ADVANCED ARCHITECTURE

COMPUTER ARITHMETIC

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Lectures 5-6

2025-2026

References

Computer Architecture - A Quantitative Approach

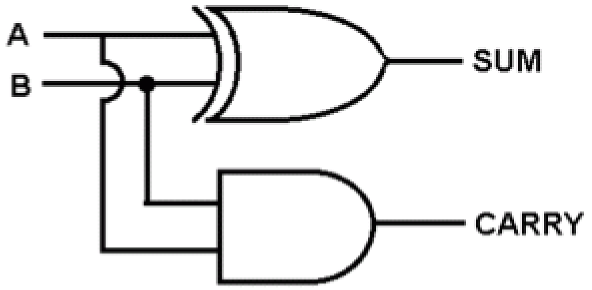
Hennessy Patterson – *Fifth Edition*

Appendix J – *Computer arithmetic* - *David Goldberg*

ADDITION AND ADDERS

Half adder and Full adder

- Adders are usually implemented by combining multiple copies of simple components
- The natural components for addition are ***half adders*** and ***full adders***
- The **half adder** takes two bits *a* and *b* as input and produces a sum bit s and a carry bit c_{out} as output
- Logic equations: $s = a\bar{b} + \bar{a}b = a \oplus b$ and $c_{out} = ab$

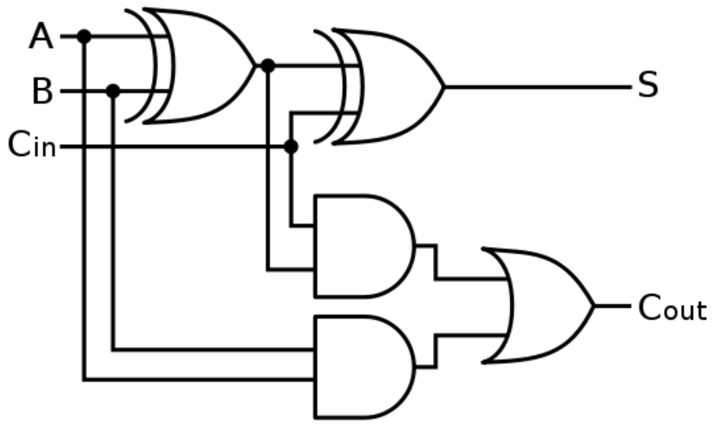


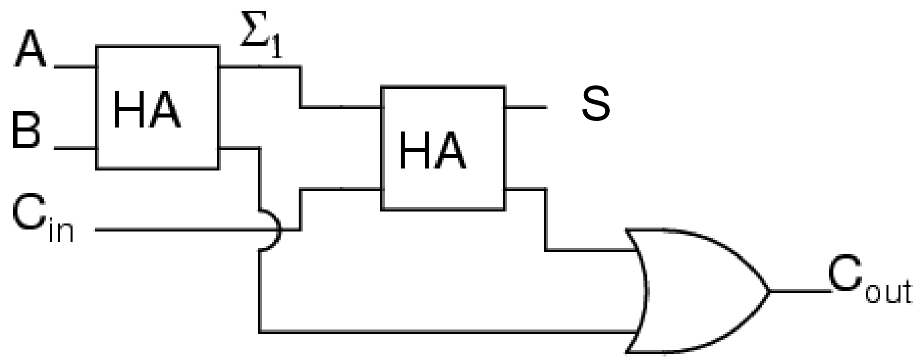
Half adder and Full adder

- The full adder takes ***three bits*** *a*, *b* and *c* as input and produces a sum bit s and a carry bit c_{out} as output
- Logic equations:

$$s = a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}c + abc = (a \oplus b) \oplus c$$

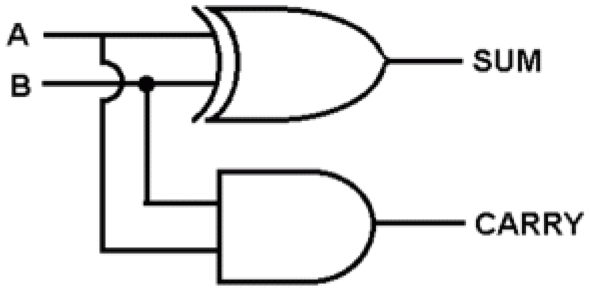
and $$c_{out} = (a \oplus b)c + ab$$

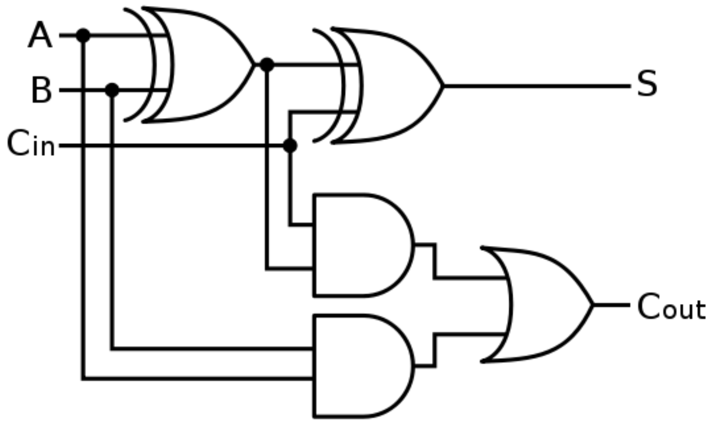


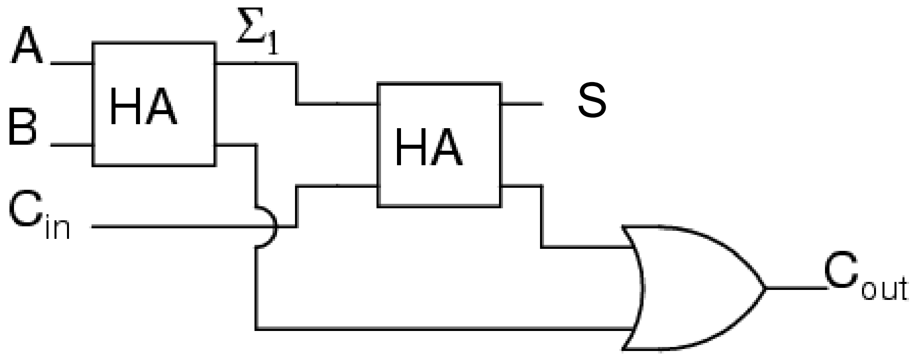


Half adder and Full adder

- The half adder is a (2,2) adder:
  - it takes two inputs and produces two outputs
- The full adder is a (3,2) adder:
  - it takes three inputs and produces two outputs

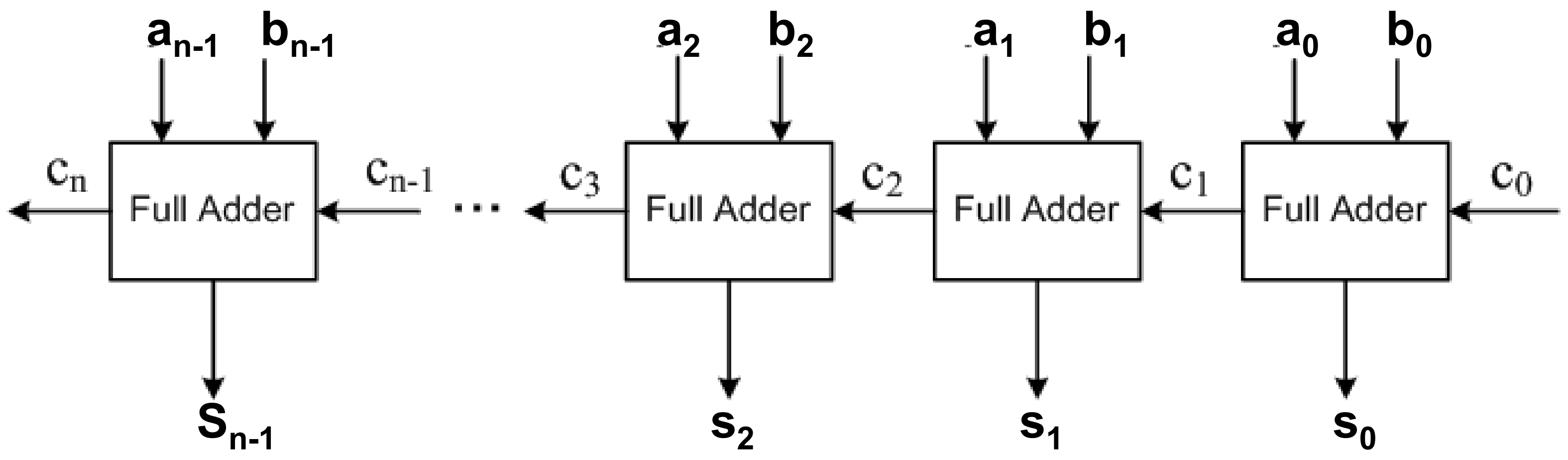






Ripple-Carry Addition

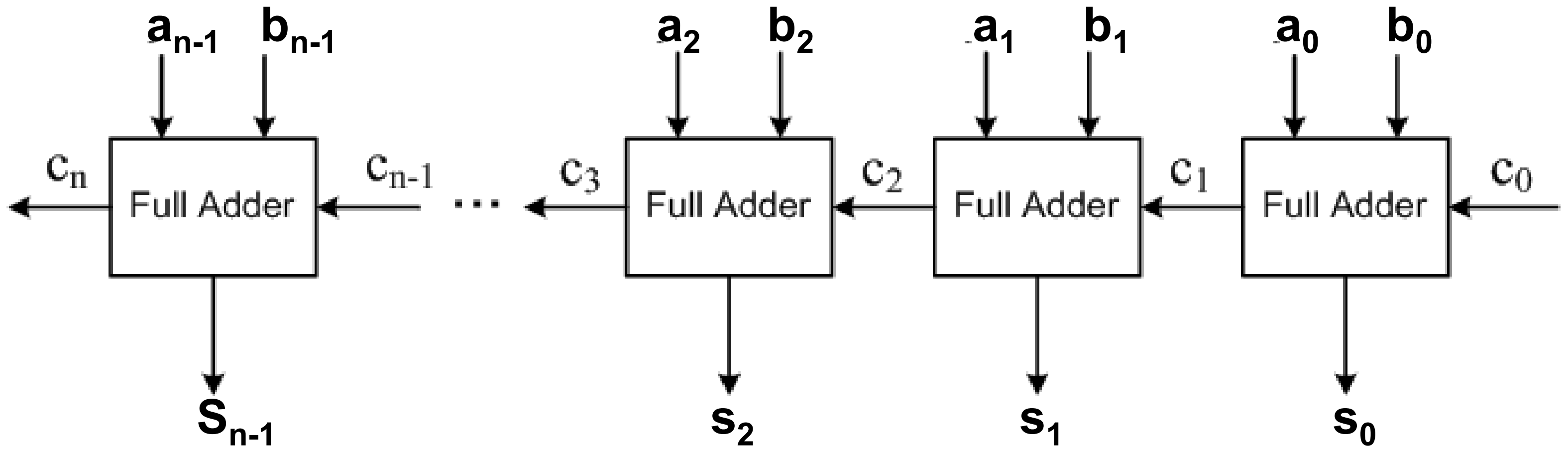
- The principal problem in constructing an adder for *n*-bit numbers out of smaller pieces is propagating the carries from one module to the next
- The most obvious way to solve this is with a ***ripple-carry adder***, consisting of *n* full adders



Ripple-Carry Addition

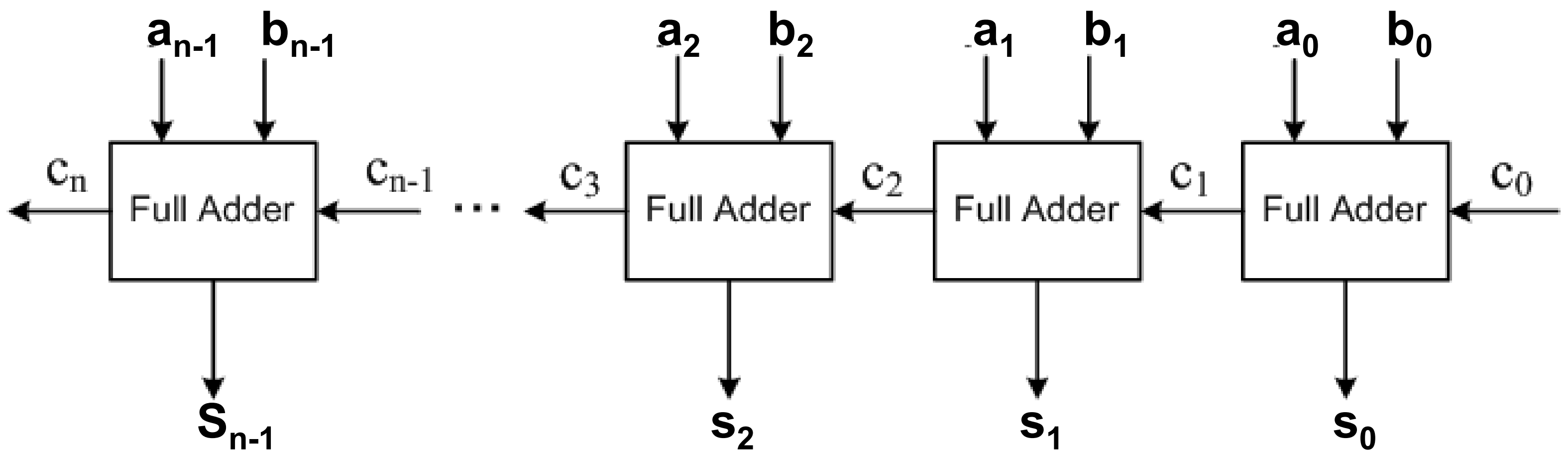
- Note that the low-order carry-in could be wired to 0, hence the **low-order adder** could be a half adder

- However, setting the low-order carry-in bit to 1 is useful for performing **subtraction**



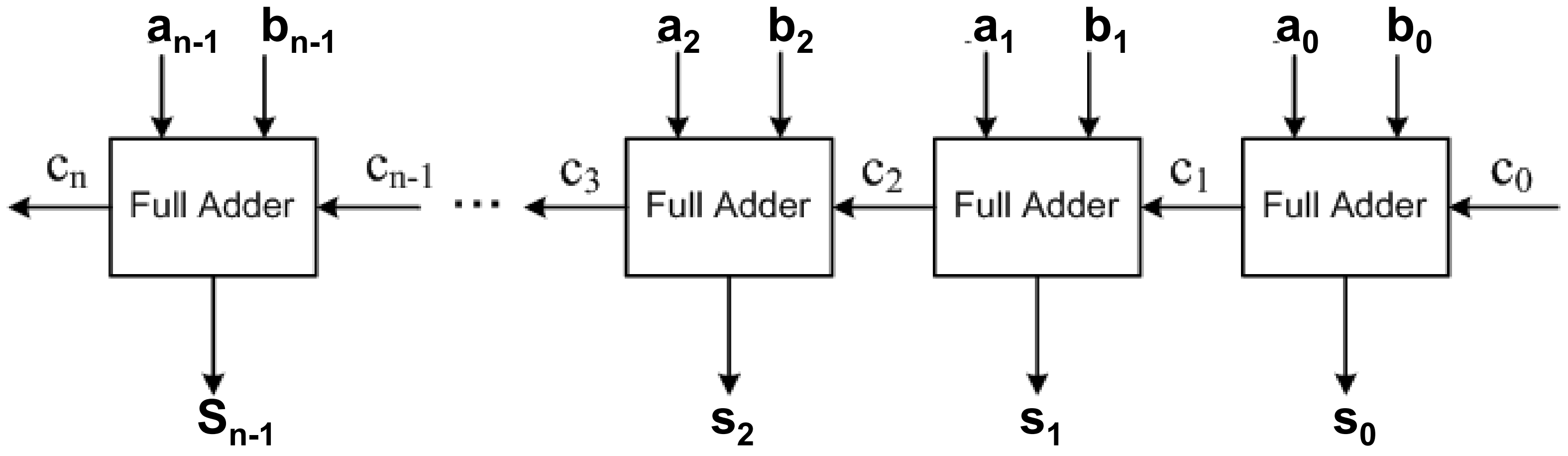
Ripple-Carry Addition

- The time a circuit takes to produce an output is proportional to the maximum number of logic levels through which a signal travels
- Determining the exact relationship between logic levels and timings is highly technology dependent



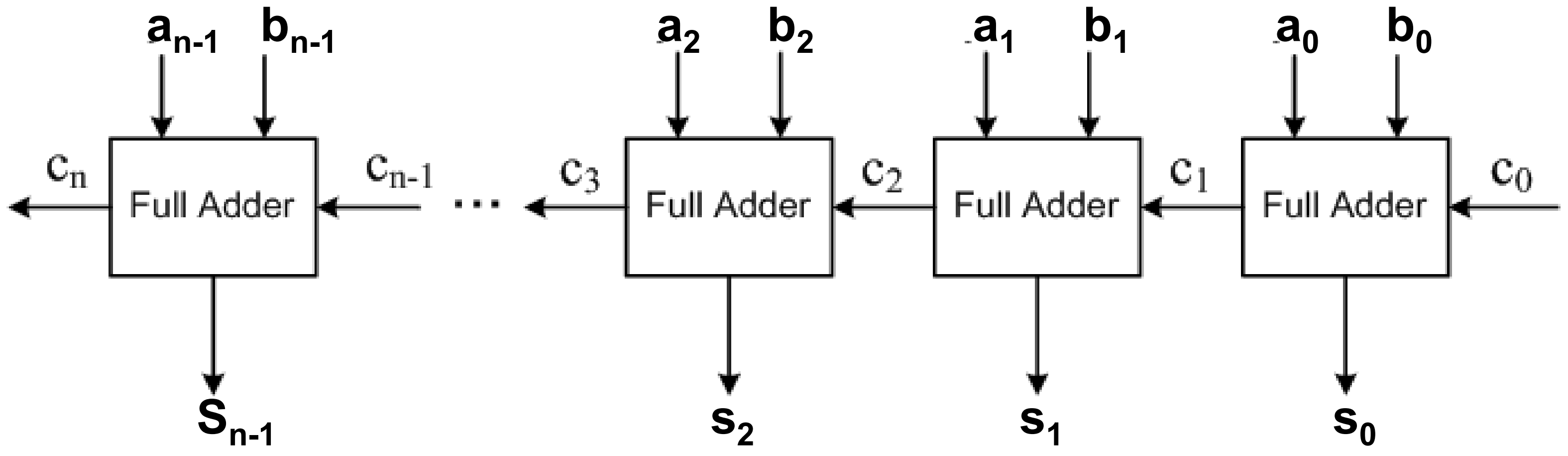
Ripple-Carry Addition

- When comparing adders we simply compare the number of logic levels in each one
- A ripple-carry adder takes:
  - two levels to compute c_1 from a_0 and b_0
  - two more levels to compute c_2 from c_1, a_1, b_1 - *and so on*, up to c_n
- So, there are a total of 2*n* levels



