

GPU - Graphics Processing Units

Intensive Computation

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Lecture 19

Programming Massively Parallel Processors

D.B. Kirk W. W. Hwu

- Chapter 3 - Introduction to Data Parallelism and CUDA C
 - **Sections 3.2 - 3.6**
- Chapter 4 - Data Parallel Execution Model
 - **Sections 4.5 - 4.7**
- Chapter 5 - CUDA Memories
 - **Sections 5.2 - 5.4**

Multicore and GPU Programming

G. Barlas

- Chapter 6 - GPU Programming
 - **Sections 6.2 - 6.7**

Computer Architecture - A Quantitative Approach, Fifth Edition

Hennessy Patterson

- Chapter 4 - Data-Level Parallelism in Vector, SIMD, and GPU Architectures
 - **Section 4.4 – Graphics Processing Units**

Graphics Processing Units

- GPUs and CPUs do **not** go back in computer architecture genealogy to a **common ancestor**
- The primary ancestors of GPUs are graphics accelerators
- Given the hardware invested to do graphics well architects ask:
how can be the design of GPUs used to improve the performance of a wider range of applications?

Graphics Processing Units

- The **challenge** for the GPU programmer
 - is **not** simply **getting good performance** on the GPU
 - but also in **coordinating the scheduling of computation** on the system processor and the GPU and the **transfer of data** between system memory and GPU memory
- GPUs have virtually **every type of parallelism** that can be captured by the programming environment:
 - **multithreading**
 - **MIMD**
 - **SIMD**
 - **instruction-level**

Programming the GPU

- NVIDIA developed a *C-like language and programming environment*: **CUDA** - **Compute Unified Device Architecture**
- CUDA produces C/C++ for the system processor - *host* - and a C and C++ dialect for the GPU - *device* (*D in CUDA*)
- **OpenCL** is a similar language, which several companies are developing as an independent multiple platforms language

Programming the GPU

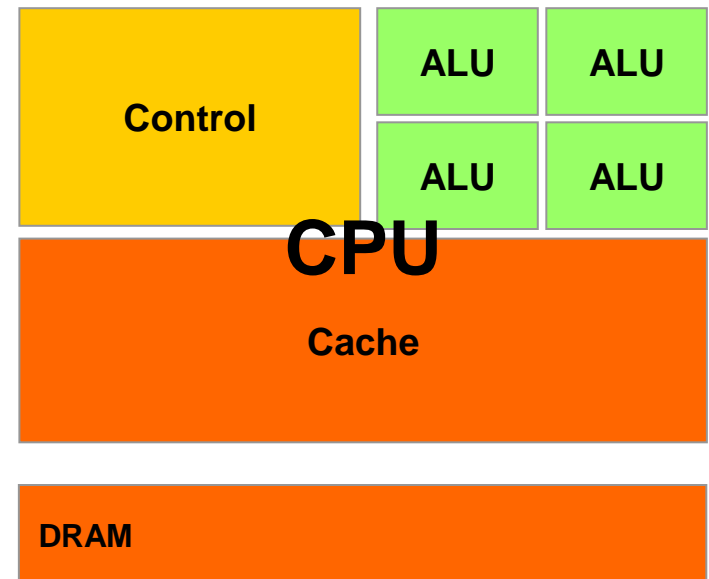
- NVIDIA unified all forms of parallelism in the **CUDA Thread**
- The compiler and the hardware can gang thousands of CUDA threads together to utilize the various styles of parallelism within a GPU (multithreading, MIMD, SIMD, ILP)
- NVIDIA classifies the CUDA programming model as **Single Instruction, Multiple Thread (SIMT)**
- Threads are blocked together - **Thread Block** - and executed in **groups of 32 threads - warp**

CUDA Programming Model

- The GPU is viewed as a compute **device** with the following features:
 - It is a coprocessor to the CPU (**host**)
 - It has its own DRAM (**device memory**)
 - It runs many **threads in parallel**
- Data-parallel portions of an application are executed on the device as **kernels** which run in parallel on many threads
- Differences between GPU and CPU threads
 - **GPU threads are extremely lightweight**
 - Very little creation overhead
 - **GPU needs 1000s of threads for full efficiency**
 - Multi-core CPU needs only a few

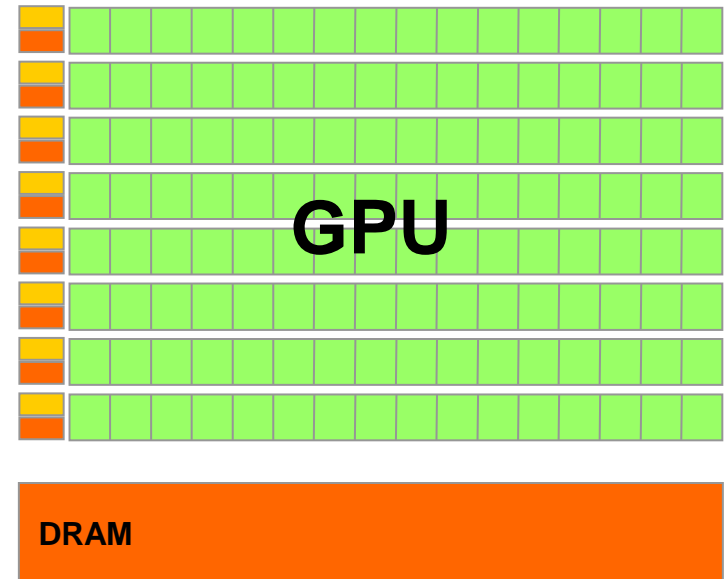
CPUs: Latency Oriented Design

- Large caches
 - Convert long latency memory accesses to short latency cache accesses
- Sophisticated control
 - Branch prediction for reduced branch latency
 - Data forwarding for reduced data latency
- Powerful ALU
 - Reduced operation latency



