector
  - ❏ use new normal in illumination model
- Perturbation function can be defined
  - ❏ analytically
  - ❏ as a lookup table
- Different effects can be achieved
  - ❏ random function gives rough surface
  - ❏ a smoother function gives more regular feature

Bump Mapping

- An example of bump mapping



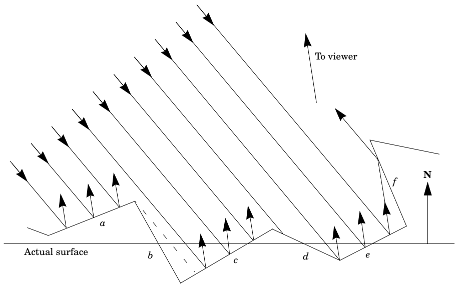
Bump Mapping

- This technique avoids explicitly modelling the geometry of the new surface
  - the *silhouette* is still smooth
  - roughening only becomes apparent when the shading model is applied
- Examples of use
  - texture of an orange
  - tread of tyre
  - etc

Cook-Torrance Model

This method models features due to small-scale roughness

- Surface modelled as collection of randomly oriented microscopic **facets**



Cook-Torrance Model

- Model accounts for three situations
  - facets which reflect light directly towards the viewer
  - facets which are in *shadow* of other facets
  - facets which reflect light which itself has been reflected from other facets - these multiple reflections contribute to the *diffuse reflection* from the surface
- Specular reflection coefficient of the overall surface given by

$$k_s = \frac{DGF}{\pi(\mathbf{N} \bullet \mathbf{V})(\mathbf{N} \bullet \mathbf{L})}$$

  - *D* is *distribution function* (Gaussian)
  - *G* is factor accounting for shadowing
  - *F* is the *Fresnel factor*

Cook-Torrance Model

- The Fresnel factor gives the fraction of light incident on a facet which is reflected rather than absorbed.
  - defined by

$$F = \frac{1}{2}\left[\frac{\sin^2(\phi - \theta)}{\sin^2(\phi + \theta)} + \frac{\tan^2(\phi - \theta)}{\tan^2(\phi + \theta)}\right]$$

  - θ and ϕ are the angles of incidence and reflection measured from the *facet normal* not the overall surface normal **N**
- Cook-Torrance model gives results similar to Phong´s except
  - for grazing angles of reflection
  - the highlight colour is not the same as light source
  - Phong is computationally less expensive

Transparency

Not all materials are opaque. Some objects allow light to be transmitted or refracted

- **Diffuse refraction**
  - transmitted light is scattered by internal and surface irregularities
  - surface appears *translucent* (frosted glass)
  - objects view through a diffuse refractor appear blurred
- **Specular refraction**
  - occurs in truly transparent materials
  - the direction of light rays are *bent* (lens)
  - objects viewed through a specular refractor are clearly seen

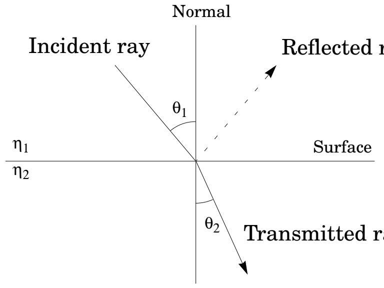
Transparency

Snell's Law

- Describes precisely how light behaves when moving from one medium to another

$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

  - η is the **refractive index** of the material
  - θ is angle of ray measured from normal



Transparency

Refractive index

- The refractive index is in general wavelength dependent
  - different colours will be bent by different amounts
  - this is called **dispersion**
  - we will ignore this effect since it is difficult to model and use an average value across the visible spectrum
- Snell's law is *significant*
  - example, light passing from air into heavy glass ($\eta = 1.5$) at $\theta = 30°$, will be bent by 11°°