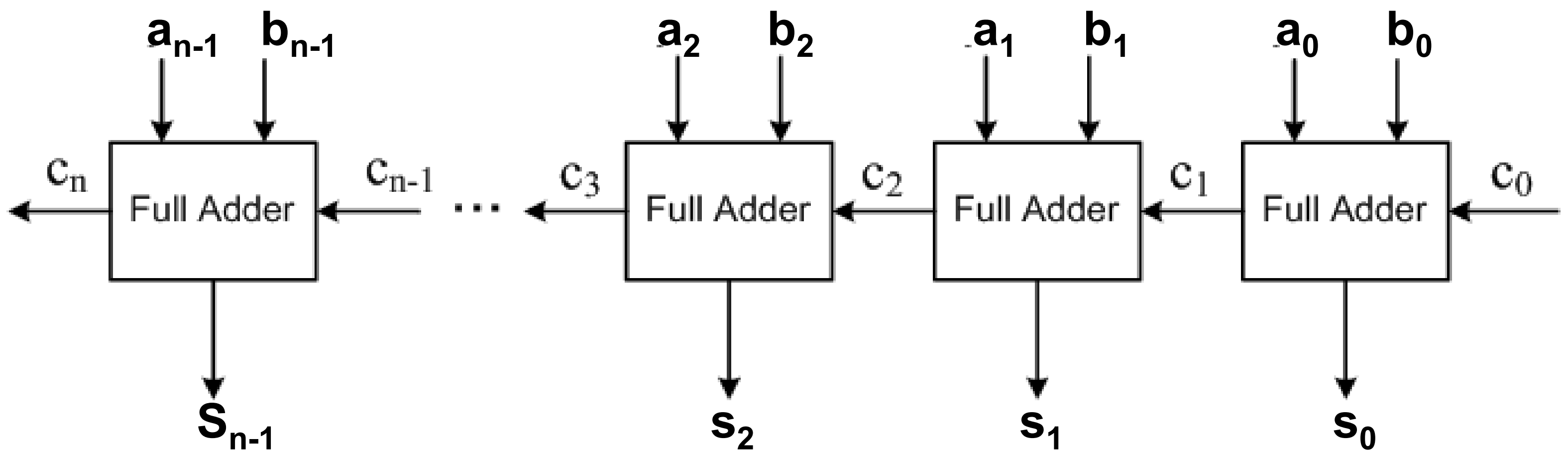
Ripple-Carry Addition

- Typical values of *n* are 32 for **integer arithmetic** and 53 for **double-precision floating point**
- The ripple-carry adder is the slowest adder, but also the cheapest
- It can be built with only *n* simple cells, connected in a simple, regular way



Ripple-Carry Addition

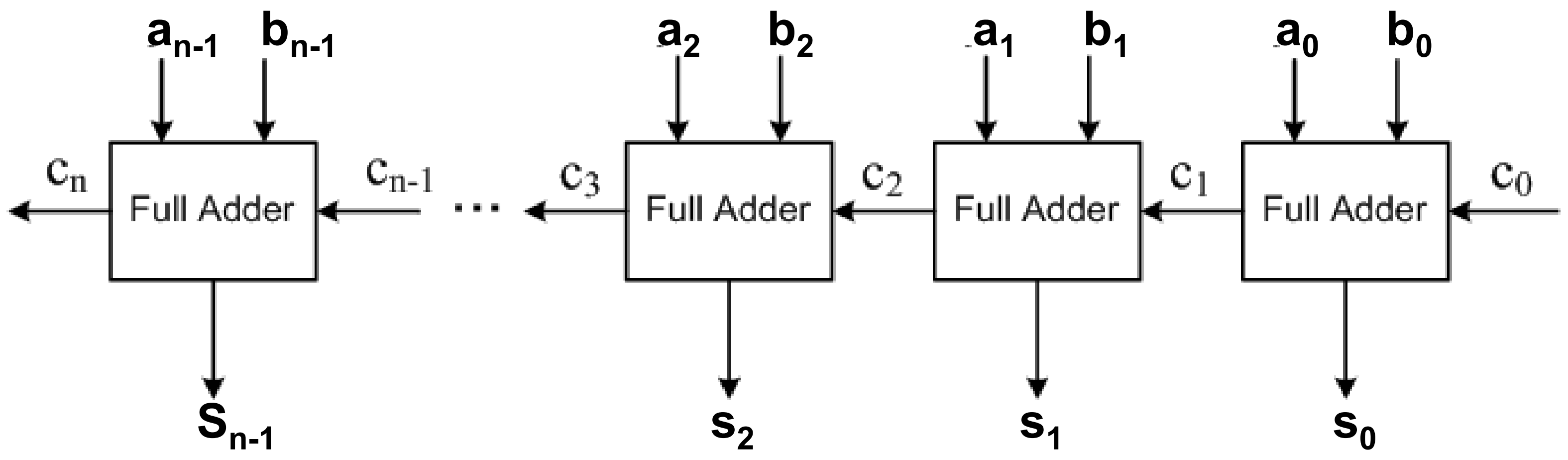
- The ripple-carry adder is relatively slow → it takes time O(*n*)
- But it is used because in technologies like CMOS, the constant factor is very small
- Short ripple adders are often used as **building blocks** in larger adders



Ripple-Carry Addition for Signed Numbers

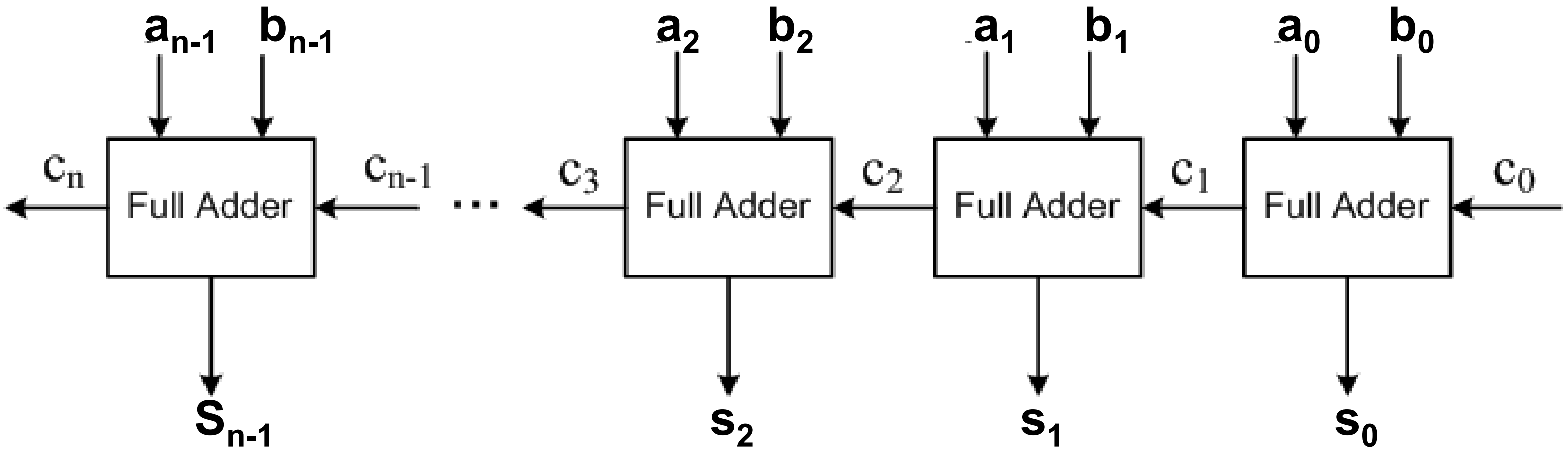
- The most widely used system for representing integers is the **two's complement**, where the **MSB** is considered associated with a negative **weight**
- The value of a two's complement number $a_{n-1}a_{n-2}\cdots a_1 a_0$ is:

$$-a_{n-1}2^{n-1} + a_{n-2}2^{n-2} + \cdots + a_1 2^1 + a_0 2^0$$



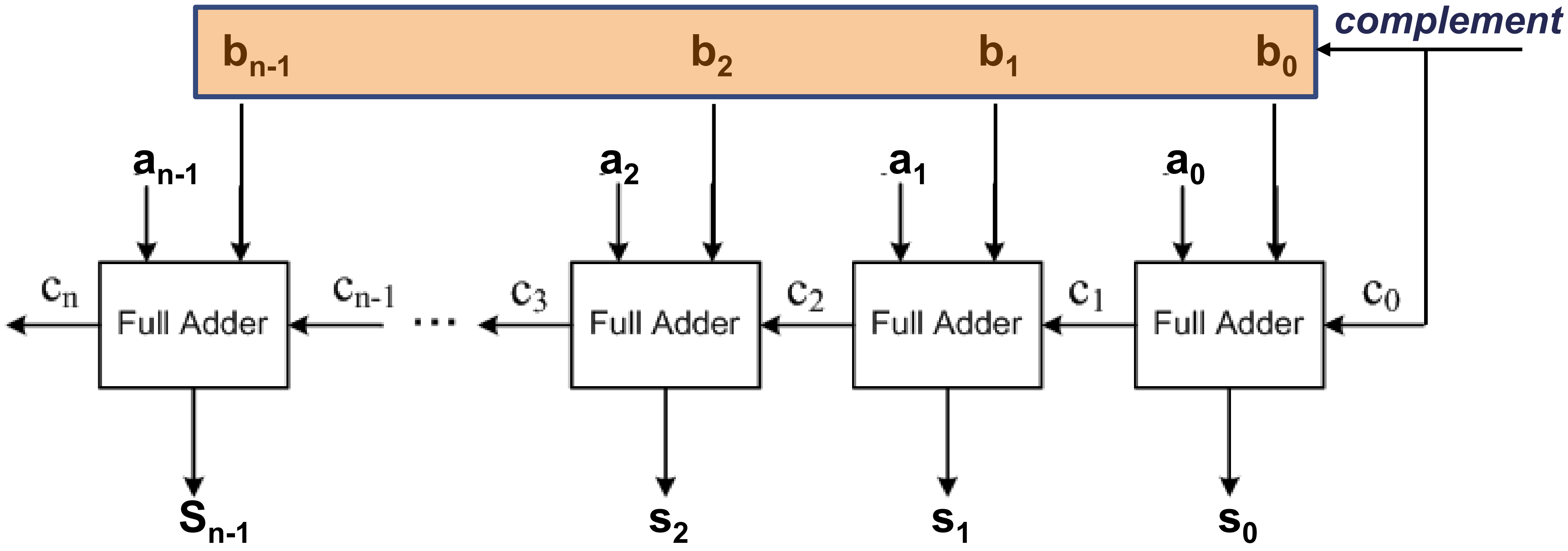
Ripple-Carry Addition for Signed Numbers

- The reasons for the popularity of two's complement are:
  - It makes **signed addition easy** → simply discard the carry-out from the high order bit
  - Subtraction is executed as an addition:
    - A-B = A+(-B), recalling that $-X = \overline{X} + 1$



Ripple-Carry Addition for Signed Numbers

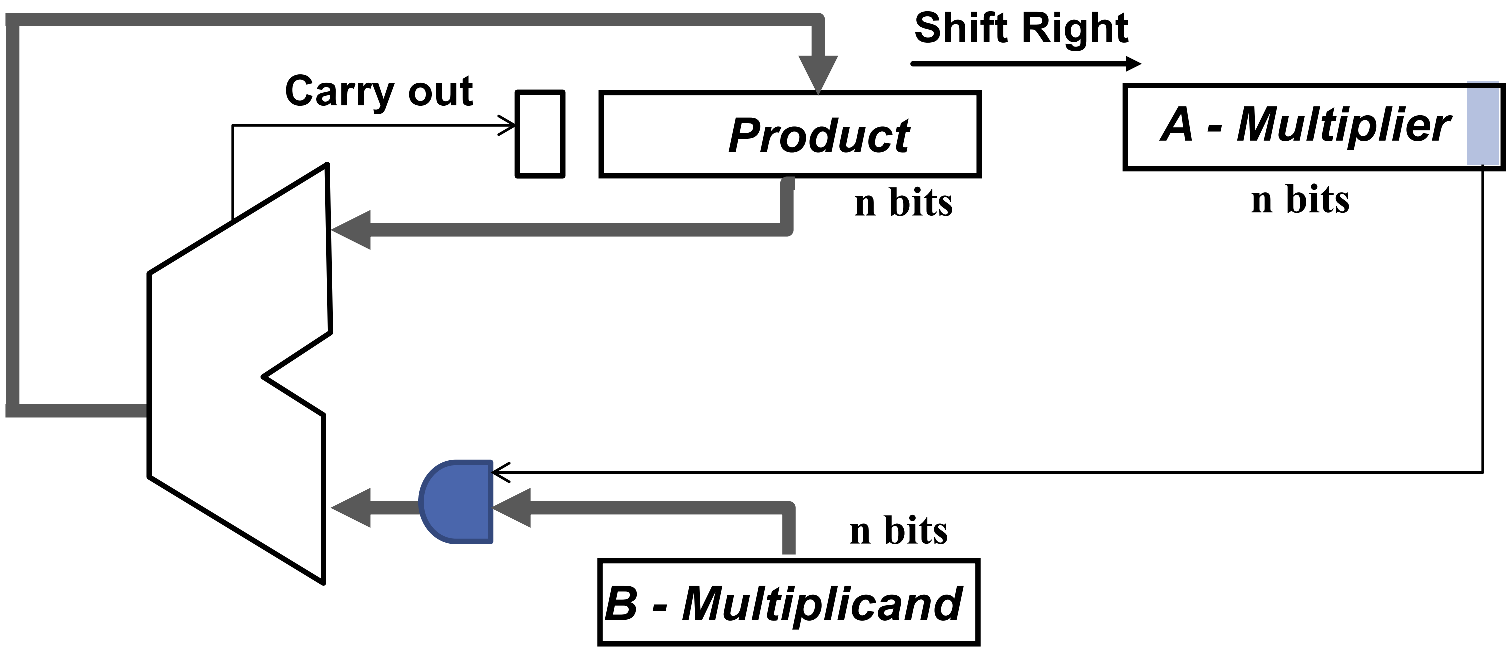
- The Ripple-Carry adder is used for subtraction acting on second operand B and on c_0
- If line ***complement*** is 1 then operand B is complemented bit wise and $c_0=1$



MULTIPLICATION AND MULTIPLIERS

Unsigned Multiplication

- The **simplest multiplier** computes the product of two unsigned numbers, $a_{n-1}a_{n-2} \cdots a_0$ and $b_{n-1}b_{n-2} \cdots b_0$ one bit at a time
- Register ***Product*** is initially 0



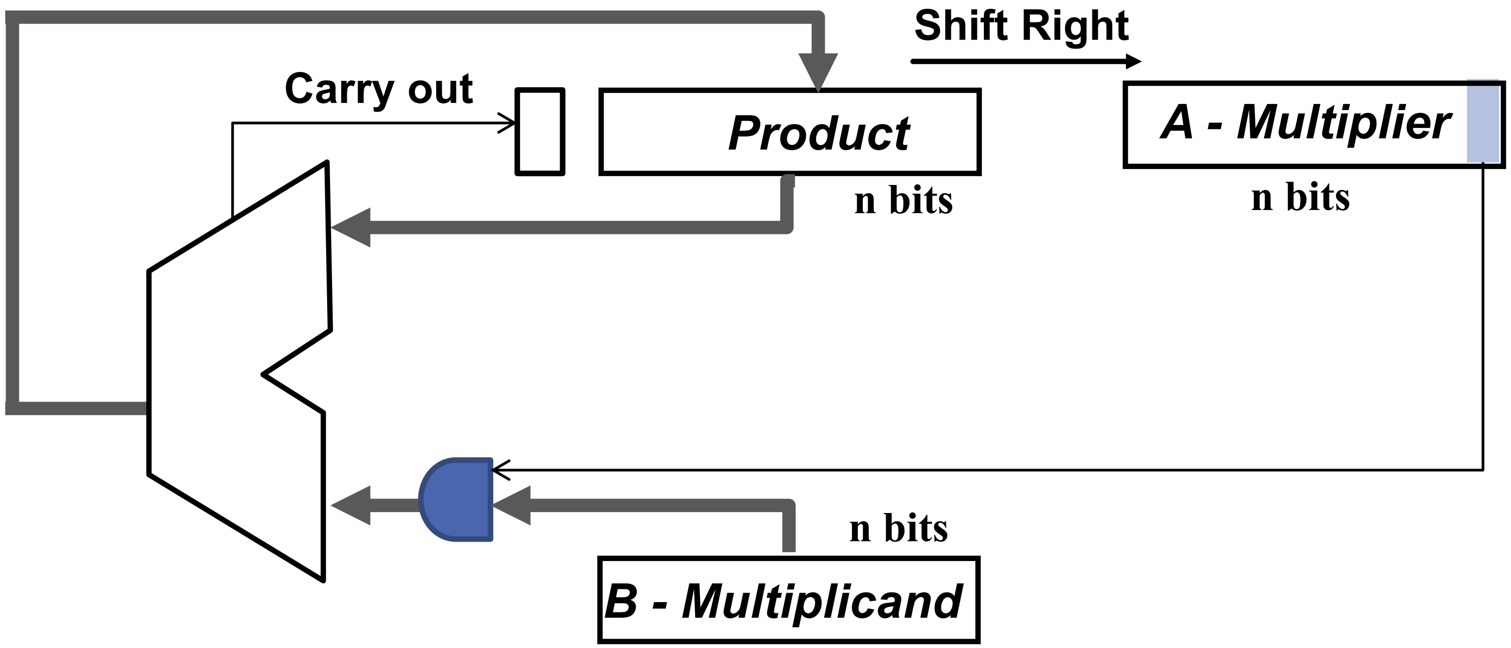
Unsigned Multiplication

- Each multiply step has two parts:

  (i) Partial product and accumulation:

  - If the lsb of A is 1, *then* register B ($b_{n-1}b_{n-2} \cdots b_0$) is added to P;

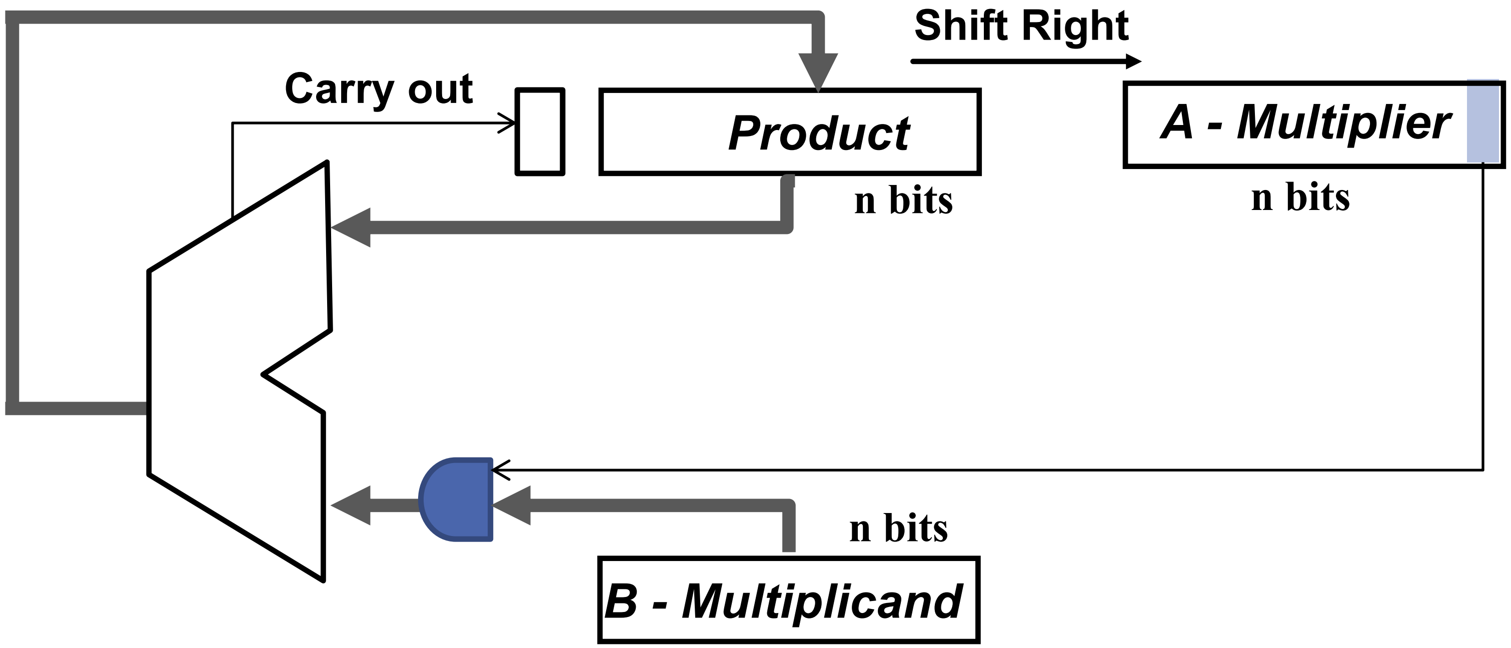
    else $0 \cdots 00$ is added to P
  - The **sum** is placed back **into P**



Unsigned Multiplication

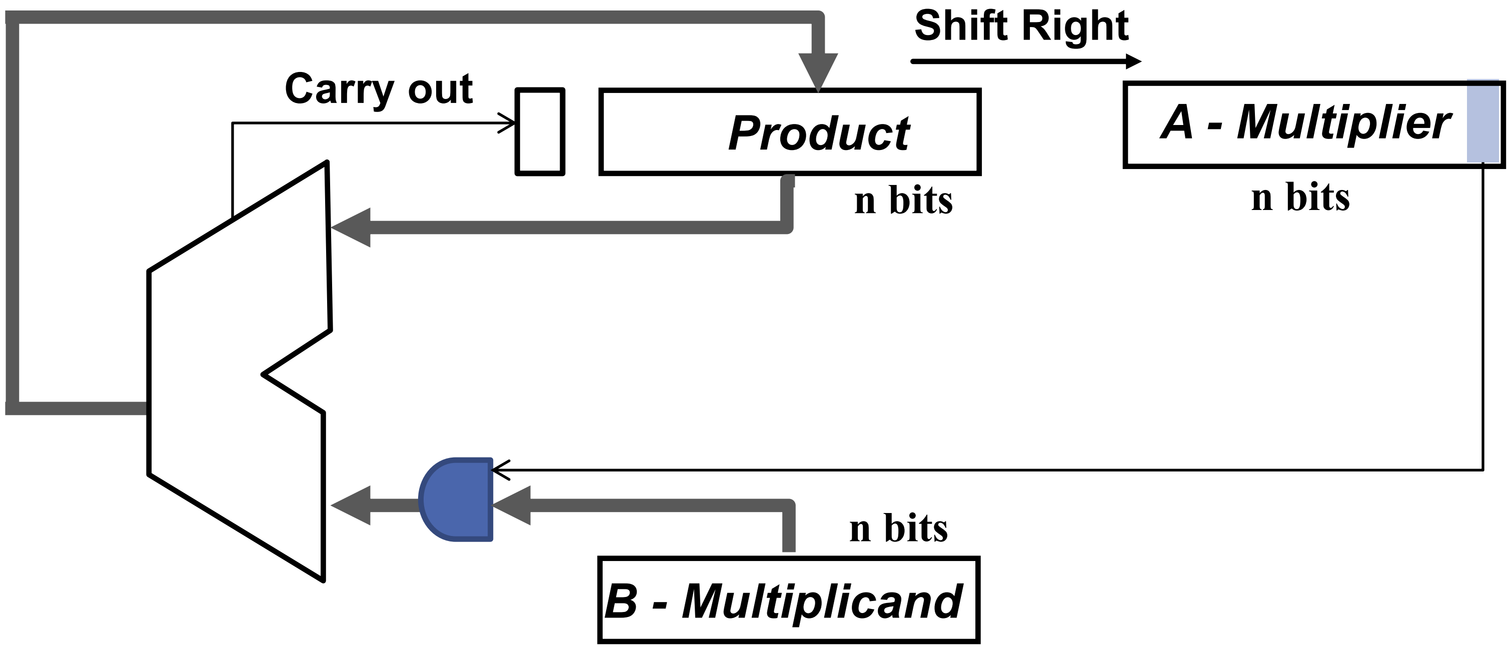
(ii) Registers **P** and **A** are **shifted right**:

- the **carry-out** of the sum is moved into the high-order bit of P
- the low-order bit of P is moved into register A,
- the rightmost bit of A (not used any more) is **shifted out**



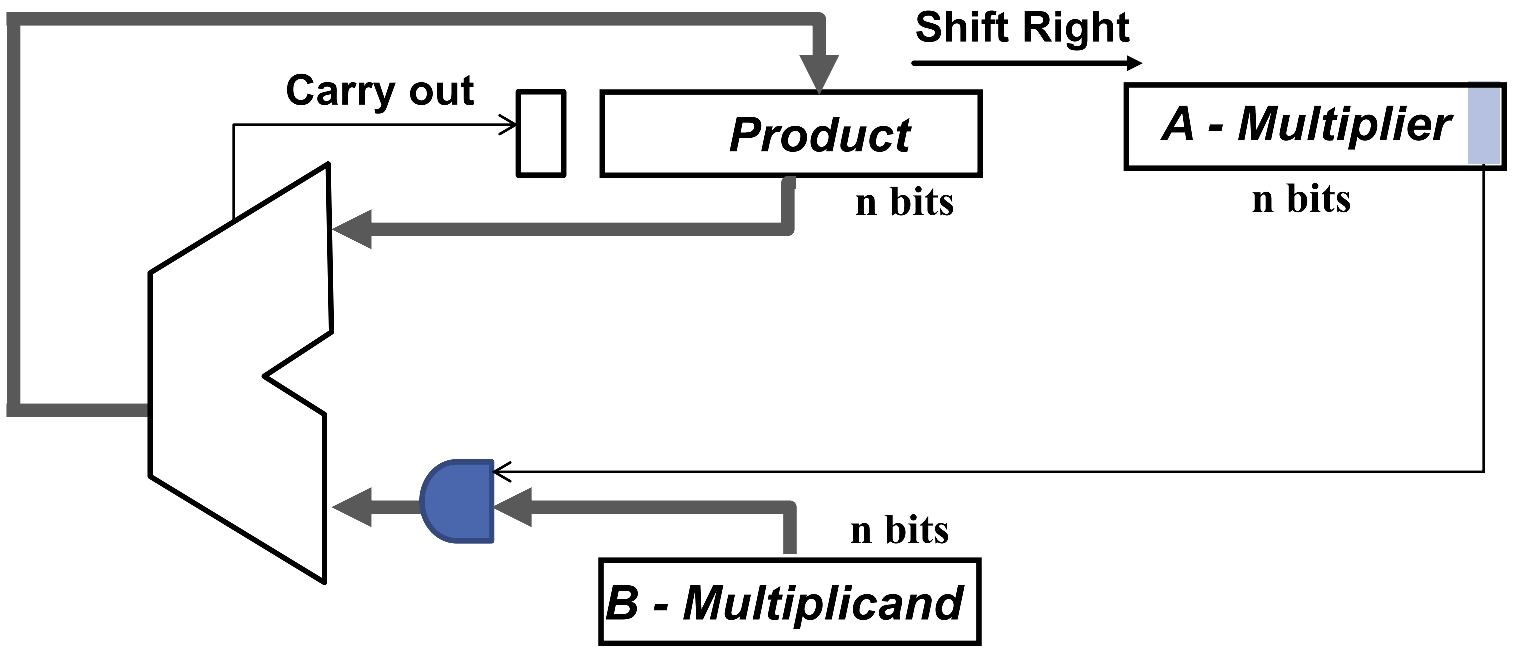
Unsigned Multiplication

- In summary, each multiplication step consists of: **adding** the contents of P to **either B or 0** (depending on the low-order bit of A), **replace** P with the sum, then **shift both P and A** one bit right
- After ***n* steps**, the product appears in registers P and A, with A holding the lower-order bits



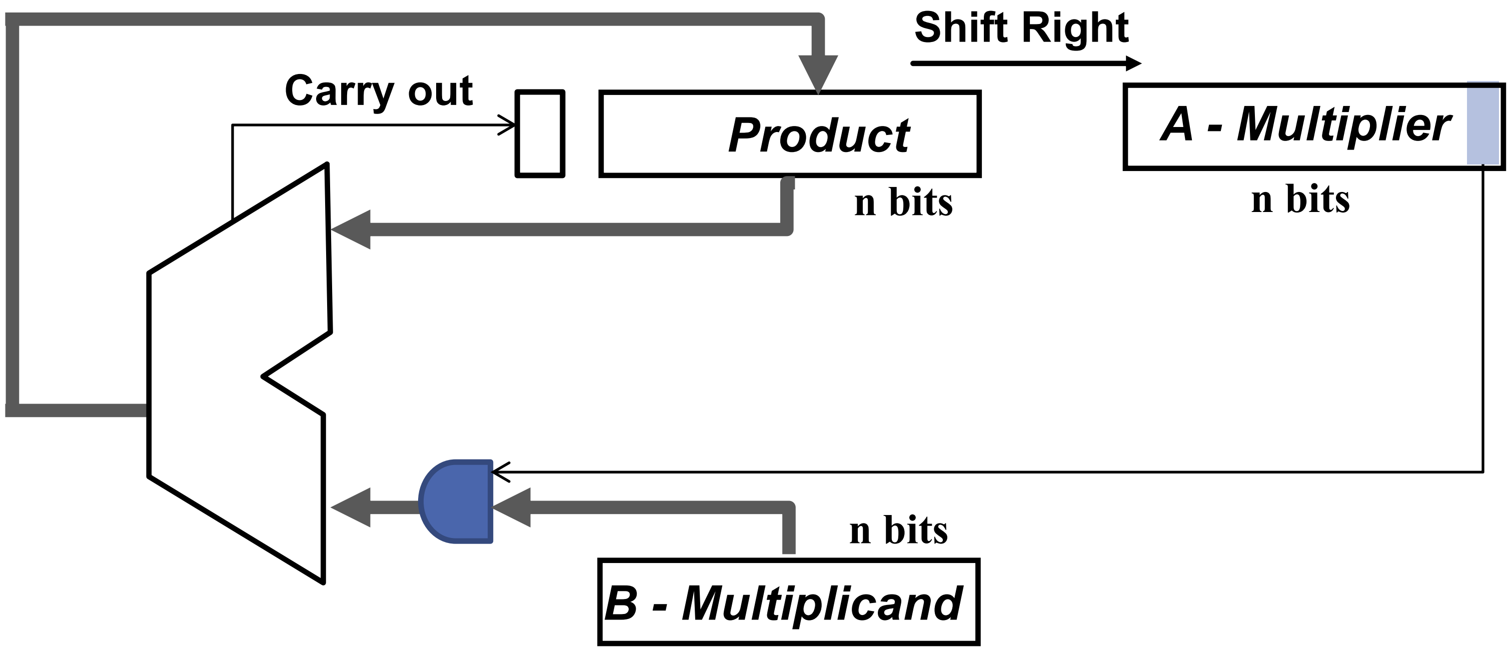
Signed Multiplication

- To multiply two's complement numbers, the **obvious approach** is to convert operands to be nonnegative, do an unsigned multiplication, and then (if the original operands were of opposite signs) negate the result
- This requires **extra time** and **hardware**



Signed Multiplication

- A better approach to multiply *A* and *B* using the hardware below:
  - If **A is nonnegative** and **B is potentially negative**, to convert the unsigned multiplication algorithm into a two's complement one we need that when P is shifted, it is **shifted arithmetically**
  - Our adder will now be **adding n-bit two's complement numbers** between -2^{n-1} and $2^{n-1} - 1$



Signed Multiplication

- If **A is negative**, the method of *Booth recoding* is used
- It is based on the fact that **any sequence of 1s** in a **binary number** can be written as: **011…11 = 100…00 – 000…01**

- **Example** If A = 7 = 0111_2, then we will successively
  - add B, add B, add B, and add 0
  - Booth recoding "recodes" the number 7 as 8 – 1 = 1000_2 – 0001_2

Multiplication

```
          0010
x         0111
--------------
+         0010      add    (1 in multiplier)
+        0010       add    (1 in multiplier)
+       0010        add    (1 in multiplier)
+      0000         shift  (0 in multiplier)
--------------
      00001110
```

Signed Multiplication

- The idea is that:
  - we ***subtract*** when we first see a 1 to ***replace a string of 1s in multiplier***
  - then later we ***add*** for the bit ***after the last one***

```
          0010
x         0111
--------------
+         0010     add   (1 in multiplier)
+        0010      add   (1 in multiplier)
+       0010       add   (1 in multiplier)
+      0000        shift (0 in multiplier)
--------------
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```

Signed Multiplication

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  - we ***subtract*** when we first see a 1 to ***replace a string of 1s in multiplier***
  - then later we ***add*** for the bit ***after the last one***

```
        0010
x       0111
------------
+       0010       add   (1 in multiplier)
+      0010        add   (1 in multiplier)
+     0010         add   (1 in multiplier)
+    0000          shift (0 in multiplier)
------------
    00001110
```

```
        0010
x       0111
------------
-       0010       sub   (first 1 in multpl)
+      0000        shift (0 in multiplier)
+     0000         shift (0 in multiplier)
+    0010          add  (prior step had last 1)
------------
    00001110
```

Signed Multiplication

- The idea is that:
  - we ***subtract*** when we first see a 1 to ***replace a string of 1s in multiplier***
  - then later we ***add*** for the bit ***after the last one***

```
        0010
x       0111
+       0010      add   (1 in multiplier)
+      0010       add   (1 in multiplier)
+     0010        add   (1 in multiplier)
+    0000         shift (0 in multiplier)
    00001110
```