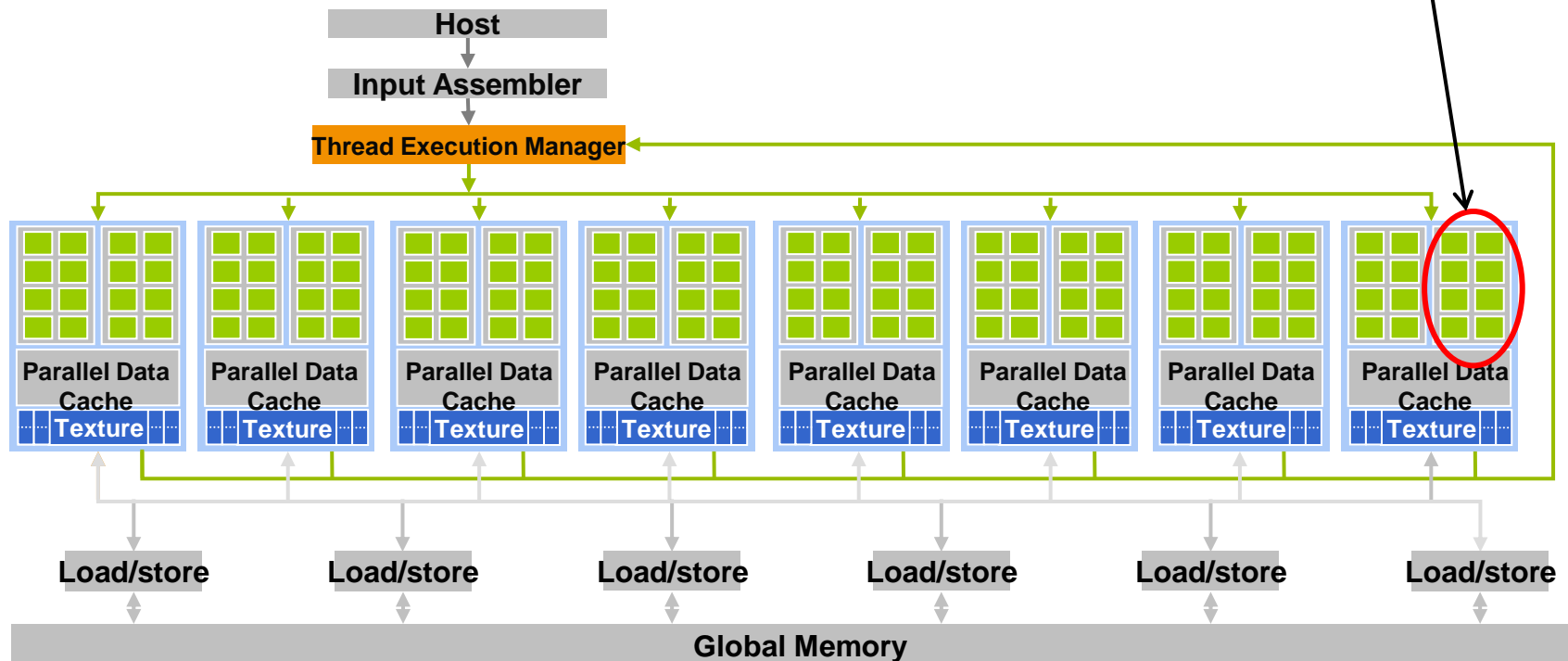
GPUs: Throughput Oriented Design

- Small caches
 - To boost memory throughput
- Simple control
 - No branch prediction
 - No data forwarding
- Energy efficient ALUs
 - Many, long latency but heavily pipelined for high throughput
- Require massive number of threads to tolerate latencies



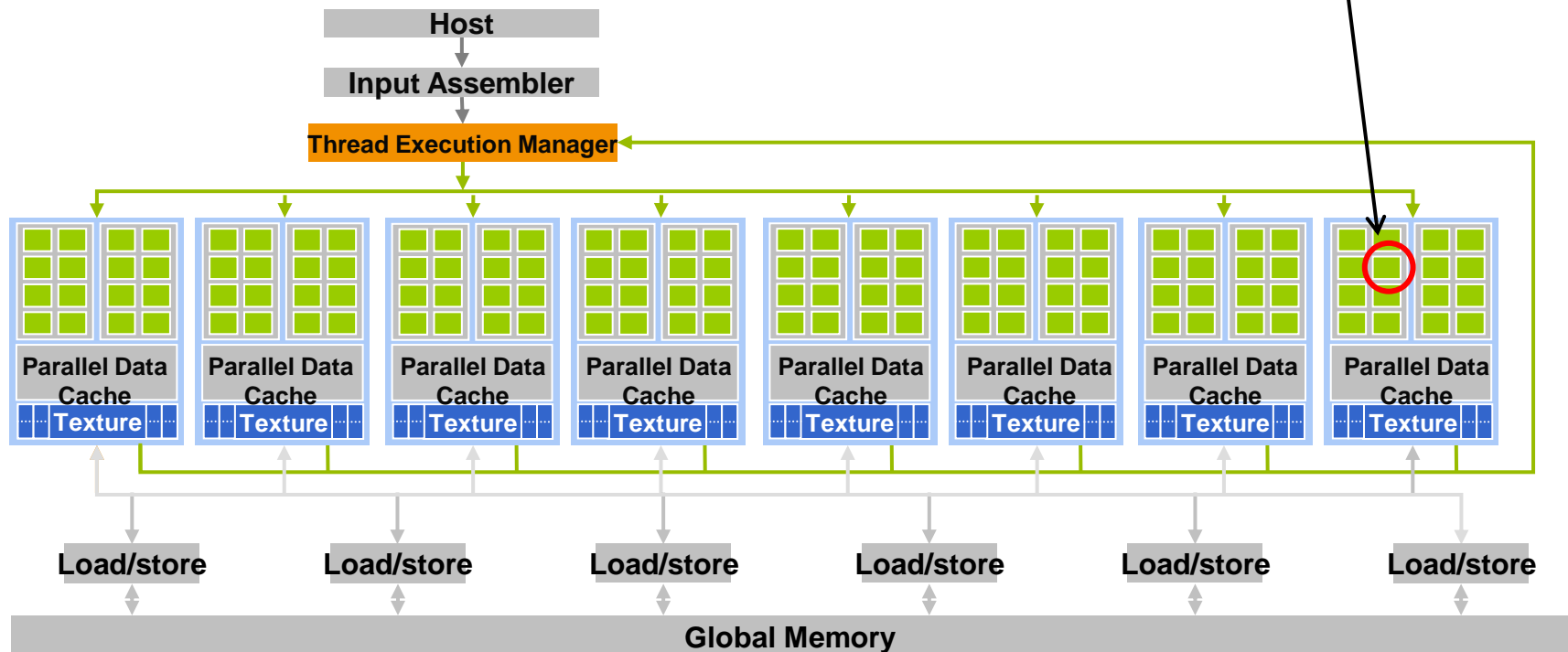
GPU Architecture

- A typical CUDA-capable GPU can be organized into
 - an array of highly threaded **streaming multiprocessors (SMs)**
 - two SMs form a building block
 - the number of SMs in a building block can vary from one generation of CUDA GPUs to another