Modelling Transparency

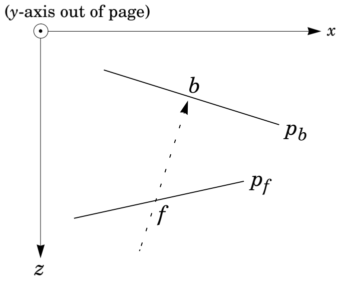
Non-refractive transparency

- Light paths are not bent
  - avoids computational overhead with trigonometric functions in Snell's law
- Then transparent objects will appear invisible!
- Introduce a **transmission coefficient** *t*, to measure transparency
  - opaque object has $t = 1$
  - perfectly transparent object has $t = 0$
  - *t* could be colour dependent (coloured glass, for example)

Modelling Transparency

Non-refractive transparency

- Example use



- The observed intensity is given by

$$I = tI_f + (1 - t)\, I_b$$

Global Illumination Models

Ray Tracing

This method simulates the global illumination distributed by specular light

- Uses the laws of **geometric optics**
  - ❑ follows light energy along *rays*
  - ❑ is a *recursive* procedure
  - ❑ is *view dependent*
- Includes an element to simulate **diffuse effects**
  - ❑ diffuse reflection from local illumination
  - ❑ otherwise surface will be black 'n' shiny
- Ray-tracing can be used on different kinds of surfaces, not just polygons
  - ❑ for example, spheres and cones

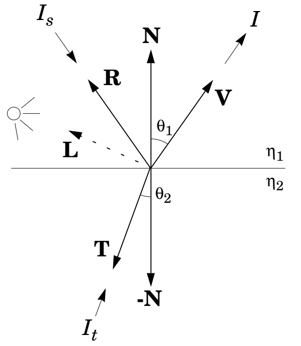
Ray Tracing

Overview of the procedure

- Rays are traced **from the view-point**
  - ❑ need to set-up *viewing system*
  - ❑ view-plane is discretized and mapped to image pixels
  - ❑ one *primary-ray* is traced through each pixel out into the scene
- Find *intersection* of primary-ray with object which is *nearest* view-point
  - ❑ object thus found represents surface visible through pixel
- At this point we have effectively performed a *hidden-surface removal* operation

Ray Tracing

Now determine the intensity of light leaving the surface intersection point



- This intensity will be due to a *local component* plus components due to *globally reflected* and *transmitted* light

Ray Tracing

- For a single point light source

$$I = k_d I_a + I_p \left[k_d (\mathbf{N} \bullet \mathbf{L}) + k_s (\mathbf{R} \bullet \mathbf{V})^n \right] + k_r I_r + k_t I_t$$

  - ❏ k_r is the **global specular reflection coefficient** (usually equal to k_s)
  - ❏ k_t is the **global specular transmission coefficient**
  - ❏ I_r *and* I_t are the intensities coming from directions **R** and **T**

- The global terms are calculated by spawning *secondary-rays* from intersection point in directions **R** and **T**
  - ❏ find intersection of secondary-rays with objects in the scene
  - ❏ apply intensity calculations again at new intersections

Ray Tracing

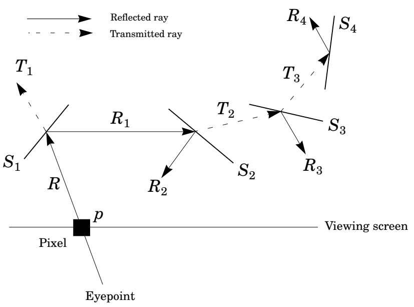
Recursive nature of ray-tracing

- We can repeat this process again and again
  - ❏ create new secondary-rays at each intersection point
  - ❏ use new rays to estimate I_r and I_t for each previous trace
- If k_r is zero, no global reflection occurs, surface is diffuse, so no need to spawn new specular rays
- Similarly for k_t, opaque surfaces have zero transmittance

Ray Tracing

Ray-tracing a particular pixel

- An schematic example



  - ❏ all surfaces are transparent except S_4
  - ❏ rays T_1, R_2, R_3, R_4 do not contribute to intensity at pixel