```
        0010
x       0111
+   11111110    sub(first 1 in multpl)
+      0000     shift (0 in multiplier)
+     0000      shift (0 in multiplier)
+    0010       add(prior step had last 1)
    00001110
```

Signed Multiplication

- Hence, to deal with **negative values of A**, all that is required is to sometimes **subtract** B from P, instead of adding either B or 0 to P

- **Rules**: If the initial content of A is $a_{n-1} \cdot \cdot \cdot a_0$, then step ($i$) in the multiplication algorithm becomes:
  - If $a_i = 0$ and $a_{i-1} = 0$, then **add 0** to P
  - If $a_i = 0$ and $a_{i-1} = 1$, then **add B** to P
  - If $a_i = 1$ and $a_{i-1} = 0$, then **subtract B** from P
  - If $a_i = 1$ and $a_{i-1} = 1$, then **add 0** to P

For the first step, when $i = 0$, take a_{i-1} to be 0

SPEEDING UP OPERATIONS

Speeding Up Operations

- Integer **addition** is the simplest operation and the most important
- Even for programs that do not do explicit arithmetic, addition must be performed to increment the program counter and to calculate addresses
- The delay of an N-bit ripple-carry adder is:

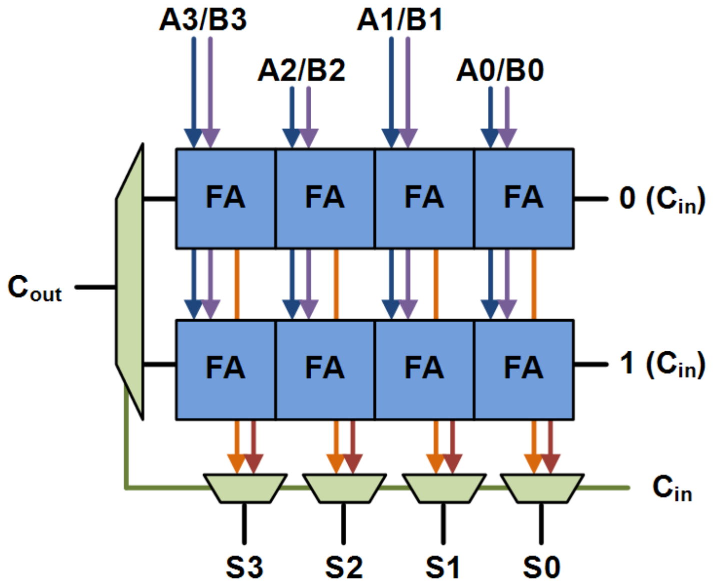
$$t_{ripple} = Nt_{FA}$$

where t_{FA} is the delay of a full adder

- There are different techniques to increase the speed of integer operations (which also lead to faster floating point operations), as the Carry Select Adder (CSA) and Carry Look-ahead Adder (CLA)

Carry-Select Adder

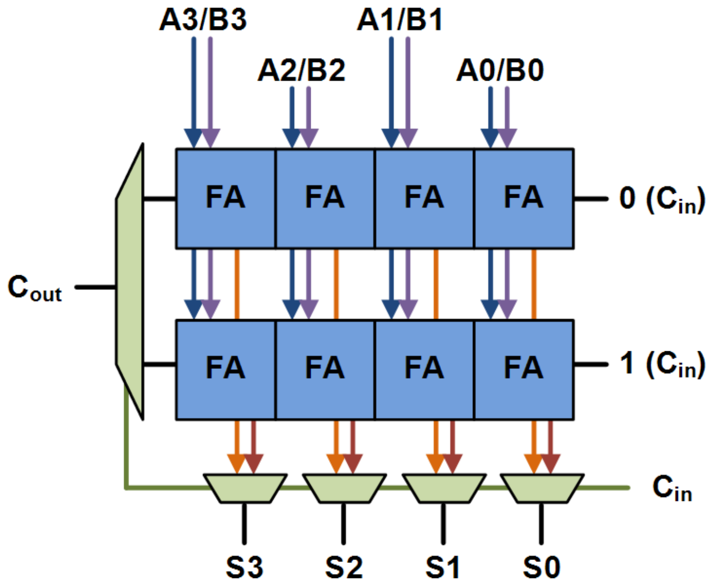
- The **carry-select adder** improves speed dividing operands bits in blocks
- A **carry-select adder** consists of:
  - short ripple carry adders acting on blocks of bits
  - multiplexers
- The two blocks of bits is added with **two ripple-carry adders**, one with the carry-in equal to 0 and the other with the carry-in equal to 1



Carry-select adder - Wikipedia

Carry-Select Adder

- In the figure below **two** 4-bit **ripple-carry adders** are **multiplexed** together
- The resulting carry and sum bits are selected by the carry-in
- The correct result is selected by the actual carry-in which selects which adder had the correct assumption



Carry-select adder - Wikipedia

Carry-Select Adder

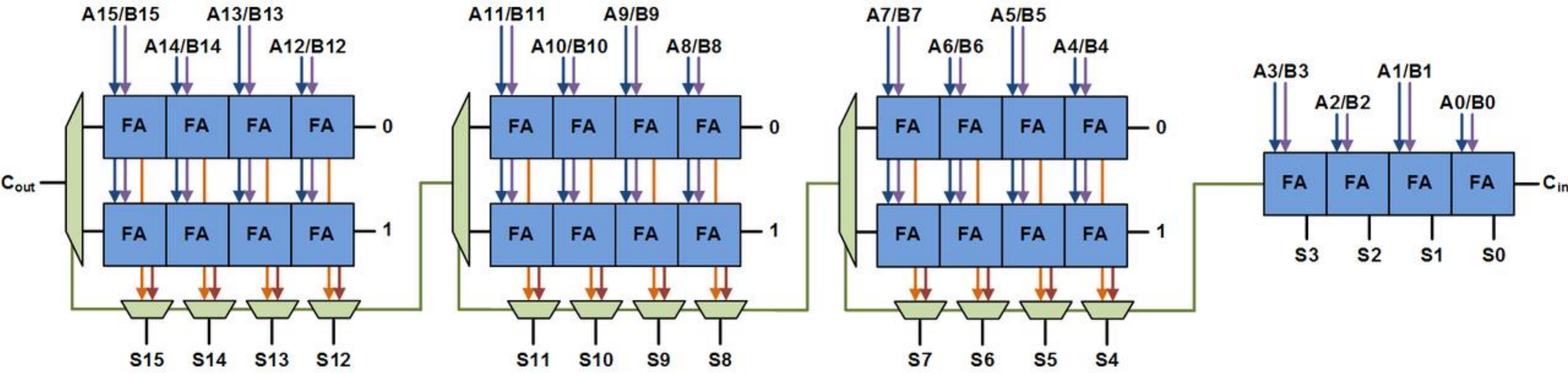
Uniform-sized adder

A 16-bit carry-select adder with a uniform block size of 4 has:

- three of these blocks
- a 4-bit ripple-carry adder

A carry select block is **not needed** for the four **LSBs**

The **delay** of this adder will be **four full adder delays** plus **three 2-to-1 MUX delays**

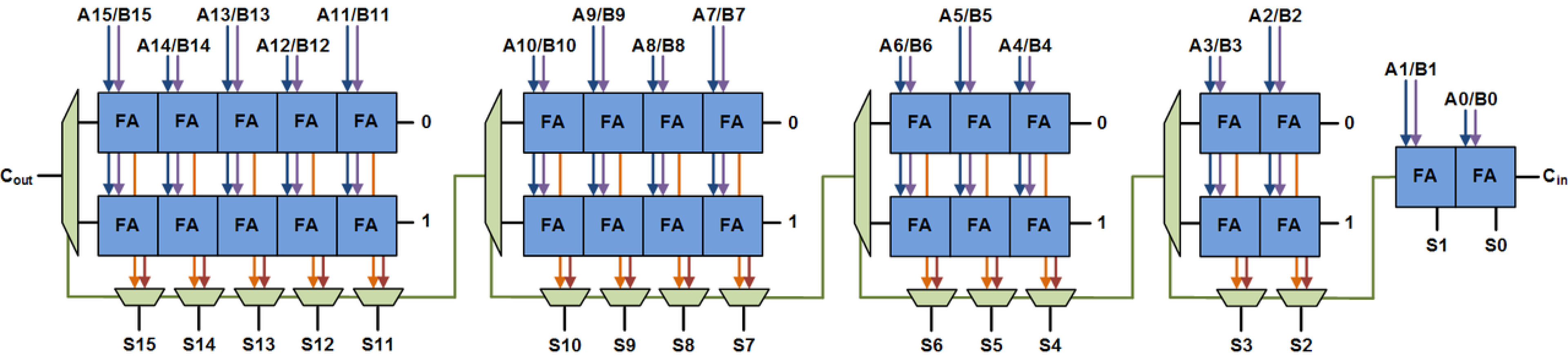


Carry-Select Adder

Variable-sized adder

A 16-bit carry-select adder with variable size can be created using block sizes of 2-2-3-4-5

- This break-up is ideal when the full-adder delay is equal to the MUX delay
- The total **delay** is **two full adder delays** plus **four mux delays**



Carry-Lookahead Adder

- A **carry-lookahead adder** improves speed by reducing the amount of time required to **determine carry bits**
- The carry-lookahead adder calculates one or more carry bits before the sum, which reduces the wait time to calculate the result of the larger-value bits of the adder
- Remember that:

$$c_{i+1} = a_i b_i + (a_i \oplus b_i) c_i$$

$$s_i = a_i \overline{b}_i \overline{c}_i + \overline{a}_i b_i \overline{c}_i + \overline{a}_i \overline{b}_i c_i + a_i b_i c_i = (a_i \oplus b_i) \oplus c_i$$

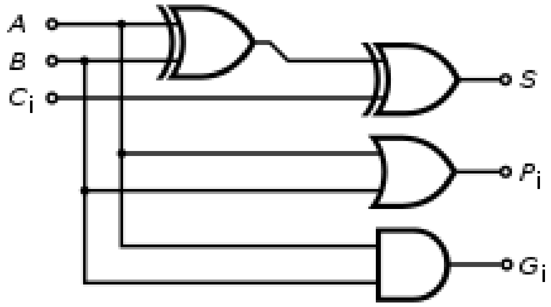
Carry-Lookahead Adder

We define:

- **Carry Generate** $g_i = a_i b_i$