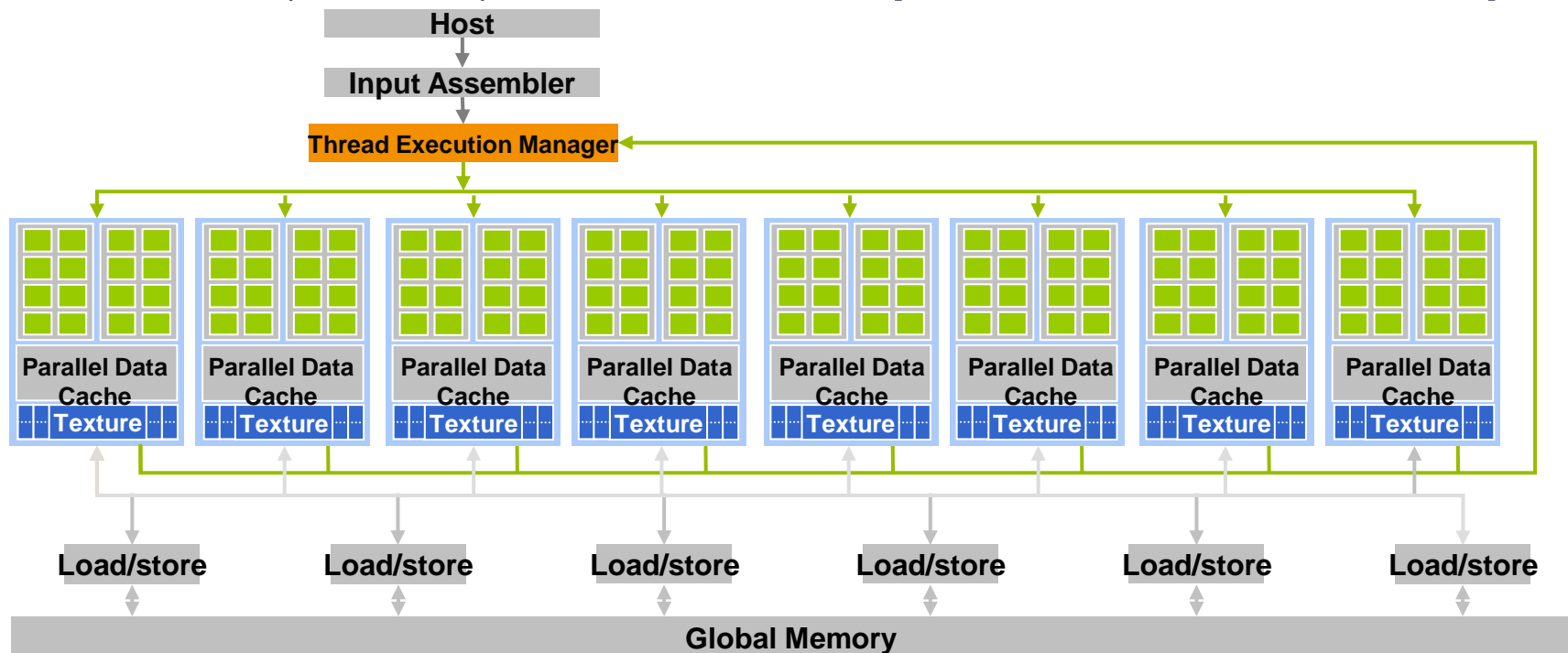
GPU Architecture

- Each SM has a number of **streaming processors (SPs)** that share control logic and instruction cache
- Each GPU currently comes with up to 4 gigabytes of graphics double data rate (GDDR) DRAM - **global memory**



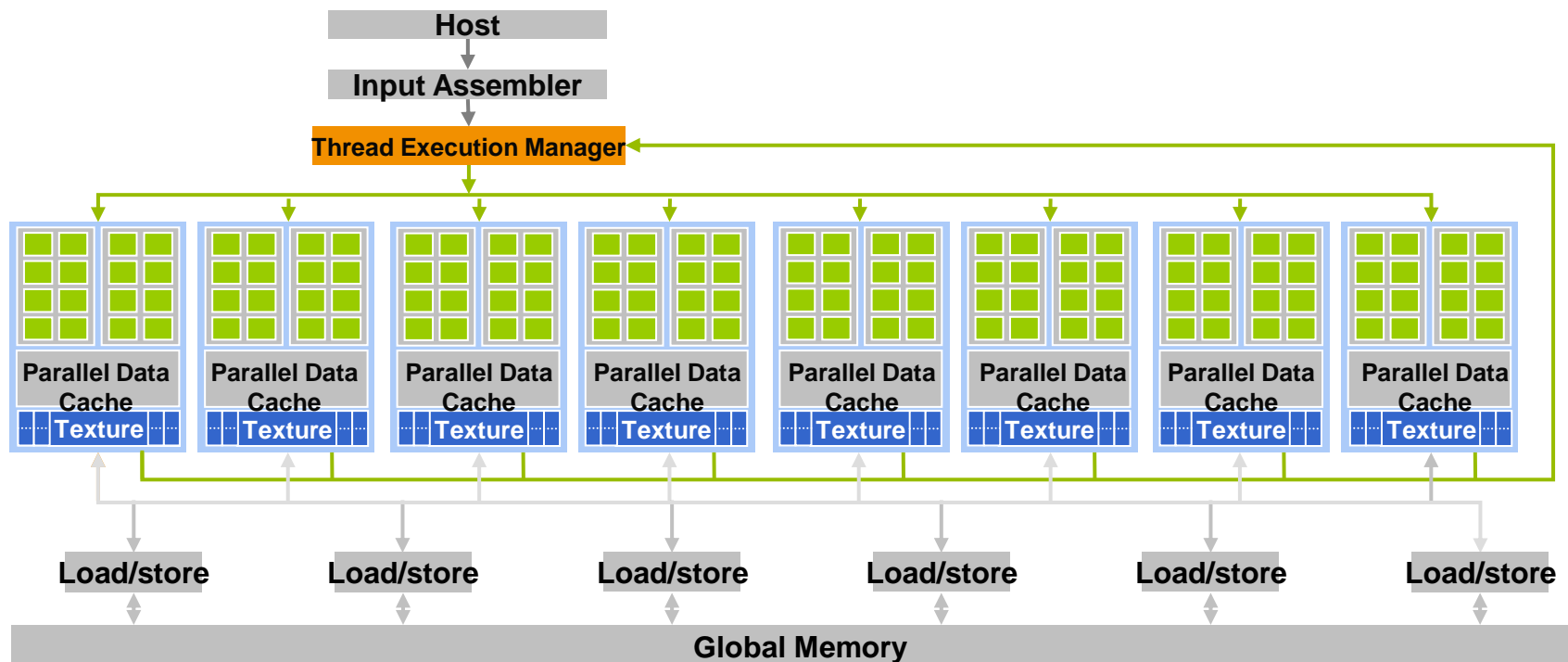
GPU Architecture

- The parallel **G80** chip has 128 SPs (16 SMs with 8 SPs)
- Each SP has a **multiply–add (MAD)** unit and an additional multiply unit
 - The **G80** (128 SPs) produces a total of **over 500 gigaflops**
 - The **GT200** (240 SPs) exceeds **1 teraflops**, the **GTX680** 1,5 teraflops



GPU Architecture

- The **G80** chip supports up to **768 threads** per SM, which sums up to about 12,000 threads for this chip
- The **GT200** supports **1024 threads** per SM and up to about 30,000 threads