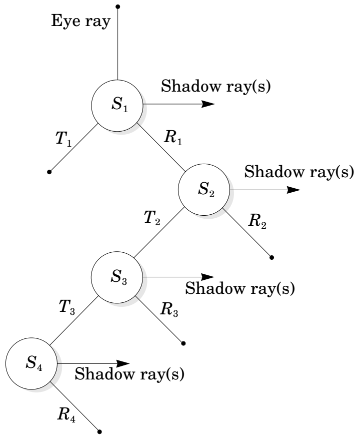
Ray Tracing

Intensity calculations at each point of intersection must take into account *shadows*

- First construct the **shadow ray**
  - origin at intersection point and direction towards the light source
- Test shadow ray against objects in scene
  - if hit is found *and* intersection is *nearer* than the light source, then point is in shadow
  - shadowed point does not contribute local illumination, except ambient
- Multiple light sources
  - shadow ray for each source

Ray Tracing

We can visualise the recursive process used in ray-tracing with a *ray-tree*



Ray Tracing

The ray-tree

- In practice, need to set-up maximum depth to which rays are traced
  - otherwise spend too much time on rays which contribute little to image
  - but if we have insufficient depth, will cause artifacts
- Three ways to control ray-tree depth
  - rays may leave scene
- return 'background' intensity
  - set absolute ray-tree depth
  - use *adaptive* depth control
- set threshold intensity returned by secondary-rays

Object Intersections

A major part of the ray-tracing algorithm is concerned with finding intersections with objects in the scene

- Each newly created ray must be tested against every object surface
  - if intersections are found, which one is the nearest?
- Need efficient intersection algorithms for all types of object
  - sphere, polygon, cone, box, cylinder, torus, etc
  - will illustrate how to calculate intersections with a sphere

Sphere Intersection

Vector equation of a ray

- Arbitrary ray defined *parametrically* as

$$\mathbf{r} = \mathbf{O} + \mathbf{D}t$$

  - **r** is position vector of point with parameter t
  - **O** is position vector of ray origin
  - **D** is *unit vector* in ray direction
- Parameter t is real-valued
  - represents a 'distance' along ray
  - for all primary rays the origin lies at the view point
- We require values of t at intersection points which are *positive*

Sphere Intersection

- Vector equation of a sphere

$$(\mathbf{r} - \mathbf{C}) \bullet (\mathbf{r} - \mathbf{C}) = R^2$$

  - **C** is position vector of sphere centre
  - R is the sphere's radius
- Substitute ray equation and solve for the parameter t, giving

$$t^2 - 2t\mu + \left[\lambda - R^2\right] = 0$$

$$\mu = \mathbf{D} \bullet \mathbf{T}$$

$$\lambda = \mathbf{T} \bullet \mathbf{T}$$

$$\mathbf{T} = \mathbf{C} - \mathbf{O}$$

- This is a quadratic in t, so

$$t = \mu \pm \sqrt{\gamma}$$

$$\gamma = \mu^2 - \lambda + R^2$$

Sphere Intersection

- Depending on the value of γ, we have one of three situations
  - $\gamma < 0$ ray misses sphere
  - $\gamma = 0$ one intersection which grazes sphere
  - $\gamma > 0$ two distinct intersections

$$t_1 = \mu + \sqrt{\gamma}$$

$$t_2 = \mu - \sqrt{\gamma}$$

- To find intersection points
  - substitute the t values back into the ray-equation

Optimisations

Naive ray-tracing can be very time consuming

- Typically 90% of computation time spent on object intersections
  - gets worse the greater number of objects in the scene
  - also get worse if we want higher resolution image (more primary-rays)
  - this slows the ray tracing method considerably
- Schemes are available to help which
  - limit the number of intersection tests
  - delay a costly test until a cheaper test has confirmed its necessity

Hierarchical Bounding Volumes

What is a bounding volume?