- **Carry propagate** $p_i = a_i \oplus b_i$ or $p_i = a_i + b_i$

Then the **expression of the carry** is:

$$c_{i+1} = a_i b_i + (a_i \oplus b_i) c_i = g_i + p_i c_i$$



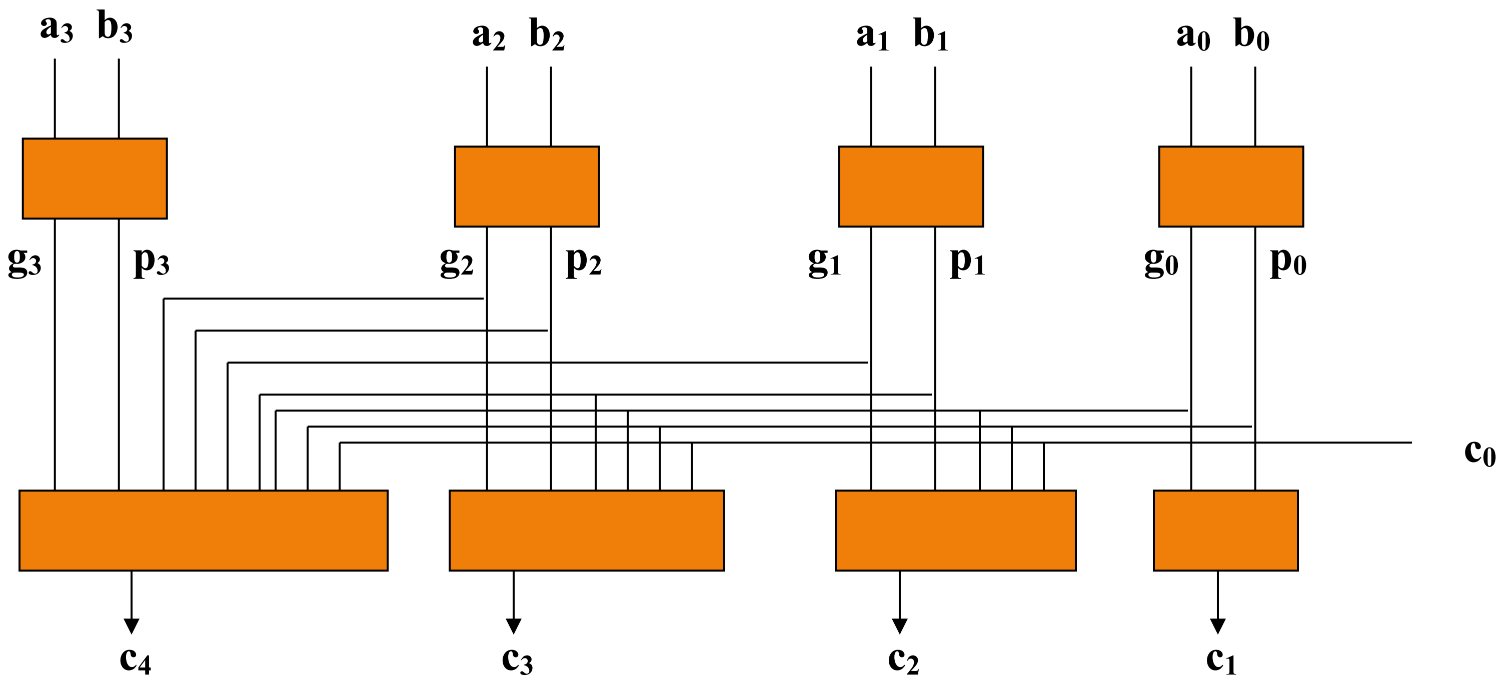
and the **expression of the sum** is:

$$s_i = (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i$$

Carry-Lookahead Adder

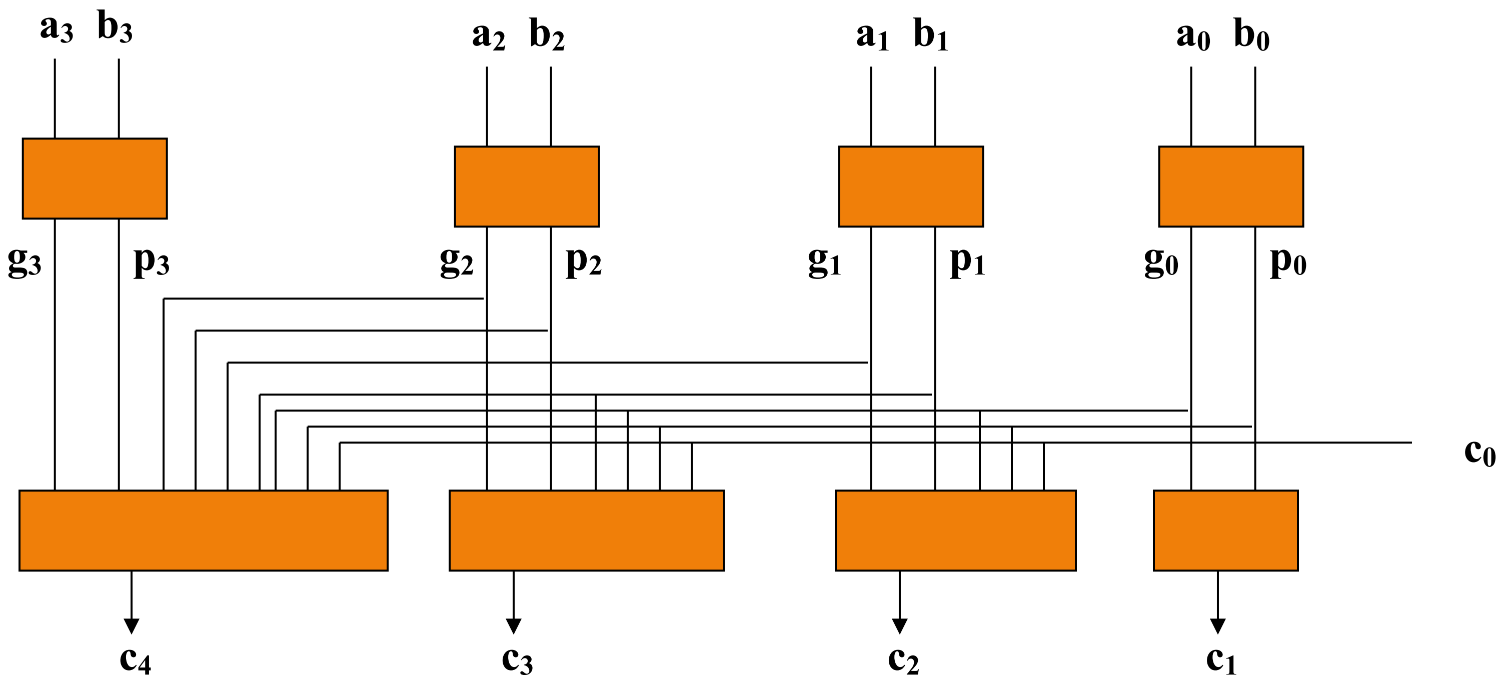
If we consider 4 bits, we have c_1, c_2, c_3, c_4, depend only on c_0:

$$c_1 = a_0b_0 + (a_0+b_0)c_0 = g_0 + p_0c_0$$
$$c_2 = a_1b_1 + (a_1+b_1)c_1 = g_1 + p_1c_1 = g_1 + p_1g_0 + p_1p_0c_0$$
$$c_3 = a_2b_2 + (a_2+b_2)c_2 = g_2 + p_2c_2 = g_2 + p_2g_1 + p_2p_1g_0 + p_2p_1p_0c_0$$
$$c_4 = a_3b_3 + (a_3+b_3)c_3 = g_3 + p_3c_3 = g_3 + p_3g_2 + p_3p_2g_1 + p_3p_2p_1g_0 + p_3p_2p_1p_0c_0$$



Carry-Lookahead Adder

- So, a carry-lookahead adder on *n* bits requires a **fan-in of *n* + 1** at the OR gate as well as at the rightmost AND gate
- The irregular structure and long wires make it impractical to build a full carry-lookahead adder when *n* is large

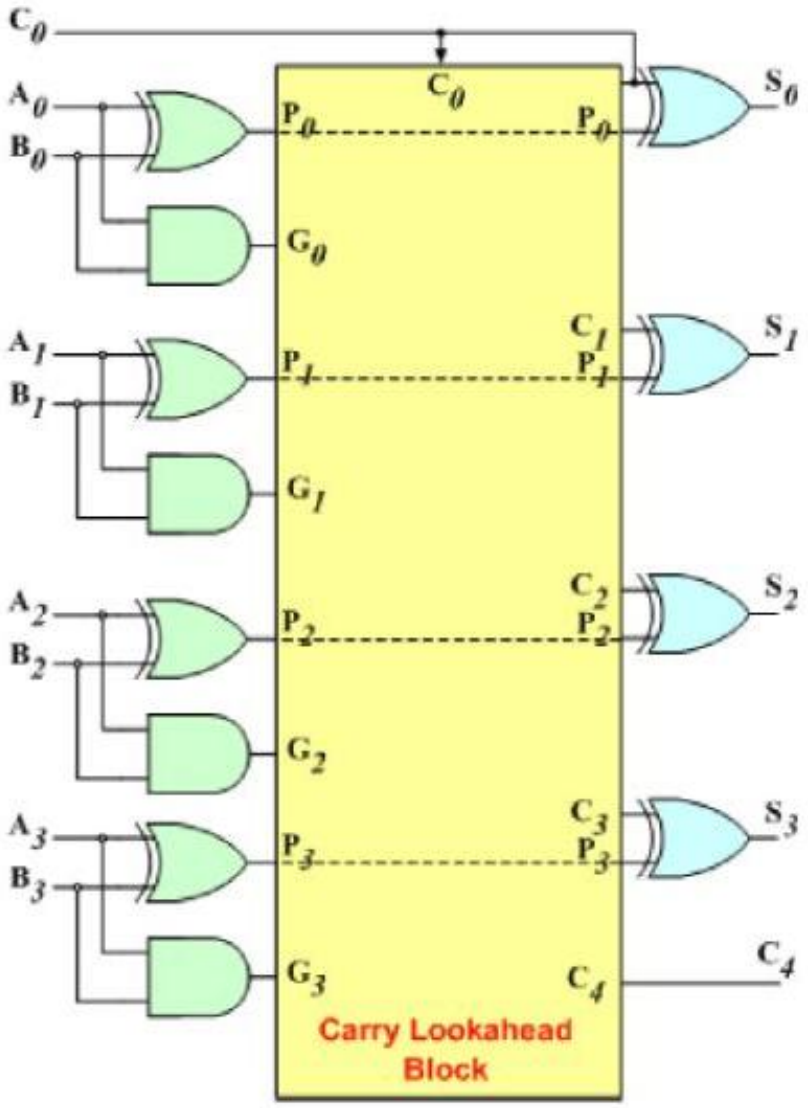


Carry-Lookahead Addition

- Structure of a **4-bit CLA**

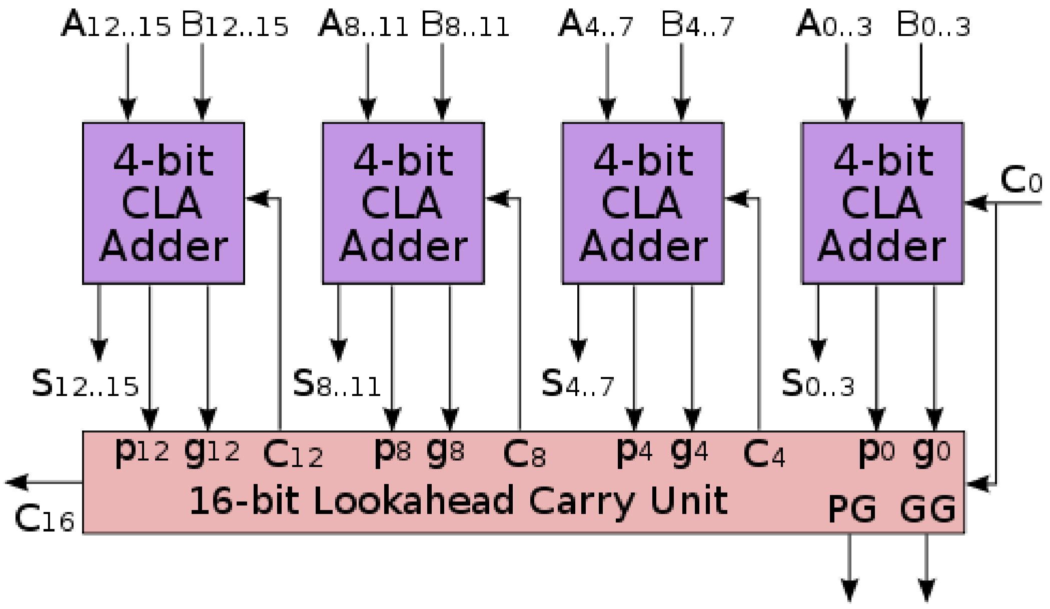
  $$s_i = (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i$$

- A CLA requires:
  - one logic level to form *p* and *g*
  - two levels to form the carries
  - two for the sum
  - for a total of **five logic levels**
- **Improvement** over the 2*n* levels required for the ripple-carry adder



Carry-Lookahead Addition

- A **16-bit adder** can be built from four 4-bit CLAs and a 4-bit Look-ahead Carry Unit (LCU) at the second level



- A **64-bit adder** can be built from four **16-bit adder** shown above, and an additional LCU that accepts bits from each LCU above and generates carry bits fed back to
- https://en.wikipedia.org/wiki/Lookahead_carry_unit

Carry-Lookahead Addition

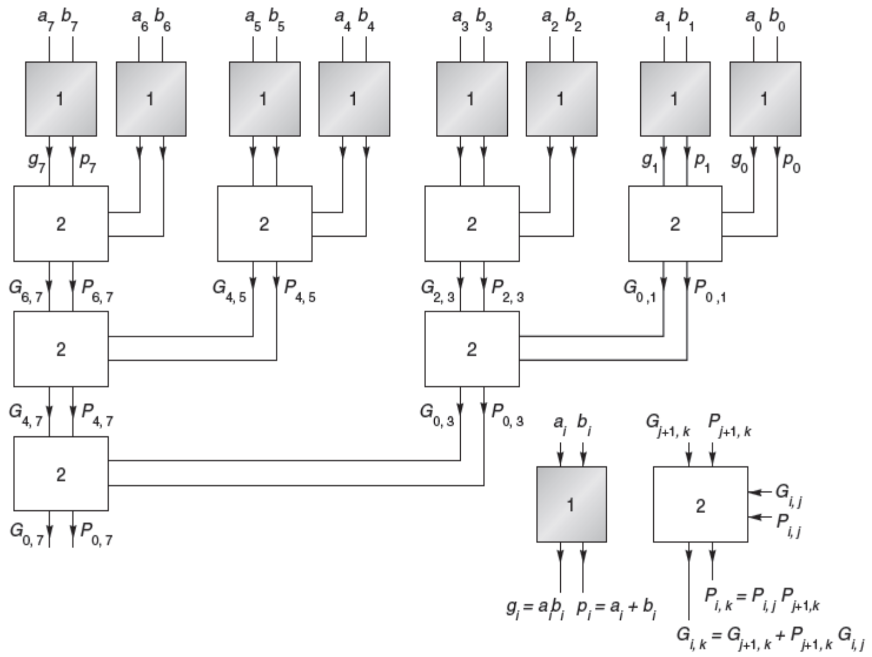
We can use the carry-lookahead idea to build an adder that has about $\log_2 n$ logic levels, as in a tree

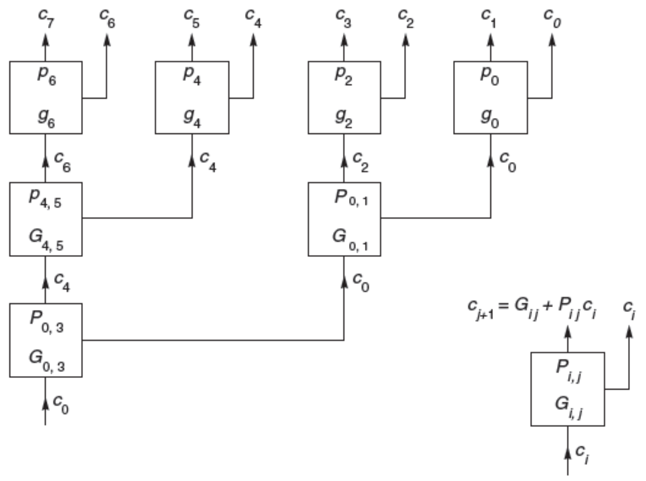
Starting from

- $G_{01} = g_1 + p_1 g_0$
- $P_{01} = p_1 p_0$

In general, for any *j* with $i < j$ and $j + 1 < k$, we have the **recursive relations**:

- $c_{k+1} = G_{ik} + P_{ik} c_i$
- $G_{ik} = G_{j+1,k} + P_{j+1,k} G_{ij}$
- $P_{ik} = P_{ij} P_{j+1,k}$


a_7 b_7 a_6 b_6 a_5 b_5 a_4 b_4 a_3 b_3 a_2 b_2 a_1 b_1 a_0 b_0
1
g_7 p_7 g_1 p_1 g_0 p_0
2
$G_{6,7}$ $P_{6,7}$ $G_{4,5}$ $P_{4,5}$ $G_{2,3}$ $P_{2,3}$ $G_{0,1}$ $P_{0,1}$
$G_{4,7}$ $P_{4,7}$ $G_{0,3}$ $P_{0,3}$
$G_{0,7}$ $P_{0,7}$
a_i b_i
$G_{j+1,k}$ $P_{j+1,k}$
$G_{i,j}$ $P_{i,j}$
$g_i = a_i b_i$ $p_i = a_i + b_i$
$P_{i,k} = P_{i,j} P_{j+1,k}$
$G_{i,k} = G_{j+1,k} + P_{j+1,k} G_{i,j}$


c_7 c_6 c_5 c_4 c_3 c_2 c_1 c_0
p_6 g_6 p_4 g_4 p_2 g_2 p_0 g_0
c_6 c_4 c_2 c_0
$P_{4,5}$ $G_{4,5}$ $P_{0,1}$ $G_{0,1}$
c_4 c_0
$P_{0,3}$ $G_{0,3}$
c_0
$c_{j+1} = G_{ij} + P_{ij} c_i$ c_i
$P_{i,j}$ $G_{i,j}$
c_i

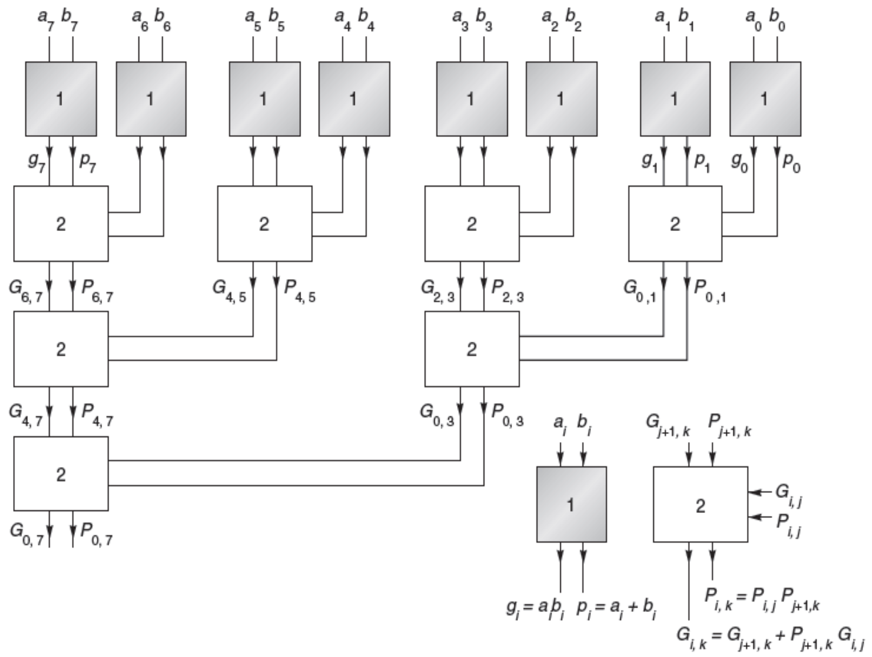
Carry-Lookahead Addition

First part of carry-lookahead tree:

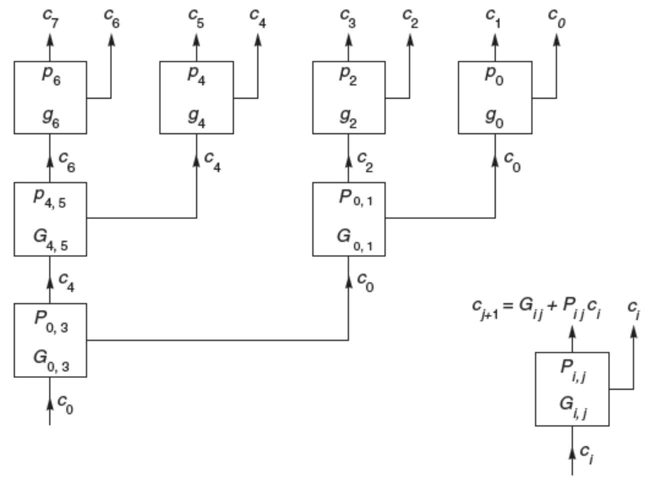
- signals flow **from the top to the bottom**
- various values of P and G are computed

Second part of carry-lookahead tree:

- signals flow **from the bottom to the top**, combining with P and G to form the carries



$a_7\ b_7$ $a_6\ b_6$ $a_5\ b_5$ $a_4\ b_4$ $a_3\ b_3$ $a_2\ b_2$ $a_1\ b_1$ $a_0\ b_0$
g_7 p_7 g_1 p_1 g_0 p_0
$G_{6,7}$ $P_{6,7}$ $G_{4,5}$ $P_{4,5}$ $G_{2,3}$ $P_{2,3}$ $G_{0,1}$ $P_{0,1}$