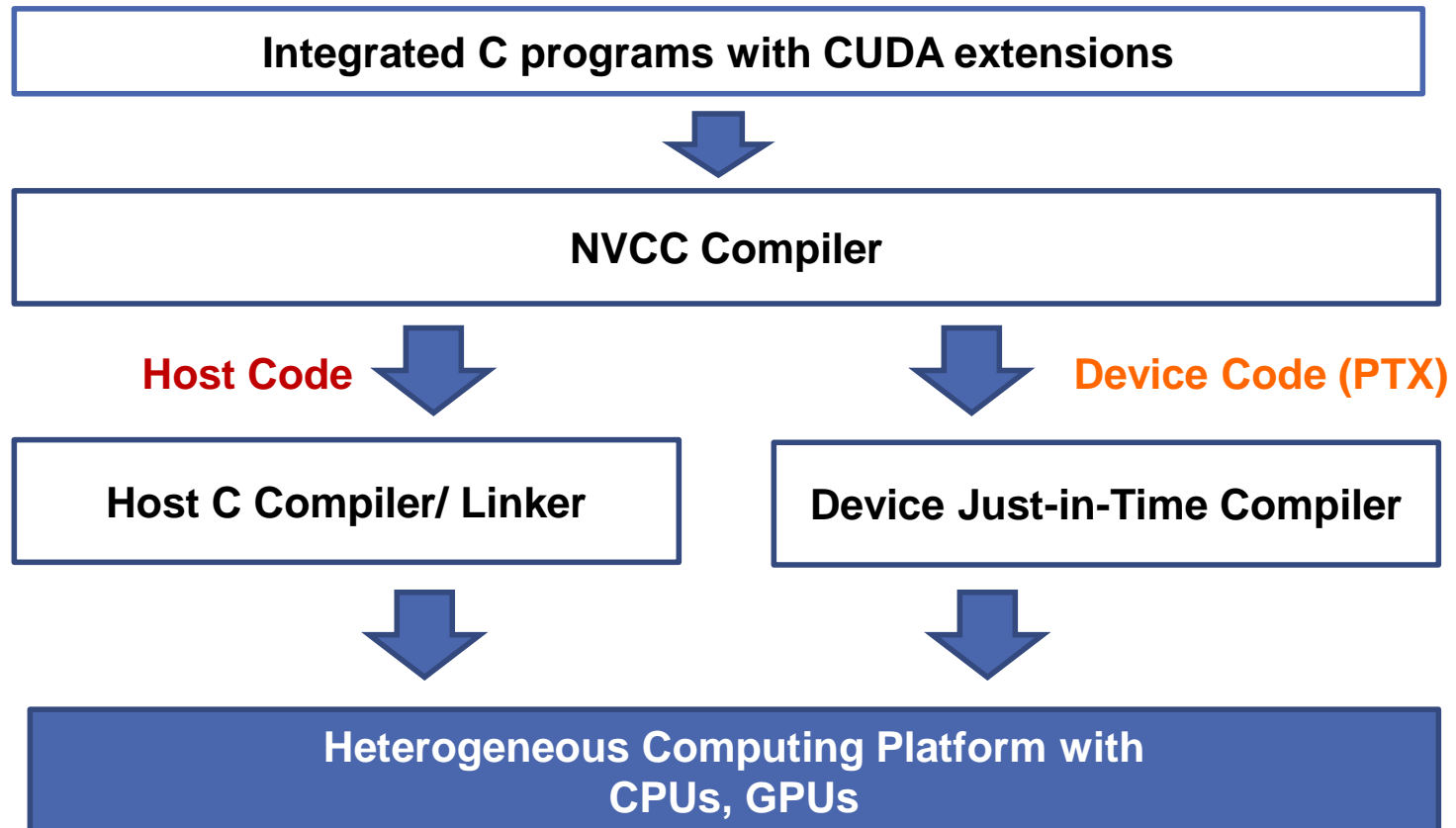
CUDA Program Structure

- The structure of a CUDA program reflects the computing system consisting of
 - a **host**, which is a traditional central processing unit (CPU)
 - one or more **devices** (GPUs)
- A **CUDA program** is a unified source code encompassing **both host and device code**
- The NVIDIA C compiler - **nvcc** - separates the two during the compilation process

CUDA Program Structure

- The **host code** is:
 - straight ANSI C code
 - it is further compiled with the host's standard C compilers and runs as an ordinary CPU process
- The **device code** is:
 - written using ANSI C extended with **keywords** for labeling data-parallel functions, called **kernels**, and their associated **data structures**
 - The device code is typically further compiled by the **nvcc** and executed on the GPU device

Compiling A CUDA Program



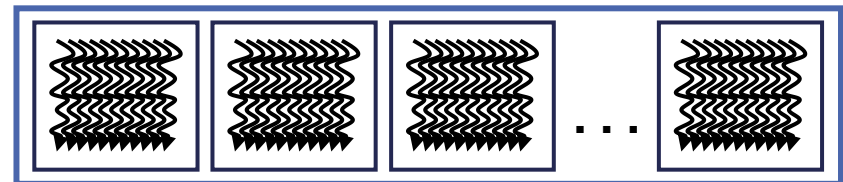
CUDA Execution Model

- The **execution starts with host (CPU) execution**
- When a **kernel function** is launched, the execution is moved to a device (GPU), where a large number of threads are generated to take advantage of abundant data parallelism

Serial Code (host)

Parallel Kernel (device)

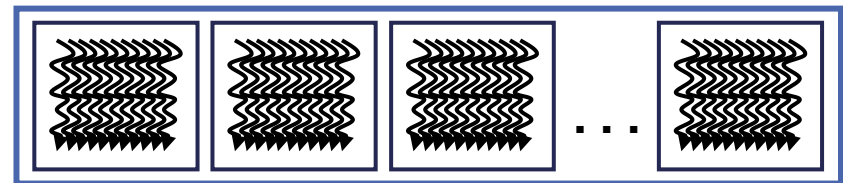
```
KernelA<<< nBlk, nTid >>>(args);
```



Serial Code (host)

Parallel Kernel (device)

```
KernelB<<< nBlk, nTid >>>(args);
```



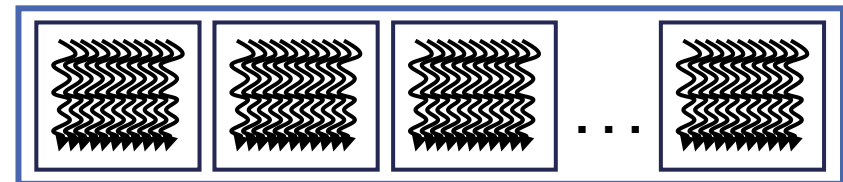
CUDA Execution Model

- All the threads that are generated by a kernel during an invocation are collectively called a **grid**
- Figure shows the execution of **two grids of threads**

Serial Code (host)

Parallel Kernel (device)

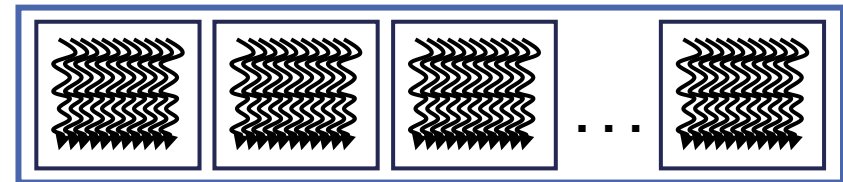
```
KernelA<<< nBlk, nTid >>>(args);
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Serial Code (host)

Parallel Kernel (device)

```
KernelB<<< nBlk, nTid >>>(args);
```



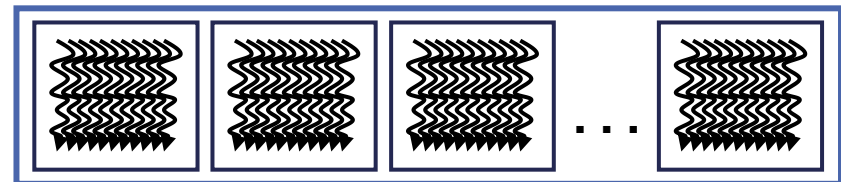
CUDA Execution Model

- When all threads of a **kernel** complete their execution:
 - the corresponding **grid terminates**
 - the **execution continues on the host** until another kernel is invoked

Serial Code (host)

Parallel Kernel (device)

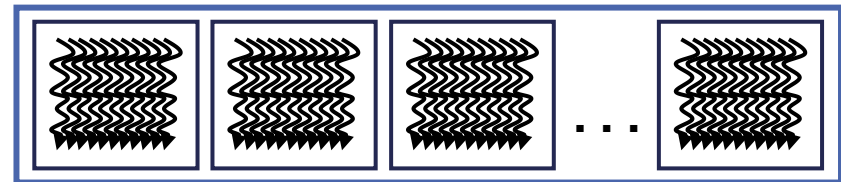
```
KernelA<<< nBlk, nTid >>>(args);
```



Serial Code (host)

Parallel Kernel (device)