- Is a simple primitive which has *smallest* volume enclosing object
  - ❏ usually spheres and axis-aligned boxes
- During ray intersection test
  - ❏ first test the bounding volume
    - usually easier and faster
  - ❏ if bounding volume intersected then test the actual object
- Spheres are very popular
  - ❏ very efficient since we only need to know if intersection takes place, NOT where the intersection points are

Hierarchical Bounding Volumes

Hierarchical scheme?

- Clusters of bounding volumes within larger bounding volume
  - ❏ intersect ray with outer volume
  - ❏ then with inner volumes if necessary
- Can have any number of levels of bounding volumes
- Hierarchical scheme efficient for scenes with non-uniform distribution of objects
  - ❏ ray doesn't do much work in areas which it is not 'looking' at
- Very good speed-up in rendering time

3D Spatial Subdivision

More powerful partitioning schemes use the idea of *voxels*

- **Voxels** are axis-aligned rectangular prisms which are like bounding volumes except...
  - ❏ fill all of space occupied by scene
  - ❏ are non-overlapping
  - ❏ do not necessarily completely enclose any particular object
- Two varieties of spatial subdivision
  - ❏ **uniform**
    - all voxels are the same size
    - stacked together
  - ❏ **non-uniform**
    - octrees
    - voxel hierarchy

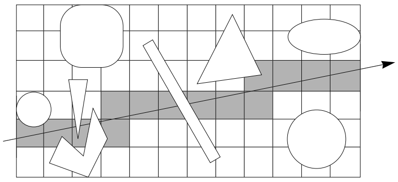
3D Spatial Subdivision

Uniform subdivision - *Voxel Grids*

- Constructing the voxel grid
  - ❏ surround the scene with a bounding cuboid
  - ❏ split cuboid into L x M x N smaller cuboids - these are the voxels
  - ❏ for each voxel keep a list of objects which encroach into its space
- Tracing a ray through the voxel grid
  - ❏ determine which voxels the ray passes through
  - ❏ only perform intersection tests on those objects which are in the voxel list
  - ❏ next, an illustration in 2D

3D Spatial Subdivision

Tracing a ray through the voxel grid



- We wish to find the first intersection
  - follow the ray voxel-by-voxel
  - if object is part of two adjacent voxels, keep results of intersection test
  - if object is hit, but intersection point is *outside* current voxel, then continue
  - only if intersection point occurs *within* the current voxel can we stop

3D Spatial Subdivision

What have we gained?

- Ray intersects objects further *down-stream* but didn't need to be tested
  - this would have been the case otherwise
  - only three voxels are considered before a valid intersection is found
  - 5 out of 8 possible intersection tests are avoided
- In practice, 3D scenes could contain 1000's of objects
  - gains are much more substantial
  - with spatial subdivision, we restrict intersection testing to objects in the *neighbourhood* of the ray's trajectory

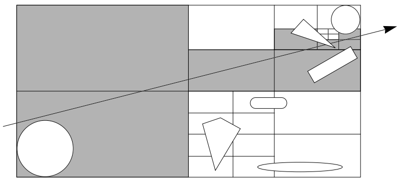
3D Spatial Subdivision

Non-uniform subdivision - *Octrees*

- Hierarchical tree of non-overlapping voxels of various sizes
  - emphasises the spatial distribution of objects in a scene
- Constructing the octree
  - surround scene with bounding cuboid
  - divide into *eight* equal-sized sub-volumes or voxels
  - keep list of objects associated with each sub-volume
  - if maximum number of objects/voxel is above threshold, then subdivide again
- Subdivision only occurs where there are lots of objects

3D Spatial Subdivision

Tracing a ray through an octree



- Procedure similar to uniform case
  - *six* voxels are considered before valid intersection is found
  - only *three* distinct intersection tests are actually made
  - ray passes through large, empty regions of the scene very quickly
  - useful work done only in high density regions

3D Spatial Subdivision

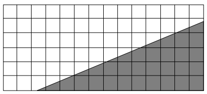
Controlling subdivision

- Trade off between voxel resolution and time spent traversing voxels
  - ❏ voxel resolution controlled by maximum number of objects/voxel
- These can be adjusted for optimum efficiency
- An example of what can be achieved
  - ❏ a uniform subdivision of a scene containing over 10,000 objects took 15 minutes to render
  - ❏ the same scene without voxelisation took 40 *days* to render!