$G_{4,7}$ $P_{4,7}$ $G_{0,3}$ $P_{0,3}$
$G_{0,7}$ $P_{0,7}$
$a_i\ b_i$
$g_i = a_i b_i$ $p_i = a_i + b_i$
$G_{j+1,k}$ $P_{j+1,k}$ $G_{i,j}$ $P_{i,j}$
$P_{i,k} = P_{i,j} P_{j+1,k}$
$G_{i,k} = G_{j+1,k} + P_{j+1,k} G_{i,j}$

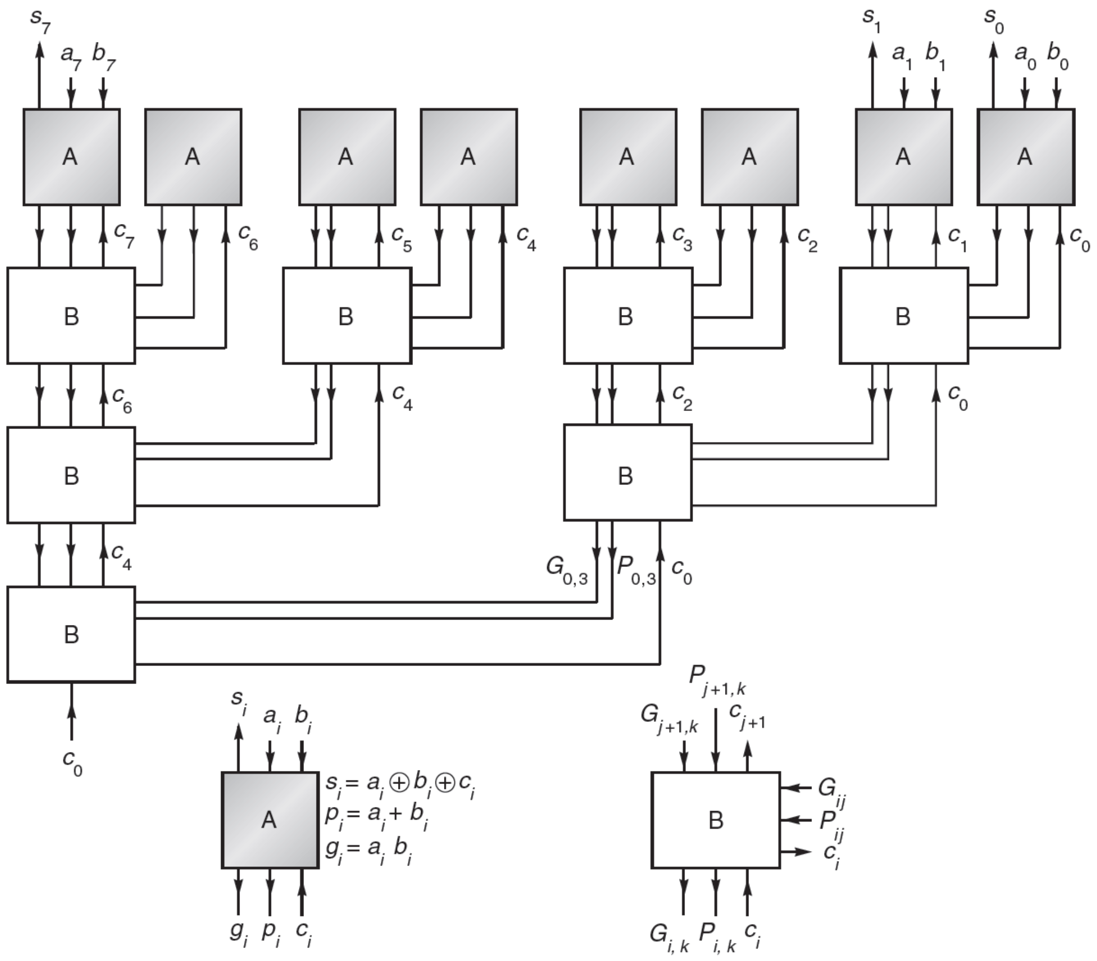


c_7 c_6 c_5 c_4 c_3 c_2 c_1 c_0
p_6 g_6 p_4 g_4 p_2 g_2 p_0 g_0
c_6 c_4 c_2 c_0
$P_{4,5}$ $G_{4,5}$ $P_{0,1}$ $G_{0,1}$
c_4 c_0
$P_{0,3}$ $G_{0,3}$
c_0
$c_{j+1} = G_{ij} + P_{ij} c_i$ c_i
$P_{i,j}$ $G_{i,j}$
c_i

Carry-Lookahead Addition

- Using the **recursive** relations, practical CLAs are designed by combining cells in a **binary tree structure**

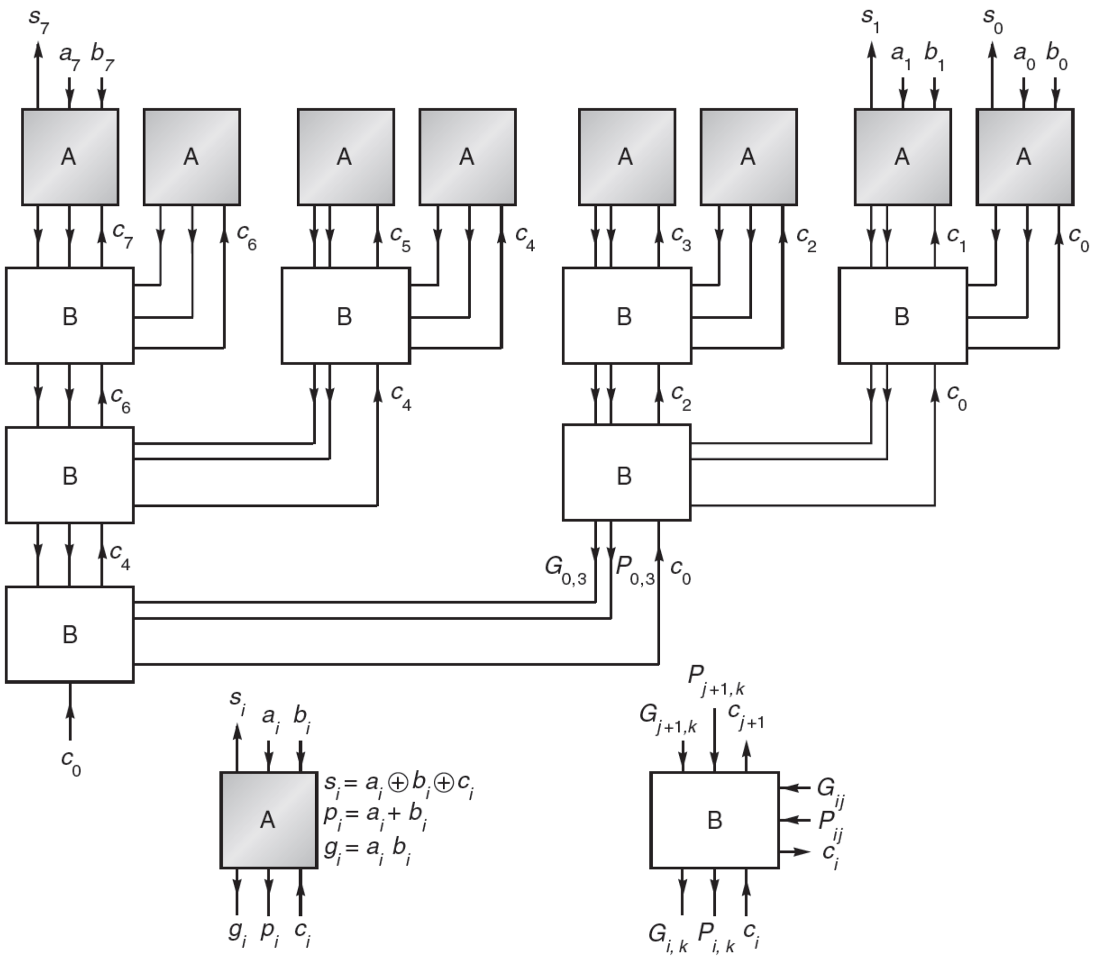
- The numbers to be added flow into the top and go downward through the tree, combining with c0 at the bottom and flowing back up the tree to form the carries



s_7 a_7 b_7
s_1 a_1 b_1
s_0 a_0 b_0
A
B
c_7 c_6 c_5 c_4 c_3 c_2 c_1 c_0
$G_{0,3}$ $P_{0,3}$ c_0
s_i a_i b_i
$s_i = a_i \oplus b_i \oplus c_i$
$p_i = a_i + b_i$
$g_i = a_i\, b_i$
g_i p_i c_i
$P_{j+1,k}$ $G_{j+1,k}$ c_{j+1}
G_{ij} P_{ij} c_i
$G_{i,k}$ $P_{i,k}$ c_i

Carry-Lookahead Addition

- The bits in a CLA pass through about $\log_2 n$ logic levels, compared with $2n$ for a ripple-carry adder
- **But** the ripple-carry adder has n cells and the CLA has $2n$ cells, even if they will take $n \log n$ space
- Speed improvement especially for a large n



Speeding Up Multiplications

- Methods that increase the speed of **multiplication** can be divided into two classes:
  - single adder
  - multiple adders

- In the simple multiplier we described, each multiplication step passes through the **single adder**
- The amount of computation in each step depends on the used adder (consider the difference between an RCA and a CLA)
- If the space for **many adders** is available, then multiplication speed can be increased thanks to the replication of resources

PIPELINED ARITHMETIC OPERATIONS

Pipelined arithmetic

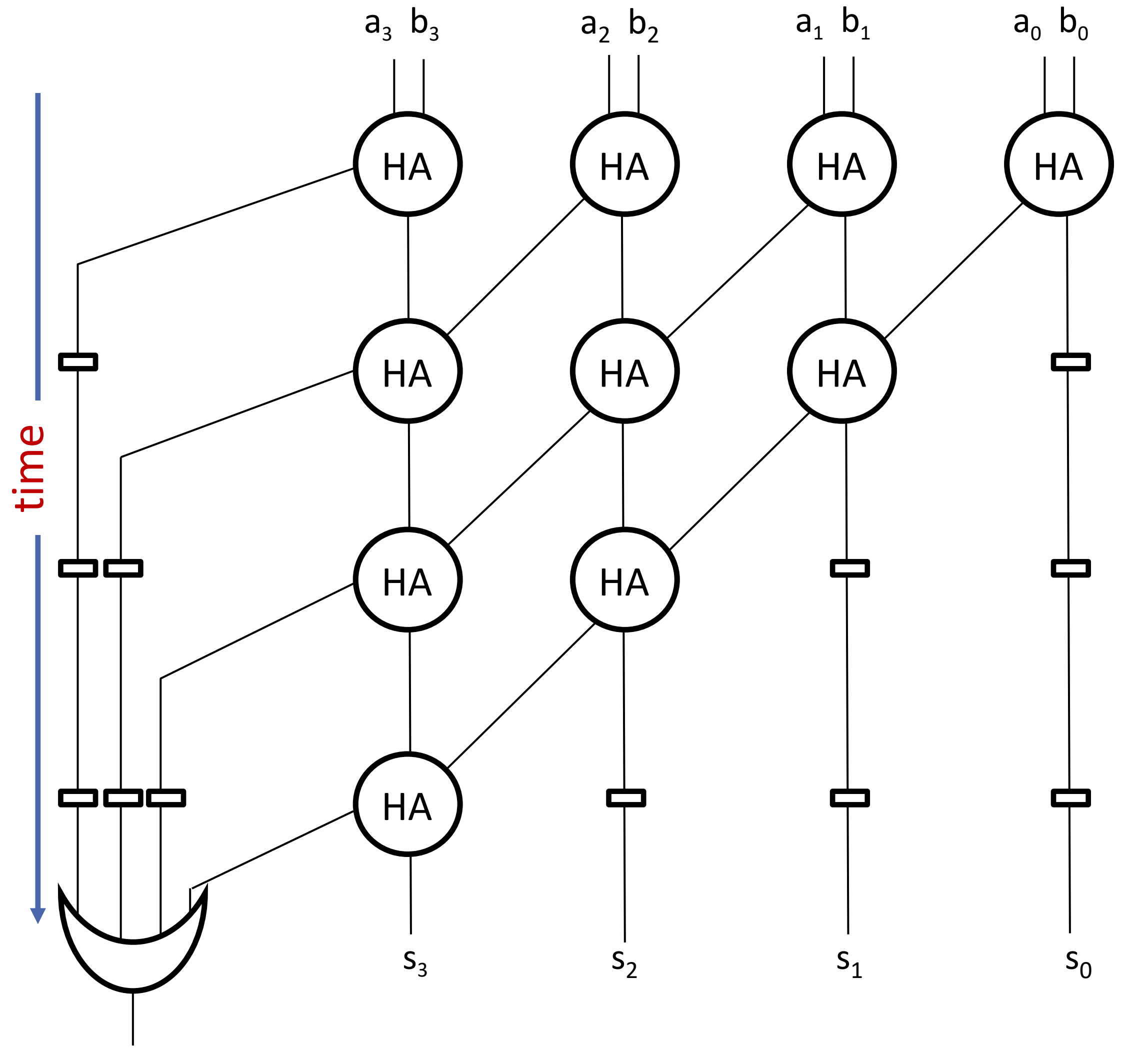
- Consider the instruction pipelining:
  - The processor goes through a repetitive cycle of fetching and processing instructions
  - In the absence of hazards:
    - the processor is continuously fetching instructions from their locations
    - the pipeline is kept full
    - a savings in time is achieved
- Similarly, a **pipelined ALU** will save time if it is fed a **stream of data** from sequential locations
- A single, isolated operation is not speeded up by pipeline
- The speedup is achieved when **a vector of operands** is presented to the units in the ALU

Pipelined arithmetic

- The relative simplicity of two-operand adders usually does not justify their implementation as pipelines
- In special-purpose design, when **many successive additions** are needed, such implementations are justifiable
- Some adders can be implemented as pipeline, such as the conditional-sum adder or the *carry-save adder* (for multiple operands)
- But, for example, some design of the *carry look-ahead adder* cannot be pipelined because some carry signals propagate backward
- There are very simple schemes for the **pipelined adders** and **multipliers** along the lines of the ripple-carry adder

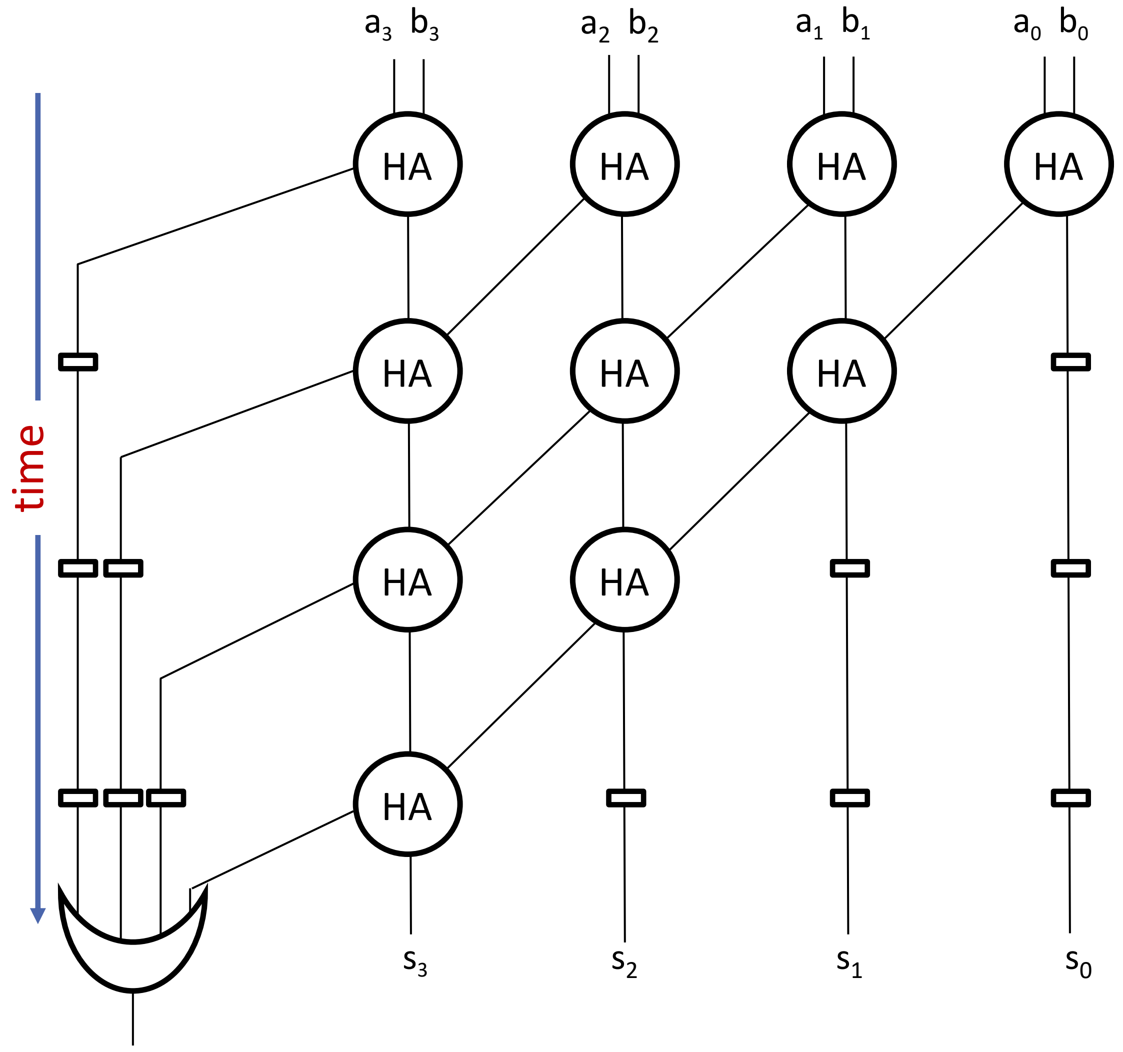
Pipelined Addition

- For n bits operands, a **pipeline adder** consists of n stages of half adders
- Registers (FF D) are inserted at each stage to synchronize the computation
- At each clock cycle a new pair of operands is applied to the inputs of the adder