```
KernelB<<< nBlk, nTid >>>(args);
```



Programming the GPU

- To distinguish between **functions** for the GPU (device) and functions for the system processor (host), CUDA uses:
 - `__device__` or `__global__` for the device
 - `__host__` for the processor
- CUDA **variables** declared as in the
 - `__device__` (or `__global__`) functionsare allocated to the GPU Memory which is accessible by all multithreaded SIMD processors

Programming the GPU

- The **call syntax** for the function *name* that runs on the GPU

```
name<<<dimGrid, dimBlock>>>( ... parameter list ... )
```

where **dimGrid** and **dimBlock** specify the dimensions of the grid (in blocks) and the dimensions of a block (in threads)

- CUDA provides keywords for:
 - the identifier for blocks per grid - **blockIdx** -
 - the identifier for threads per block - **threadIdx** -
 - the number of threads per block - **blockDim** - which comes from the **dimBlock** parameter

Example

- Consider the DAXPY example

```
// Invoke DAXPY
daxpy(n, 2.0, x, y);
// DAXPY in C
void daxpy(int n, double a, double *x, double *y)
{
    for (int i = 0; i < n; ++i)
        y[i] = a*x[i] + y[i];
}
```

Example

- In the CUDA version, we launch:
 - **n threads**, one per vector element
 - with **256 CUDA Threads per thread block**

```
// Invoke DAXPY with 256 threads per Thread Block
__host__
int nblocks = (n + 255) / 256;
    daxpy<<<nblocks, 256>>>(n, 2.0, x, y);
// DAXPY in CUDA
__device__
void daxpy(int n, double a, double *x, double *y)
{
    int i = blockIdx.x*blockDim.x + threadIdx.x;
    if (i < n) y[i] = a*x[i] + y[i];
}
```

Example

- The GPU function calculates the corresponding element **index i** based on the **block ID**, the **number of threads per block**, and the **thread ID**
- If this index is within the array ($i < n$), it performs the multiply and add

```
// Invoke DAXPY with 256 threads per Thread Block
__host__
int nblocks = (n+ 255) / 256;
    daxpy<<<nblocks, 256>>>(n, 2.0, x, y);
// DAXPY in CUDA
__device__
void daxpy(int n, double a, double *x, double *y)
{
    int i = blockIdx.x*blockDim.x + threadIdx.x;
    if (i < n) y[i] = a*x[i] + y[i];
}
```


Threads and Blocks

- The C version has:
 - A loop where each iteration is independent of the others
 - This allows the loop to be transformed into a parallel code
 - **Each loop iteration** becomes an **independent thread**
- The programmer determines the parallelism in CUDA explicitly by specifying
 - the **grid dimensions**
 - the **number of threads per Streaming Processor**
- By assigning a single thread to each element, there is **no need to synchronize** among threads when writing results to memory

Threads and Blocks

- A **thread** is associated with **each data element**
 - *CUDA threads*, with thousands of which for various styles of parallelism
- Threads are organized into **blocks**
 - **Thread Blocks**: groups of up to 512 elements
 - **Streaming Processor**: hardware that executes a whole thread block (32 elements executed per thread at a time)
- Blocks are organized into a **grid**
 - **Blocks are executed independently** and in any order
 - Different **blocks cannot communicate directly** but can **coordinate** using memory operations in GPU Global Memory

Vector Addition – Traditional C Code

```
// Compute vector sum C = A+B
void vecAdd(float* A, float* B, float* C, int n)
{
    for (i = 0, i < n, i++)
        C[i] = A[i] + B[i];
}

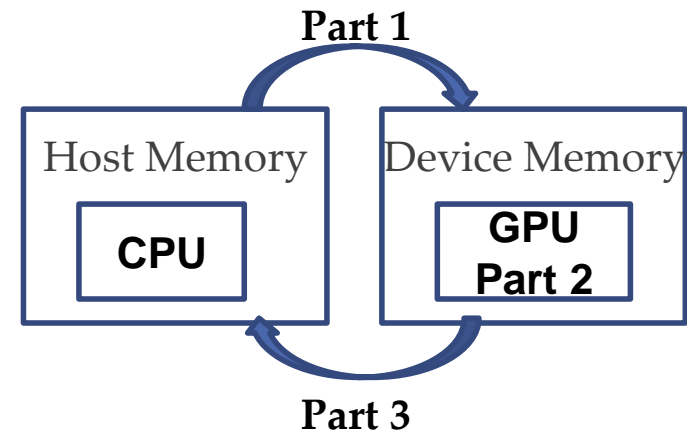
int main()
{
    // Memory allocation for A_h, B_h, and C_h
    // I/O to read A_h and B_h, N elements
    ...
    vecAdd(A_h, B_h, C_h, N);
}
```

Vector Addition – Kernel

```
void vecAdd(float* h_A, float* h_B, float* h_C, int n)
{
    int size = n* sizeof(float);
    float* d_A, d_B, d_C;
    ...
1. // Allocate device memory for A, B, and C
   // Copy A and B to device memory

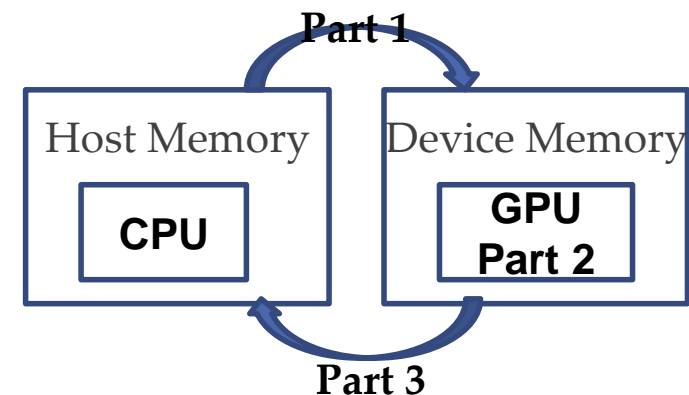
2. // Kernel launch code - to have the device
   // to perform the actual vector addition