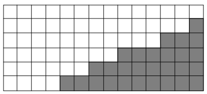
Image Aliasing

What is aliasing?

- Eye-rays passing through image plane **samples** the light distribution
  - ❏ discrete representation is only approximate
  - ❏ if we try to *reconstruct* the light distribution, distortions occur
  - ❏ this is **aliasing**



reality

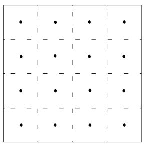


image

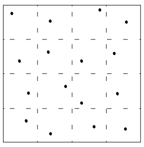
Image Aliasing

Anti-aliasing techniques

- Anti-aliasing attempts to reduce the effects of aliasing
  - ❏ usual way is to sample each pixel more than once - **supersampling**
  - ❏ two ways to supersample



Uniform



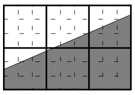
Jittered

  - ❏ **uniform** sampling passes eye-rays through set of *sub-pixels*
  - ❏ **jittered** sampling randomly displaces the uniform sampling array

Image Aliasing

Reconstruction

- We must combine the intensity information gathered by the supersampled rays
  - ❏ this is called **filtering**
- Many ways of filtering, for example
  - ❏ **box** - average the intensities
  - ❏ **gaussian** - average weighted by distance from centre of pixel



Pixel values

0	1/9	5/9
7/9	1	1

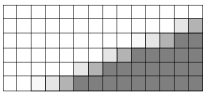


Image Aliasing

Adaptive supersampling

- Supersampling every pixel is generally not necessary
  - ❏ aliasing only noticeable along edges or other *high contrast* boundaries
  - ❏ adaptive schemes attempt to locate pixels which need special attention
- This is one way of doing this
  - ❏ first shoot rays through corners of pixel

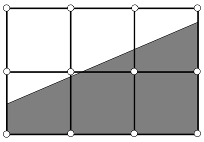
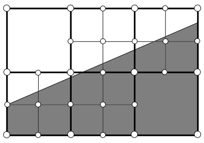


Image Aliasing

- ❏ for each pixel examine variation of these 4 samples
- ❏ if contrast too large (threshold user defined) split into 4 again



- ❏ repeat as required

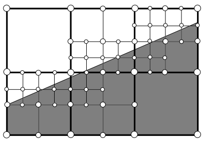


Image Aliasing

Adaptive supersampling

- **Contrast parameter**

$$C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

  - ❏ I_{max} and I_{min} largest and smallest sample intensity
  - ❏ required for each colour band (R, G. B)
  - ❏ supersampling performed if any value exceeds pre-set values
  - ❏ for example, 0.25, 0.20, 0.40
- Adaptive supersampling is *very* efficient
  - ❏ many samples can be reused
  - ❏ in example, only 62 out of a possible total of 117 eye-rays are traced

Radiosity

This method simulates the global illumination distributed by diffuse light

- Attempts to accurately model the effects of ambient light
  - ❏ before we just had $I_a k_d$ with I_a constant
- Uses the laws of *energy conservation*
  - ❏ scenes are usually *closed* environments
- We assume all surfaces are purely diffuse reflectors
- Radiosity is a *view-independent* technique
  - ❏ we calculate the diffuse light leaving all surfaces
  - ❏ then render from any particular view point