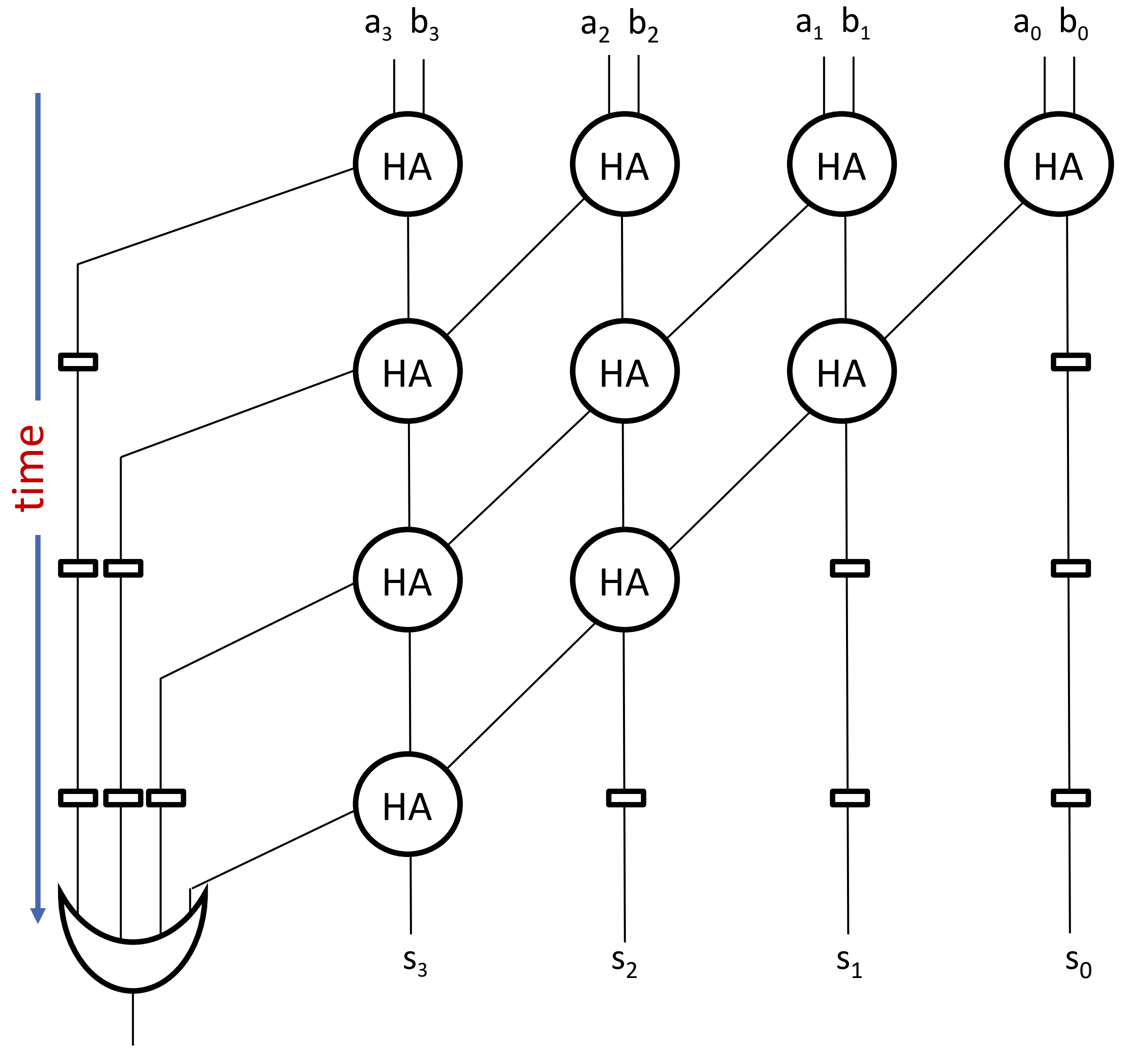
Pipelined Addition

- After ***n* clock cycles**, the sum of the first pair of operands is obtained
- The computing time for a single sum is the same of the carry-ripple adder
- A new sum is obtained at each clock cycle starting from the (*n*+1)-th clock cycle



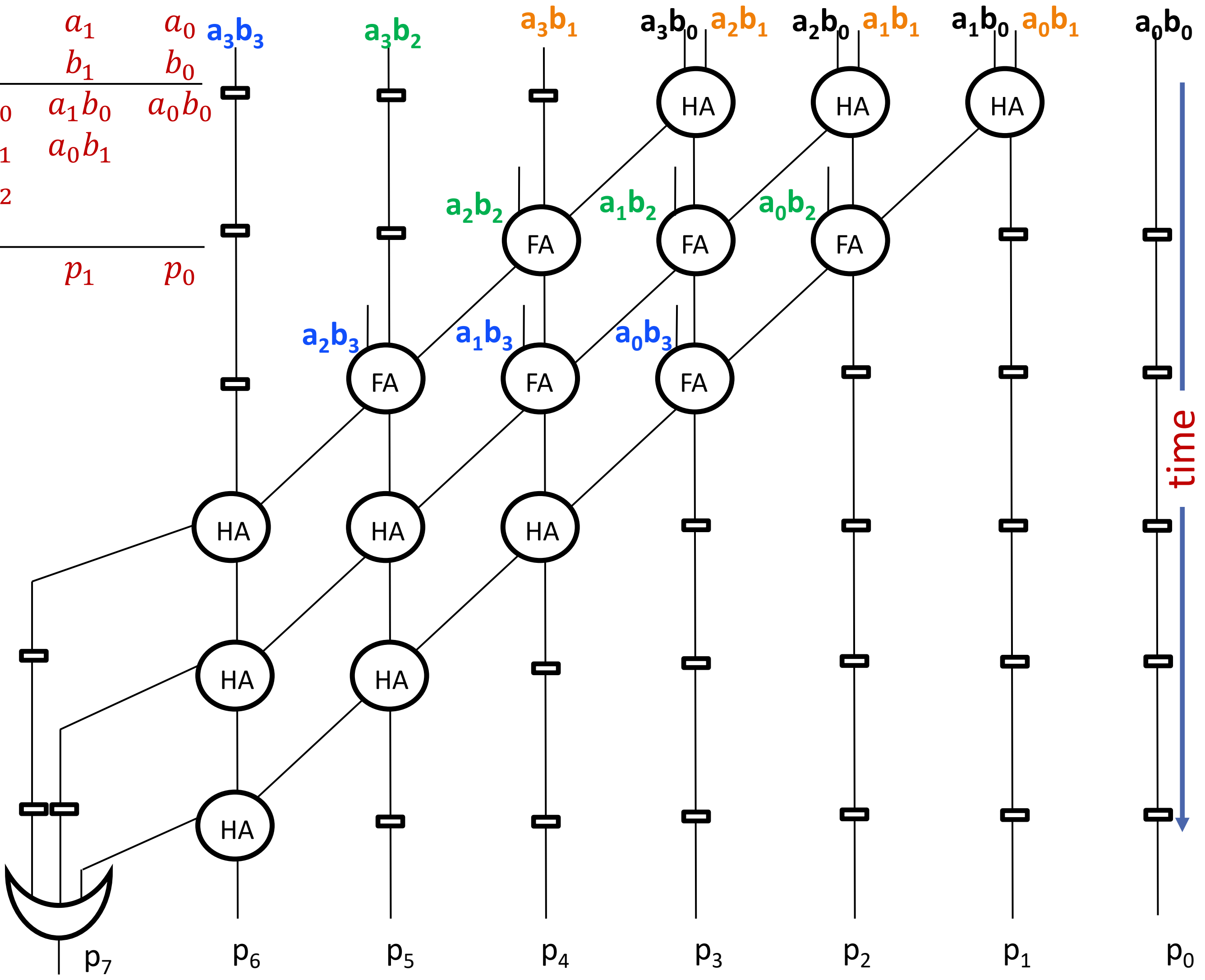
Pipelined Addition

- The number of **HA** is **O(n^2)**, whereas the circuit complexity of the carry-ripple adder is O(n)
- The added circuit complexity pays off if long sequences of numbers are being added



Pipelined Unsigned Multiplication

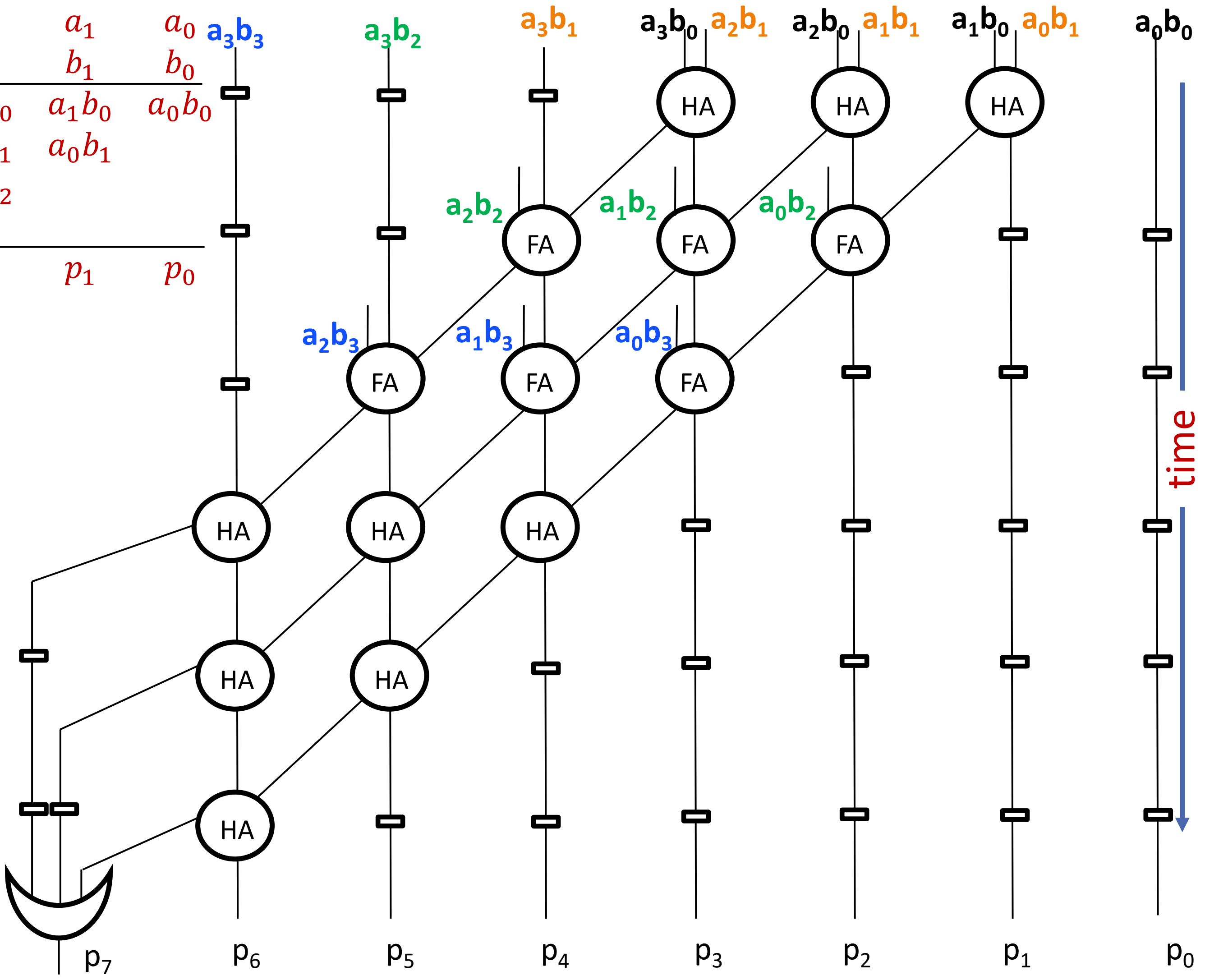
$$
\begin{array}{cccccccc}
 & & & & a_3 & a_2 & a_1 & a_0 \\
 & & & & b_3 & b_2 & b_1 & b_0 \\
\hline
 & & & & a_3b_0 & a_2b_0 & a_1b_0 & a_0b_0 \\
 & & & a_3b_1 & a_2b_1 & a_1b_1 & a_0b_1 & \\
 & & a_3b_2 & a_2b_2 & a_1b_2 & a_0b_2 & & \\
 & a_3b_3 & a_2b_3 & a_1b_3 & a_0b_3 & & & \\
\hline
p_7 & p_6 & p_5 & p_4 & p_3 & p_2 & p_1 & p_0
\end{array}
$$



- The product of two *n* bit operands has length 2*n*
- Result is obtained by executing n-1 sums

Pipelined Unsigned Multiplication

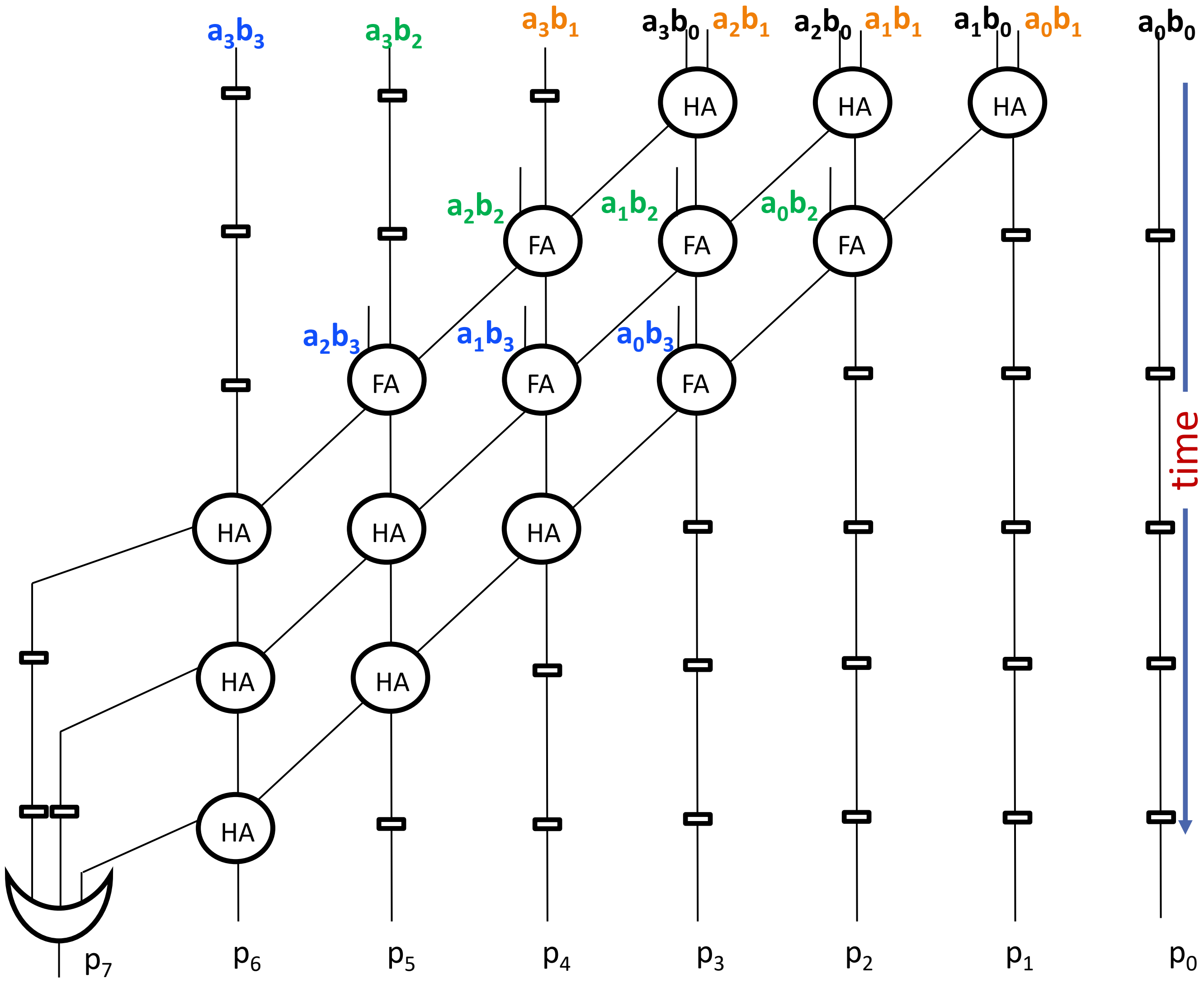
$$
\begin{array}{cccccccc}
 & & & & a_3 & a_2 & a_1 & a_0 \\
 & & & & b_3 & b_2 & b_1 & b_0 \\
\hline
 & & & & a_3b_0 & a_2b_0 & a_1b_0 & a_0b_0 \\
 & & & a_3b_1 & a_2b_1 & a_1b_1 & a_0b_1 & \\
 & & a_3b_2 & a_2b_2 & a_1b_2 & a_0b_2 & & \\
 & a_3b_3 & a_2b_3 & a_1b_3 & a_0b_3 & & & \\
\hline
p_7 & p_6 & p_5 & p_4 & p_3 & p_2 & p_1 & p_0
\end{array}
$$



- Inputs to the multiplier are logical AND among pairs of bits
- There are 2(n-1) stages of FA or HA

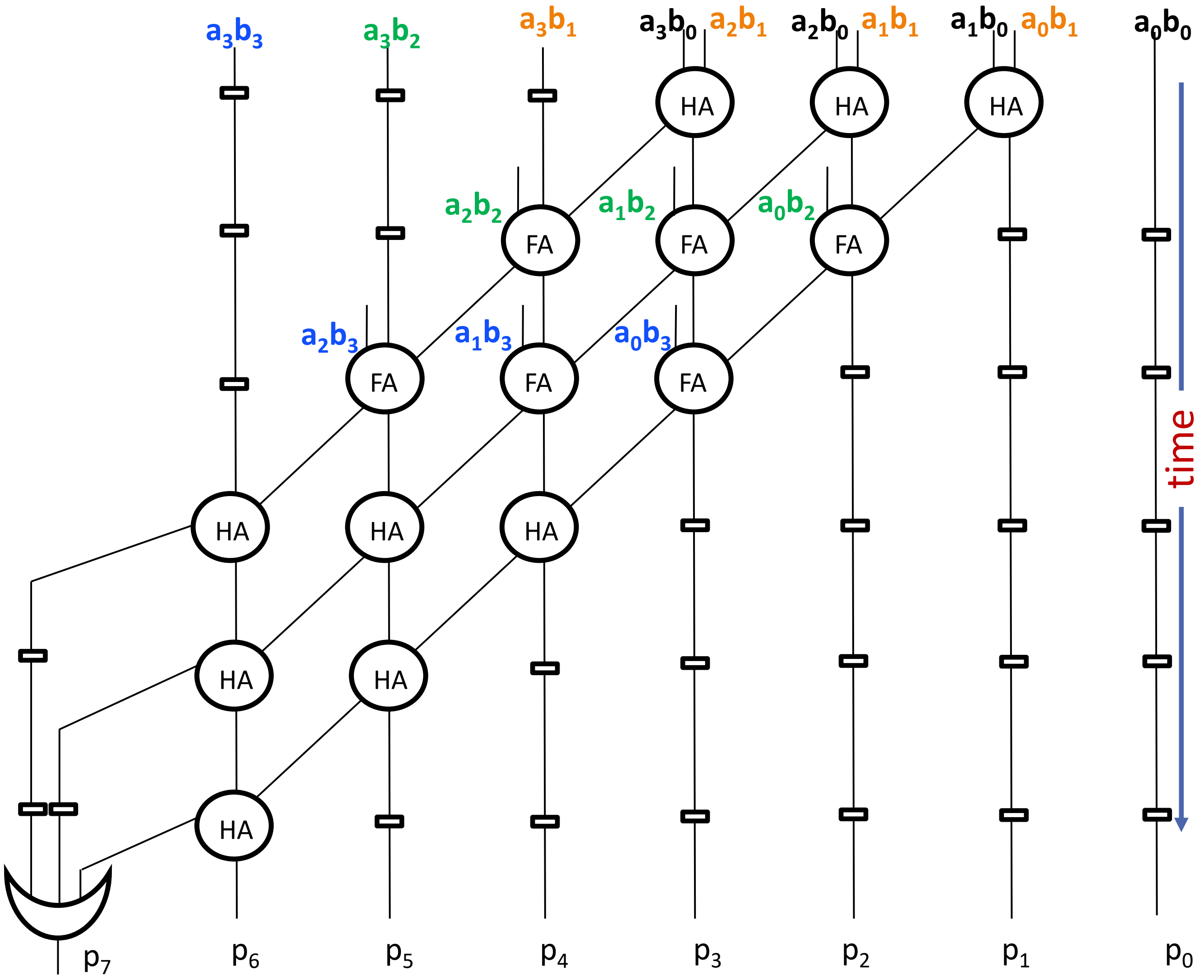
Pipelined Unsigned Multiplication

- After stage (n-1) all bit products (AND) are added
- Last (n-1) stages represent a pipelined adder
- Bit p_{2n-1} of the result is obtained as OR among the carries generated by the most left HA of each stage



Pipelined Unsigned Multiplication

- After 2(*n*-1) clock cycles, the product of the first pair of operands is obtained
- A new result is obtained at each clock cycle starting from the **(2*n*-1)-th** clock cycle

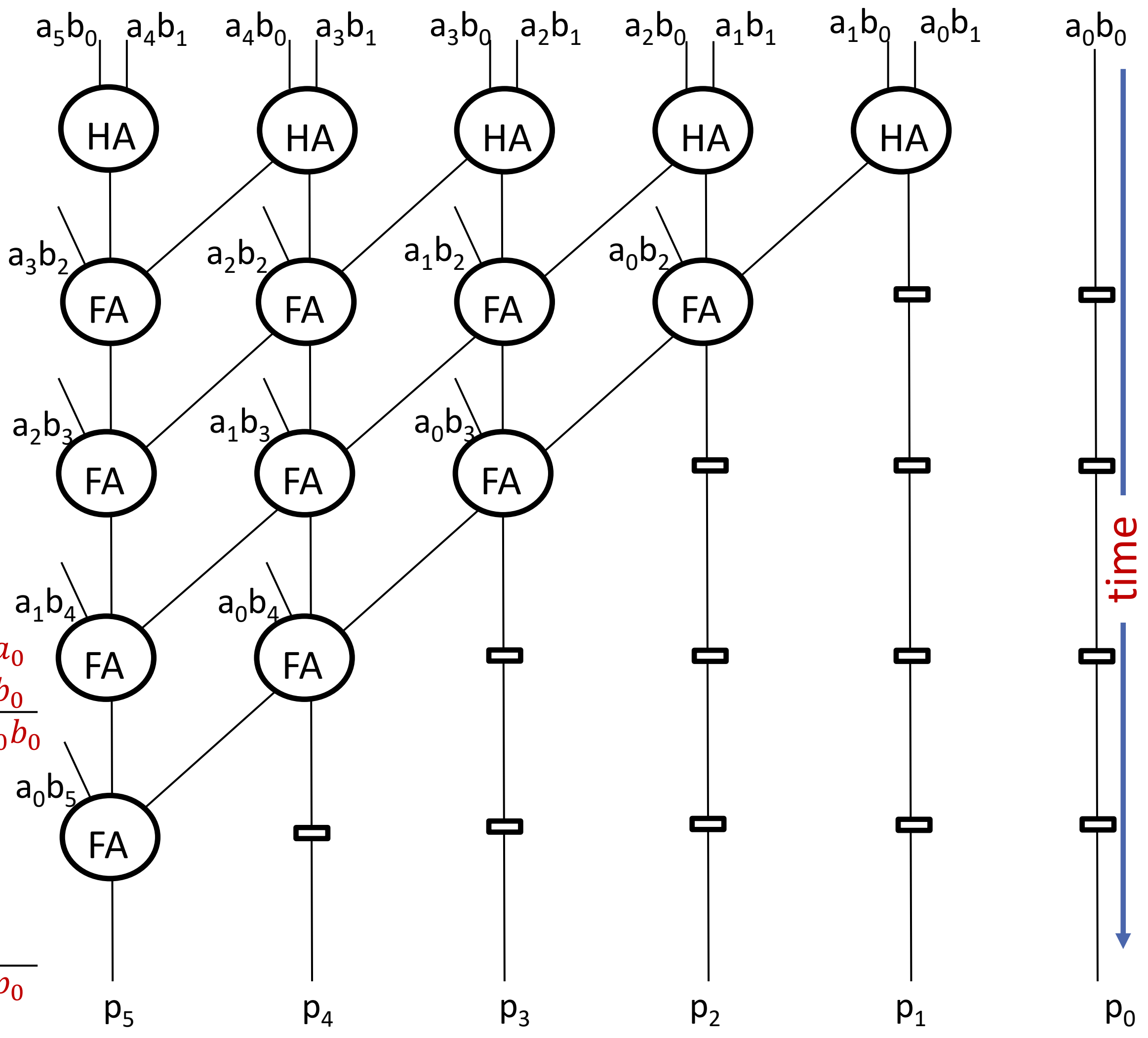


Pipelined Signed Multiplication

- **Signed numbers** are arithmetically extended to the length 2*n* of the product

- **Example with 3-bit operands**

		a_5	a_4	a_3	a_2	a_1	a_0
		b_5	b_4	b_3	b_2	b_1	b_0
		a_5b_0	a_4b_0	a_3b_0	a_2b_0	a_1b_0	a_0b_0
	a_5b_1	a_4b_1	a_3b_1	a_2b_1	a_1b_1	a_0b_1	
a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2		
a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3			
a_3b_4	a_2b_4	a_1b_4	a_0b_4				
a_2b_5	a_1b_5	a_0b_5					
		p_5	p_4	p_3	p_2	p_1	p_0

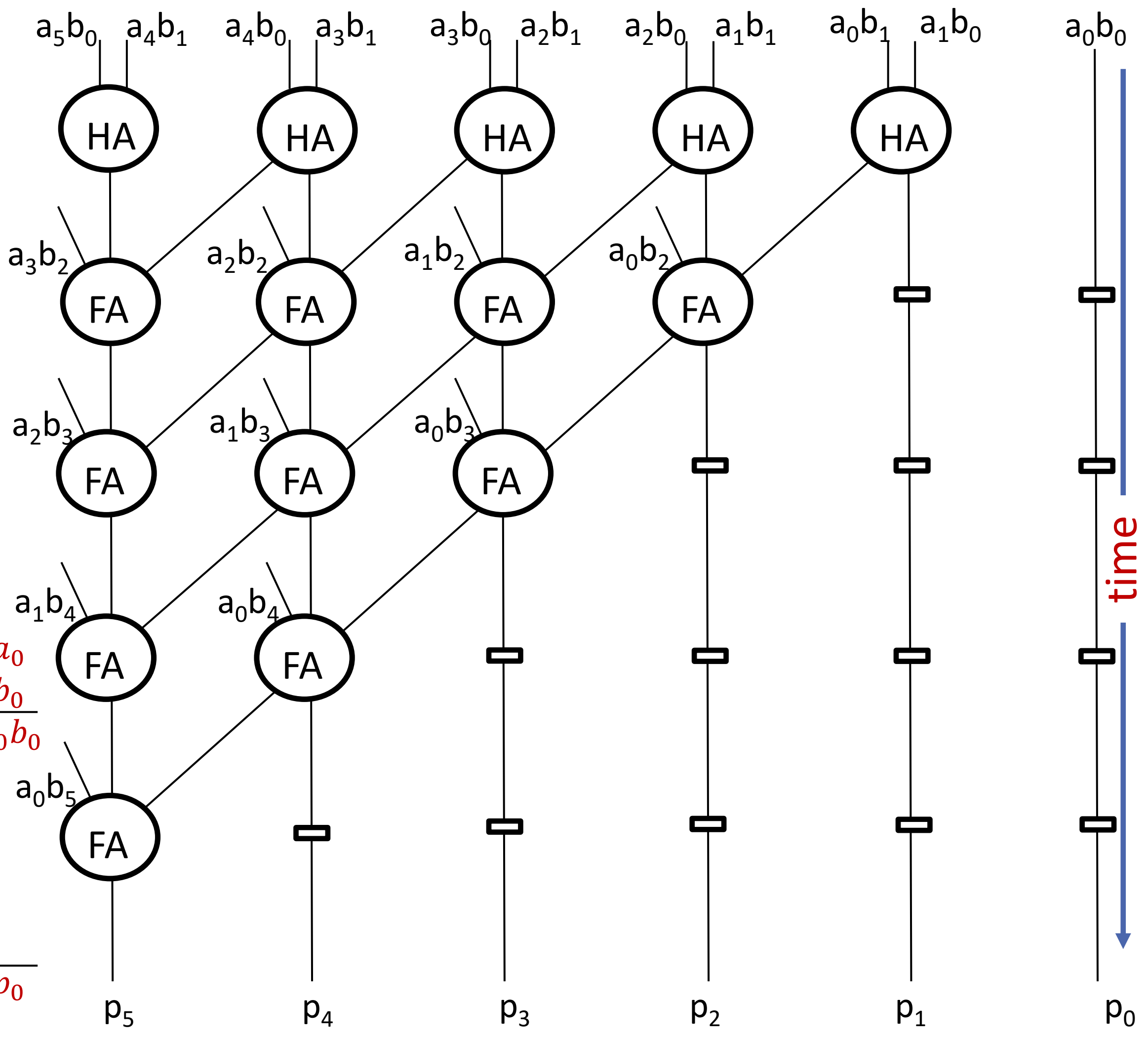


Pipelined Signed Multiplication

- Partial products of length 2*n* are considered (the remaining part is ignored)
- All stages except the first consist of FAs

- **Example with 3-bit operands**

		a_5	a_4	a_3	a_2	a_1	a_0
		b_5	b_4	b_3	b_2	b_1	b_0
		a_5b_0	a_4b_0	a_3b_0	a_2b_0	a_1b_0	a_0b_0
	a_5b_1	a_4b_1	a_3b_1	a_2b_1	a_1b_1	a_0b_1	
a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2		
a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3			
a_3b_4	a_2b_4	a_1b_4	a_0b_4				
a_2b_5	a_1b_5	a_0b_5					
		p_5	p_4	p_3	p_2	p_1	p_0



CIRCUIT AREA AND TIME EVALUATION

Circuit area and time

- To discuss about the time and area, it is useful the analytical model called **unit-gate model** presented in
  - A. Tyagi, *A reduced-area scheme for carry-select adders*, IEEE Trans. Comput., 1993