3. // Copy C from the device memory
   // Free device vectors
}
```



Device Memory and Data Transfer

- The **host** and **devices** have **separate memory** spaces
- To execute a kernel on a device
 - the programmer needs to **allocate memory on the device**
 - **transfer data** from the host memory to the allocated device memory
 - this corresponds to *Part 1 of Figure*
- After device execution
 - the programmer needs to **transfer result data** from the device memory back to the host memory
 - **free up** the device memory
 - this corresponds to *Part 3 of Figure*

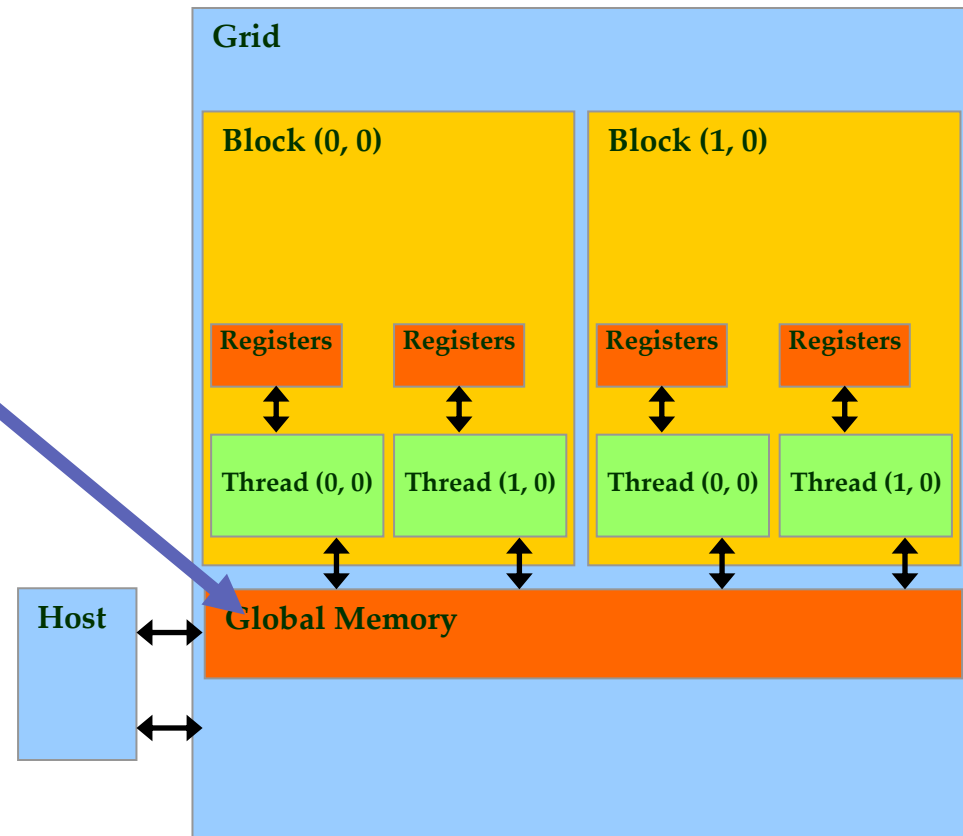


Device Memory and Data Transfer

- The CUDA memory model is supported by API functions that help programmers to manage data in memories
- The function `cudaMalloc()` :
 - called **from the host code** to allocate object in the device global memory
 - Two parameters:
 - **address of a pointer** variable to the allocated object after allocation
 - **size** of the allocated object in terms of bytes
- The function `cudaFree()` :
 - Frees object from device global memory
 - Pointer to freed object
- The function `cudaMemcpy()` for memory data transfer

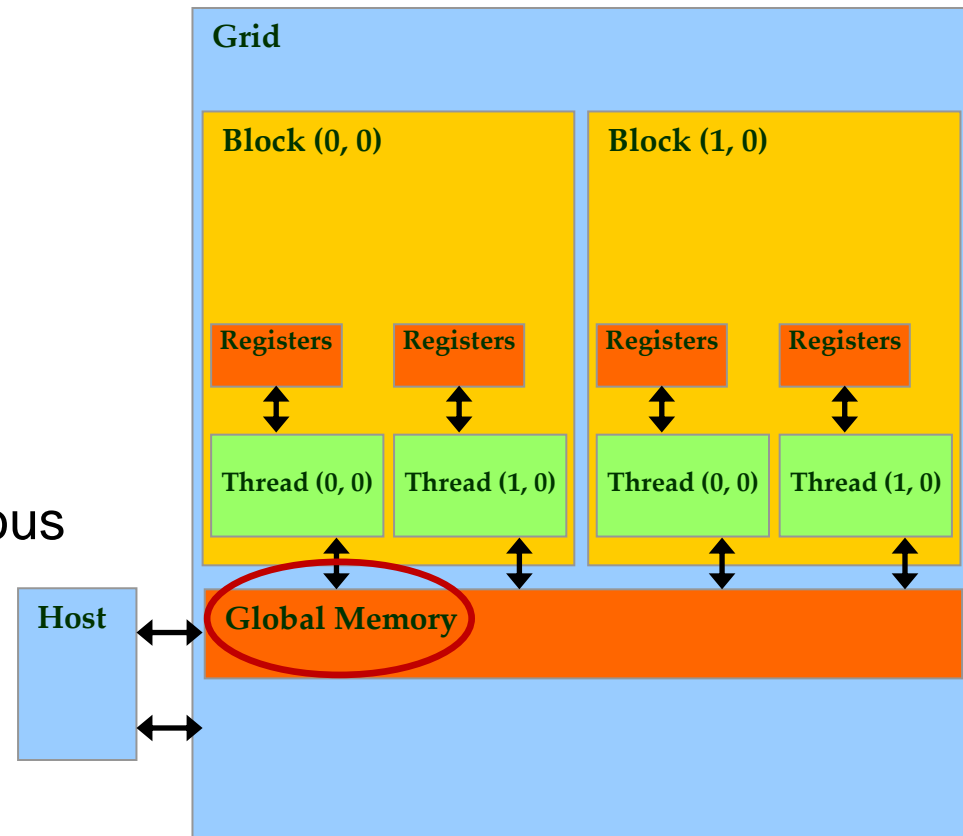
CUDA Device Memory Management API

- `cudaMalloc()`
 - Allocates object in the device **global memory**
 - Two parameters
 - **Address of a pointer** to the allocated object
 - **Size** of allocated object in terms of bytes
- `cudaFree()`
 - Frees object from device global memory
 - **Pointer** to freed object



Host-Device Data Transfer API functions

- `cudaMemcpy()`
 - memory data transfer
 - requires four parameters
 - **Pointer** to destination
 - **Pointer** to source
 - Number of bytes copied
 - Type/Direction of transfer
- Transfer to device is asynchronous

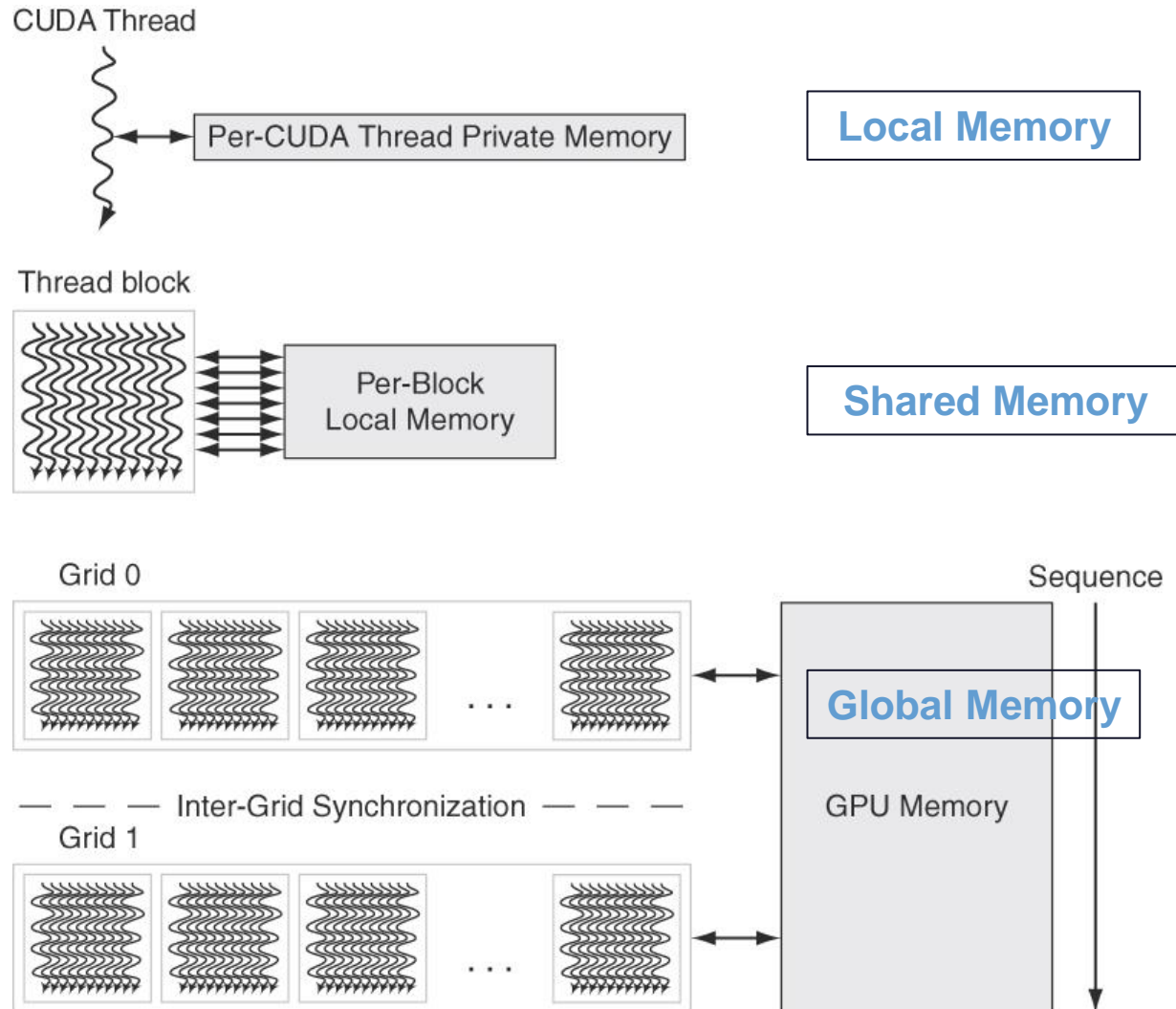


NVIDIA GPU Memory Structures

Local Memory is private to a single CUDA Thread

Shared Memory is shared by all threads of SIMD instructions within a thread block

GPU Global Memory is shared by all Grids



Vector Addition