Radiosity

The Radiosity Equation

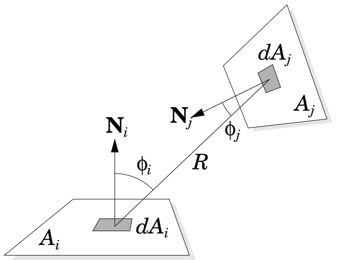
- Assume scene is comprised of discrete patches
- Energy equilibrium gives for each patch

$$A_iB_i = A_iE_i + \rho_i \sum_{j=1}^{n} B_jF_{ji}A_j$$

  - ❑ B_i is the **radiosity** of the patch
    - energy per unit time per unit area
  - ❑ E_i is the energy emitted (same units as B)
  - ❑ F_{ji} is the **form-factor**
    - energy leaving patch j which directly reaches patch i
  - ❑ ρ_i is the **diffuse reflectivity**(like k_d)
  - ❑ A_i is the **area** of the patch

Radiosity

Form-factors



- Assuming radiosity is constant across patch

$$F_{ji} = \frac{1}{A_j}\int_{A_j}\int_{A_i}\frac{\cos\phi_i\cos\phi_j}{\pi R^2}H_{ji}dA_idA_j$$

  - ❑ H_{ji} is a visibility factor equal to 1 if dA_j can see dA_i, otherwise 0

Radiosity

Form-factors

- **Reciprocity relation** for form-factors

$$A_iF_{ij} = A_jF_{ji}$$

  - ❑ applies to enclosed environment
  - ❑ useful for calculating form-factors
    - we can get F_{ij} easily once F_{ji} is known
- Also, for an enclosed environment

$$\sum_{j=1}^{n} F_{ij} = 1$$

  - ❑ follows from energy conservation
  - ❑ useful as a test of form-factor accuracy

Radiosity

Solving the equation

- With the reciprocity relation we can write

$$B_i = E_i + \rho_i \sum_{j=1}^{n} B_jF_{ij}$$

- This is a set of simultaneous equations in the unknown quantities B_i
  - ❑ for example, a 3-patch scene

$$B_1 = E_1 + \rho_1 (F_{11}B_1 + F_{12}B_2 + F_{13}B_3)$$
$$B_2 = E_2 + \rho_2 (F_{21}B_1 + F_{22}B_2 + F_{23}B_3)$$
$$B_3 = E_3 + \rho_3 (F_{31}B_1 + F_{32}B_2 + F_{33}B_3)$$

  - ❑ we calculate the F_{ij} and we know the E_i hence we can determine the radiosities of each patch

Computing Form-Factors

There are no known analytical solutions to the form-factor integral equation

$$F_{ji} = \frac{1}{A_j}\int_{A_j}\int_{A_i}\frac{\cos\phi_i\cos\phi_j}{\pi R^2}H_{ji}dA_i dA_j$$

- We need a numerical technique
  - ❏ but double integrals are still tough
- Assume the distance between patches is large compared to their size
  - ❏ inner integral is approximately constant

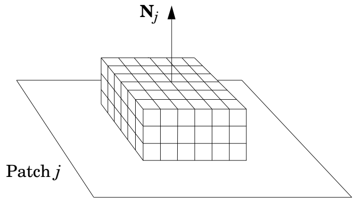
$$F_{ji} = \int_{A_i}\frac{\cos\phi_i\cos\phi_j}{\pi R^2}H_{ji}dA_i$$

  - ❏ form-factor from dA_j to A_i

Computing Form-Factors

We evaluate the simplified form-factor integral using a *projection method*

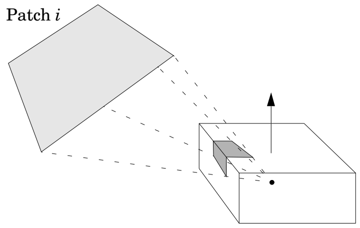
- The **hemi-cube algorithm** is one such method
  - ❏ half a cube of side 2 is centred about a patch *j*
  - ❏ each face is discretised uniformly into a number of pixels (user controllable)
  - ❏ commonly 50x50 or 100x100



Computing Form-Factors

The hemi-cube algorithm

- Next step is to project every other patch onto the surface of the hemi-cube...