  is commonly used

- They propose a simplistic model for **gate-count** and **gate-delay**:
  - Each gate except **EX-OR** counts as **one** elementary gate
  - An **EX-OR** gate is counted as **two** elementary gates, because in static CMOS, an **EX-OR** gate is implemented as two elementary gates **(NAND)**
  - The **delay** through an elementary gate is counted as **one** gate-delay unit, but an **EX-OR** gate is **two** gate-delay units

Circuit area and time

- If the ***fan-in*** and ***fan-out*** of the gates are ignored, unfair comparisons are produced for circuits containing gates with a large difference in *fan-in* or *fan-out*
  - For example, gates in the CLA adder have different *fan-in*
  - A carry-ripple adder has no gates with *fan-in* and *fan-out* greater than 2

- The **gate-count** and **gate-delay** comparisons may not always be consistent with the area-time comparisons if the *fan-in* of gates is not taken into account

- The best comparison for a VLSI implementation is **actual** area and time

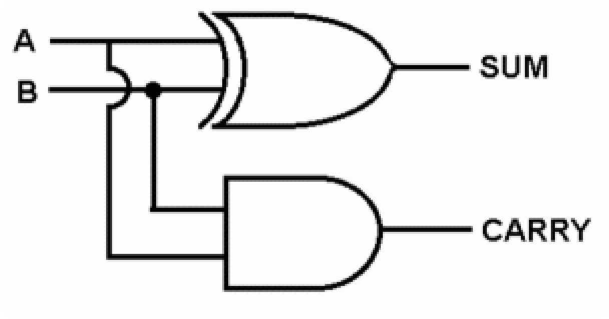
Circuit area and time

- In summary, we consider:
  - ***Any gate*** (but the EX-OR) counts as **one gate** for both area and delay $\rightarrow$ A_{gate} and T_{gate}
  - An ***exclusive-OR gate*** counts as **two elementary gates** for both area and delay $\rightarrow$ $A_{EX\text{-}OR} = 2A_{gate}$ and $T_{EX\text{-}OR} = 2T_{gate}$

- To take into account the ***fan-in*** and ***fan-out***, we consider that an ***m*-input gate** counts as:
  - $m - 1$ **gates** for area $\rightarrow$ $A_{m\text{-}gate} = (m-1)A_{gate}$
  - $\log_2 m$ **gates** for delay $\rightarrow$ $T_{m\text{-}gate} = \log_2 m \; T_{gate}$

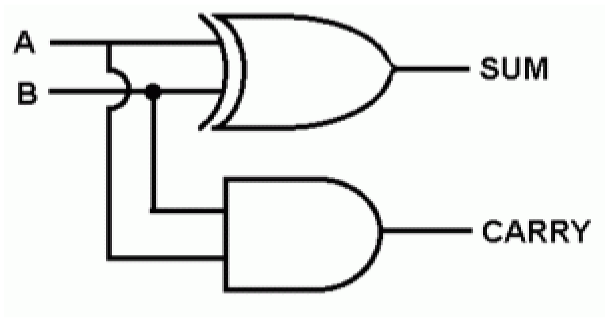
Circuit area and time

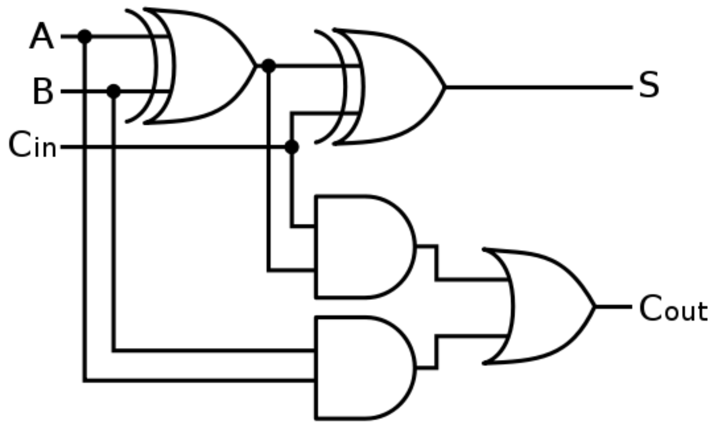
- A half adder (HA) has:
  - **Delay:** 2 unit gates – $T_{HA} = 2\ T_{gate}$
  - **Area:** 3 unit gates – $A_{HA} = 3\ A_{gate}$



Circuit area and time

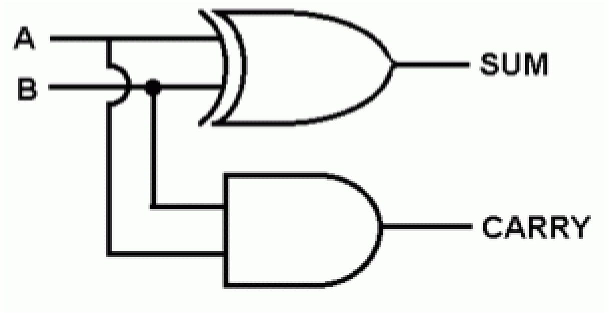
- A **half adder** (HA) has:
  - **Delay:** 2 unit gates – $T_{HA} = 2\ T_{gate}$
  - **Area:** 3 unit gates – $A_{HA} = 3\ A_{gate}$
- A **full adder** (FA) has:
  - **Delay:** 4 unit gates – $T_{FA} = 4\ T_{gate}$
  - **Area:** 7 unit gates – $A_{FA} = 7\ A_{gate}$

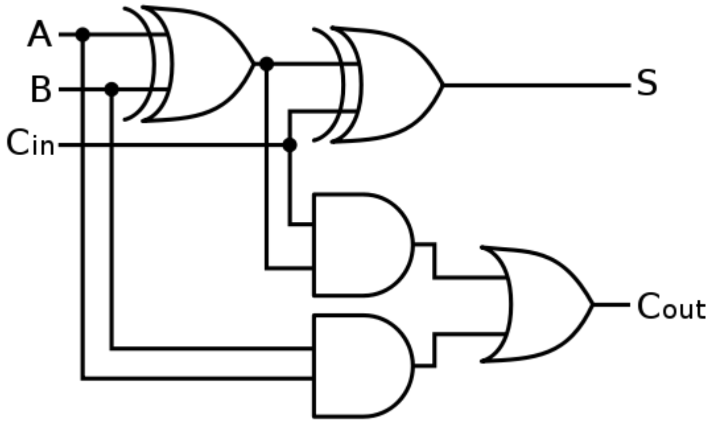




Circuit area and time

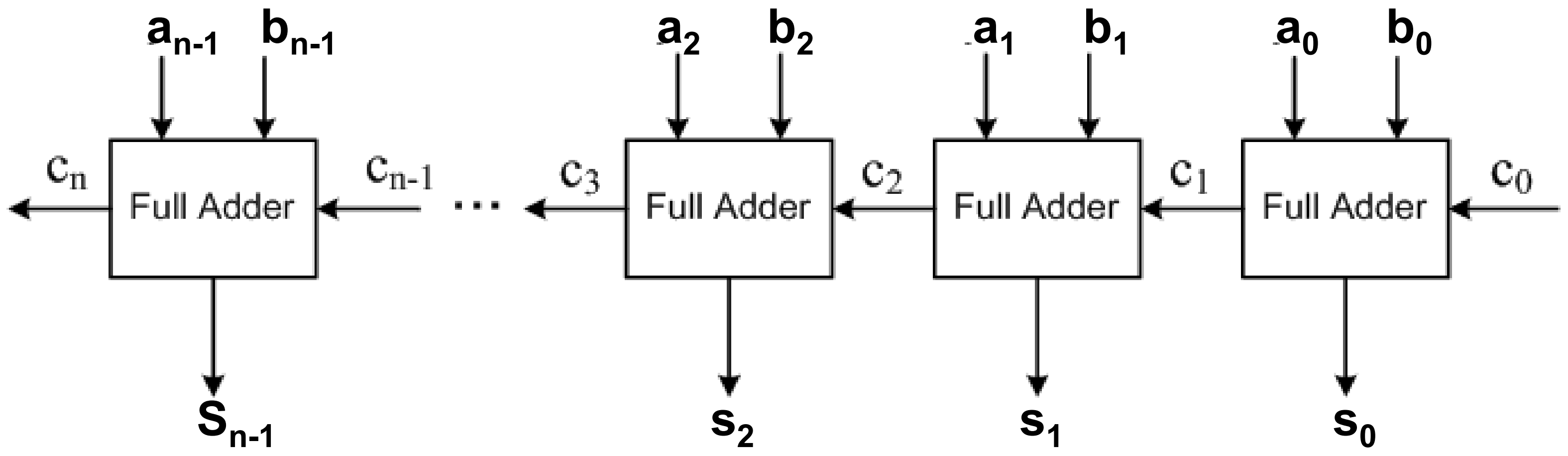
- A **half adder** (HA) has:
  - **Delay:** 2 unit gates – $T_{HA} = 2\ T_{gate}$
  - **Area:** 3 unit gates – $A_{HA} = 3\ A_{gate}$
- A **full adder** (FA) has:
  - **Delay:** 4 unit gates – $T_{FA} = 4\ T_{gate} = 2\ T_{HA}$
  - **Area:** 7 unit gates – $A_{FA} = 7\ A_{gate} = 2\ A_{HA} + A_{gate}$





Circuit area and time

- A ripple-carry adder for n-bits operands has:
  - **Delay:** $T_{RC\text{-}adder}$ → $T_{RC\text{-}adder} = n\ T_{FA} = 2n\ T_{HA} = 4n\ T_{gate}$
  - **Area:** $A_{RC\text{-}adder}$ → $A_{RC\text{-}adder} = n\ A_{FA} = 2n\ A_{HA} + n\ A_{gate} = 7n\ A_{gate}$

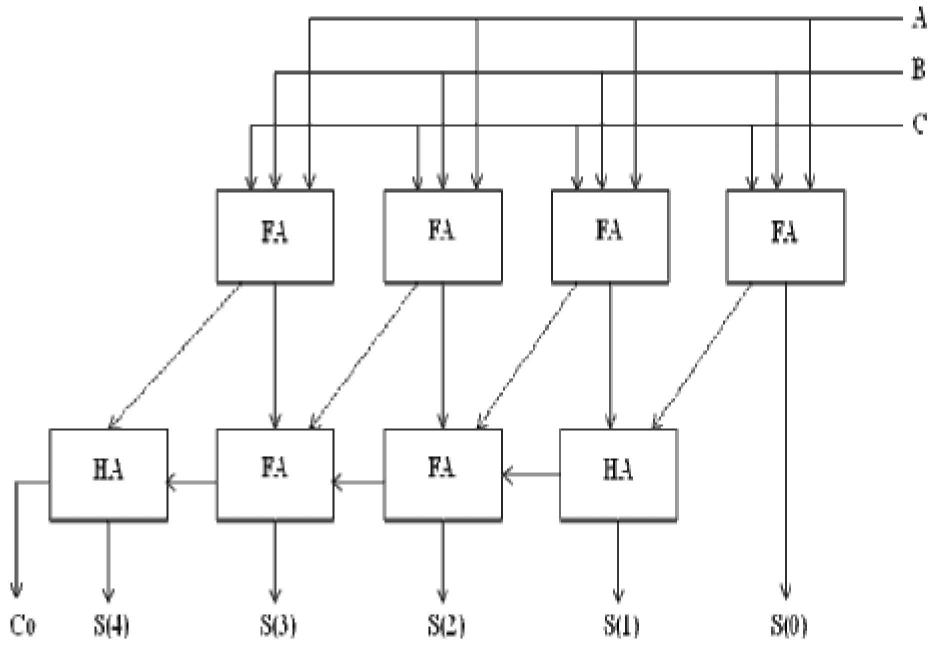


Circuit area and time

Exercise

Compute the time (propagation delay) and area required by the 4-bits Carry-Save-Adder, that is an adder for three values A, B and C, shown here below.

Compute the speedup of 4-bits Carry-Save-Adder with respect to the standard binary ripple-carry adder.



Delay and Area for the Ripple-carry adder

- $T_{RC\text{-}adder} = n\ T_{FA} = 2n\ T_{HA} = 4n\ T_{gate}$
- $A_{RC\text{-}adder} = n\ A_{FA} = 2n\ A_{HA} + n\ A_{gate} = 7n\ A_{gate}$