```
void vecAdd(float* h_A, float* h_B, float* h_C, int n)
{
    int size = n * sizeof(float);
    float* d_A, d_B, d_C;

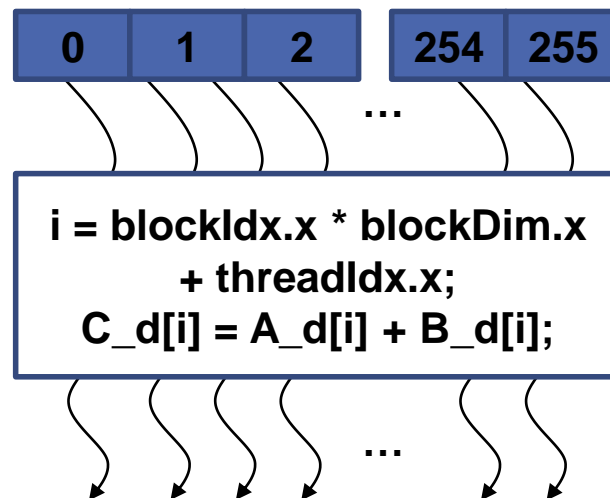
1. // Transfer A and B to device memory
    cudaMalloc((void **) &d_A, size);
    cudaMemcpy(d_A, h_A, size, cudaMemcpyHostToDevice);
    cudaMalloc((void **) &d_B, size);
    cudaMemcpy(d_B, h_B, size, cudaMemcpyHostToDevice);

    // Allocate device memory for C
    cudaMalloc((void **) &d_C, size);