- ... and determine which pixels are covered
  - ❏ if two patches project to the same pixel, the nearest one is stored
  - ❏ this accounts for the H_{ji} term

Computing Form-Factors

The hemi-cube algorithm

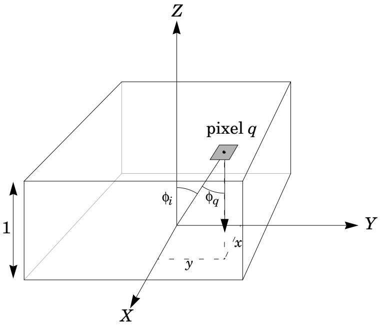
- Finally, we determine the form-factors by summing the **delta-form-factors** of the pixels which each patch projects to

$$F_{ji} = \sum_q \Delta F_q$$

  - ❏ delta-factors can be pre-calculated
- This gives us *n* form-factors relative to one patch
  - ❏ repeat operation with another hemi-cube centred about another patch
  - ❏ do this for all patches in scene

Computing Form-Factors

Delta-form-factors

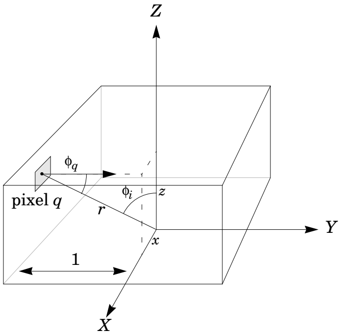


- For a pixel on the top-face

$$\Delta F_q = \frac{1}{\pi\left[x^2+y^2+1\right]^2}\Delta A$$

Computing Form-Factors

Delta-form-factors



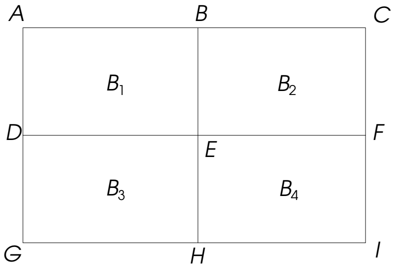
- For a pixel on a side-face

$$\Delta F_q = \frac{z}{\pi\left[x^2+z^2+1\right]^2}\Delta A$$

Rendering

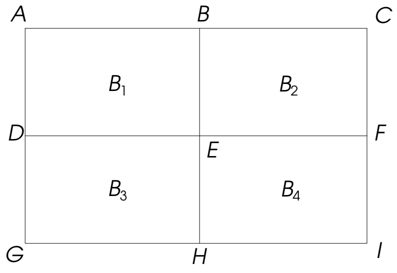
We now have *n* radiosity values, so how do we render a particular view?

- We want to smooth shade the surface
  - ❏ use Gouraud method
  - ❏ need radiosities at each patch vertex and then interpolate across the patch
  - ❏ For example



Extrapolation

- We **extrapolate** the radiosity values at the centre of the patches to the patch vertices
- There are 3 distinct cases:
  - ❏ internal vertices (e.g. vertex *E*)
  - ❏ edge vertices (e.g. vertex *B*)
  - ❏ corner vertices (e.g. vertex *A*)

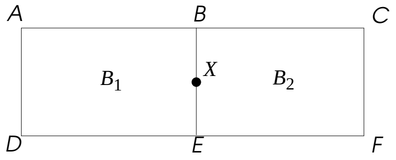


Extrapolation

- **Internal vertices**: average the radiosities of all the patches containing the vertex

$$B_E = (B_1 + B_2 + B_3 + B_4)/4$$

- **Edge vertices**: find nearest *internal* vertex and note that B_X can be expressed in two ways



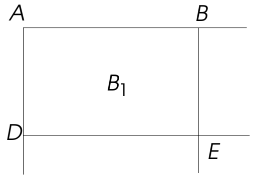
$$\frac{B_B + B_E}{2} = \frac{B_1 + B_2}{2}$$

  - hence

$$B_B = B_1 + B_2 - B_E$$

Extrapolation

- **Corner vertices**: find nearest *internal* vertex and note average of corner vertex and its internal vertex is the patch radiosity



  - for example

$$B_1 = (B_A + B_E)/2$$

  - hence

$$B_A = 2B_1 - B_E$$

Progressive Refinement Radiosity

So far we have covered the full-matrix radiosity method

- This requires
  - $O(n^2)$ storage for the form-factors
  - $O(n^2)$ time to solve radiosity equation
- **Progressive refinement** attempts to overcome these problems
  - gives $O(n)$ storage costs
  - gives $O(n)$ computation time costs *against some initial image*
- Progressive refinement combines image realism with *user interactivity*

Progressive Refinement

Formulation

- Recall the energy equilibrium equation...

$$A_iB_i = A_iE_i + \rho_i \sum_{j=1}^{n} B_jF_{ji}A_j$$

- ...and consider the interaction between two patches *i* and *j*

$$B_i(\text{due to } B_j) = \rho_i B_j F_{ji}\frac{A_j}{A_i}$$

- So applying a single hemi-cube at patch *j* we can find the contribution of this patch to the rest of the scene
  - but to be used in some *iterative* scheme, it's better to consider *changes* in radiosity

Progressive Refinement

Formulation

- Write this as

$$B_i(\text{due to } \Delta B_j) = \rho_i \Delta B_j F_{ji} \frac{A_j}{A_i}$$

  - ❏ ΔB_j is the *unshot* radiosity of patch j
    - due to its emission of light
    - or light received from other patches
- We use the progressive refinement method in the following way
- For each patch keep track of two radiosity values
  - ❏ current radiosity estimate, B_j
  - ❏ unshot radiosity, ΔB_j
  - ❏ initially both these are equal to E_j

Progressive Refinement

Formulation

- Choose patch with *largest* unshot energy
  - ❏ unshot energy is $\Delta B_j A_j$
- *Shoot* this energy to all other patches as described before
  - ❏ each patch will received a certain amount of energy which is added to both B_j and ΔB_j
- Set the unshot radiosity of the shooting patch to *zero*
- With new estimates of B_j, render an image
- Repeat this cycle again and again, rendering an image after each step

Progressive Refinement

Formulation

- After each step, all the ΔB_j will be *underestimates*
  - ❏ but each step reduces the relative size of the ΔB_j
  - ❏ hence the radiosity estimates, B_j, slowly *converge* to their full-matrix values
- The cycle is repeated until the *total unshot energy* in the whole scene falls below some predefined value

Progressive Refinement

Ambient contribution

- With progressive refinement, first few images will generally be *dark*
  - ❏ not all the energy has been distributed
  - ❏ only surfaces in the direct line of sight of the light source(s) are illuminated
- An **ambient radiosity term**, B_{amb}, is introduced *for display purposes only*
  - ❏ based upon how much unshot energy remains and how this could be distributed
  - ❏ when rendering, use $\rho_j B_{amb} + B_j$, instead of B_j
  - ❏ this value plays no part in subsequent refinements of B_j

Progressive Refinement

Calculating the ambient term

- First estimate the form-factors

$$F_{ji}^{est} = \frac{A_i}{\sum_{k=1}^{n} A_k}$$

- Next, determine average reflectance of the scene

$$\rho_{av} = \frac{\sum_{j=1}^{n} \rho_j A_j}{\sum_{j=1}^{n} A_j}$$

  - ❏ from this we can write the overall inter-reflection coefficient as

$$1 + \rho_{av} + \rho_{av}^2 + \rho_{av}^3 + \ldots = \frac{1}{1 - \rho_{av}}$$

Progressive Refinement

Calculating the ambient term

- Then ambient radiosity term is given by

$$B_{amb} = \frac{1}{1 - \rho_{av}} \sum_{i=1}^{n} \Delta B_i F_{ji}^{est}$$

- Note how the ambient term gracefully diminishes as the refinement progresses
  - ❏ $B_{amb} \rightarrow 0$ as $\Delta B_i \rightarrow 0$

These materials have been produced as part of the Information Technology Training Initiative, funded by the Information Systems Committee of the Higher Education Funding Councils.

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