2. // Kernel invocation code - to be shown later
    ...
3. // Transfer C from device to host
    cudaMemcpy(h_C, d_C, size, cudaMemcpyDeviceToHost);
    // Free device memory for A, B, C
    cudaFree(d_A); cudaFree(d_B); cudaFree(d_C);
}
```

Arrays of Parallel Threads

- A **kernel** function specifies the code to be executed by all threads during a parallel phase
 - All of these threads execute the same code
- A CUDA kernel is executed by a grid (array) of threads
 - All threads in a grid run the same kernel code
 - Each thread has an index that it uses to compute memory addresses and make control decisions



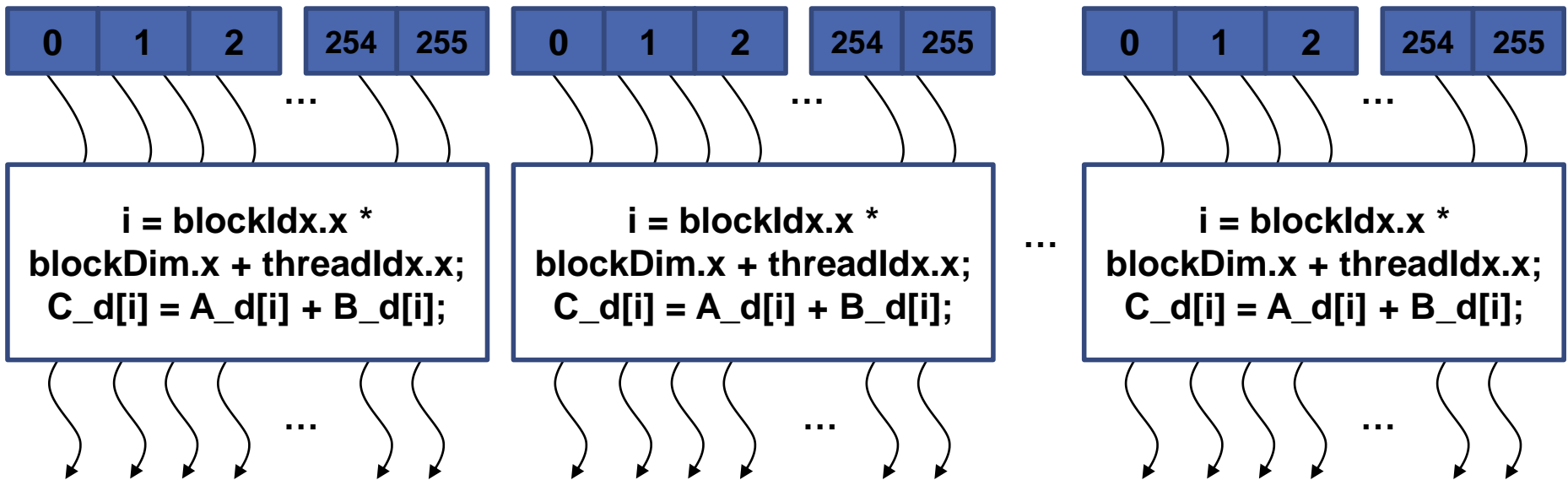
Thread Blocks: Scalable Cooperation

- Thread array is divided into multiple blocks
 - Threads within a block cooperate via **shared memory**, **atomic operations** and **barrier synchronization**
 - Threads in different blocks cannot cooperate

Thread Block 0

Thread Block 1

Thread Block N-1

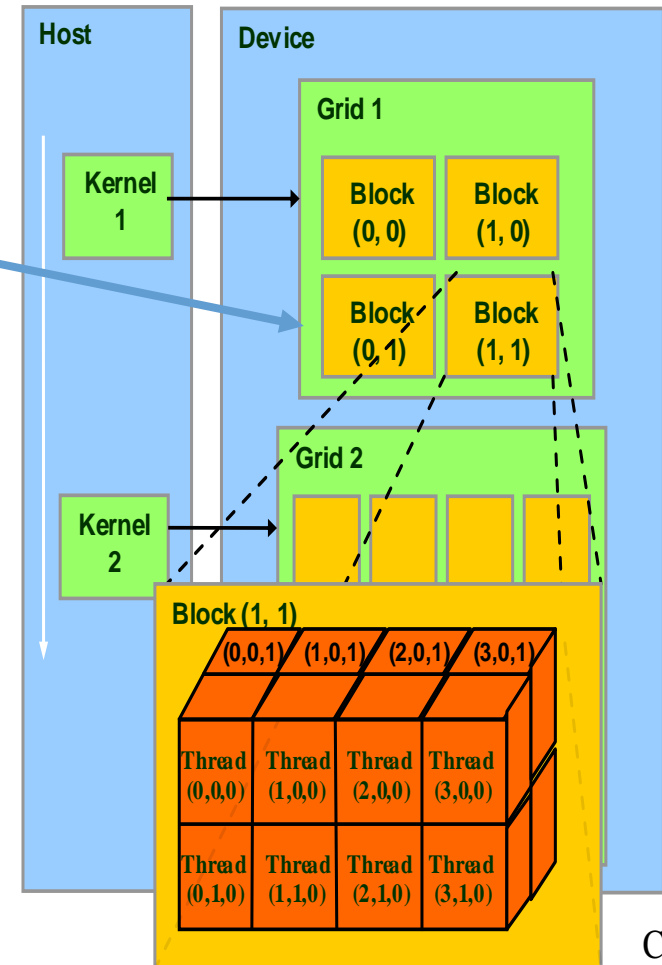


Arrays of Parallel Threads

- When a **kernel** is invoked, it is executed as **grid** of parallel threads
- Each CUDA thread grid typically is comprised of **thousands to millions** of lightweight GPU threads per kernel invocation
- Creating enough threads to fully utilize the hardware often requires a large amount of data parallelism

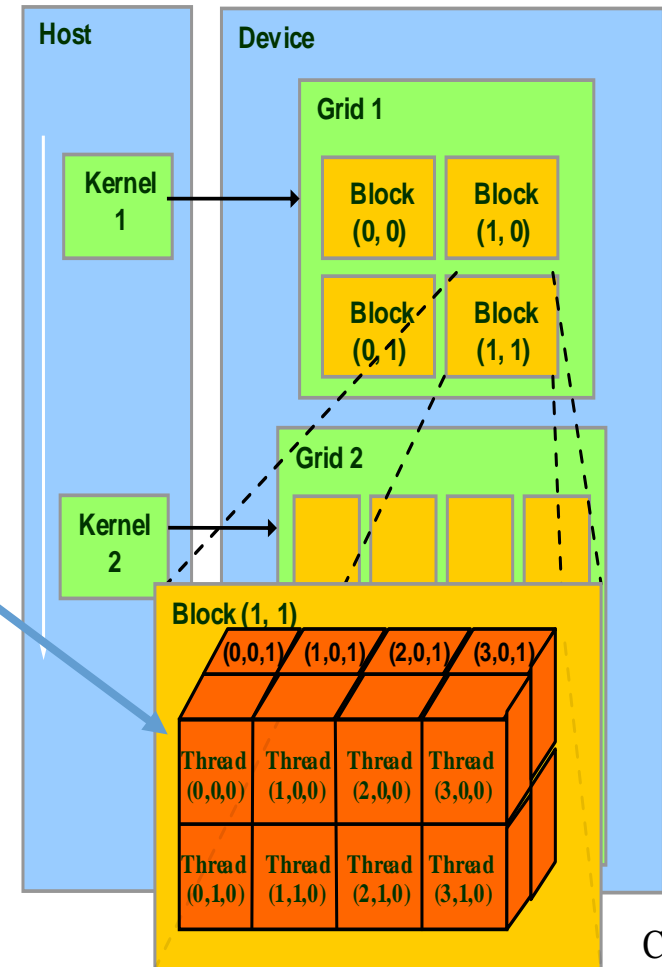
blockIdx and threadIdx

- Threads in a grid are organized into a **two-level hierarchy**
 - top level**, each **grid** consists of one or more **thread blocks**
 - All **blocks** in a grid have the **same number of threads** organized in the same manner
 - Each **grid** is organized as a **three-dimensional array** of blocks
 - Each **block** has a unique **three dimensional coordinate** given by the CUDA specific keywords **blockIdx.x**, **blockIdx.y** and **blockIdx.z**



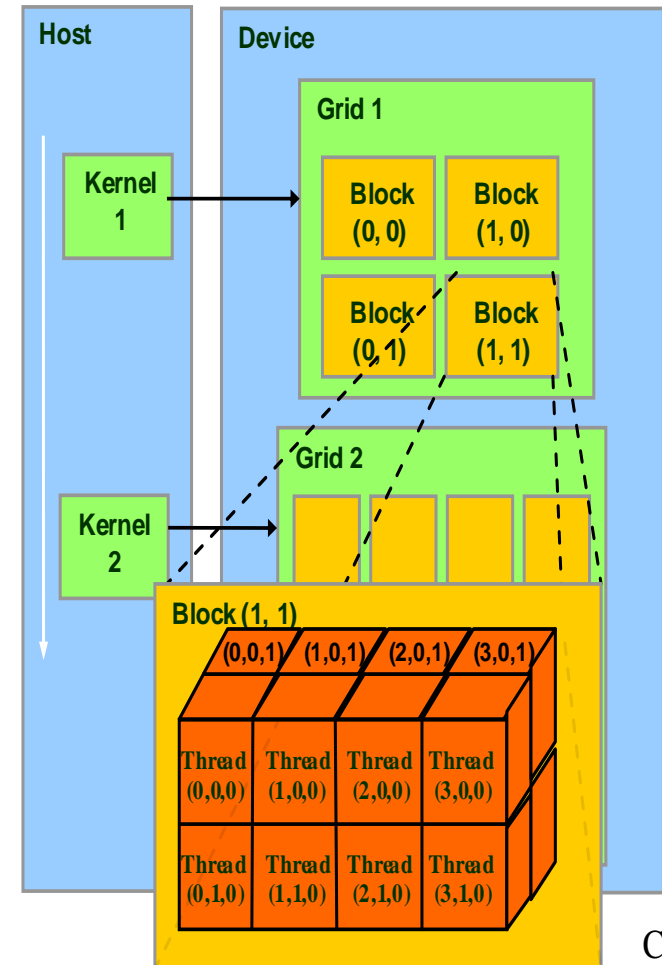
blockIdx and threadIdx

- Threads in a grid are organized into **a two-level hierarchy**
 - Each thread **block** is organized as a **three-dimensional array** of threads with a total size of up to 512 threads
 - The coordinates of threads in a block are uniquely defined by three thread indices: **threadIdx.x**, **threadIdx.y**, and **threadIdx.z**
 - Not all applications will use all three dimensions of a thread block



blockIdx and threadIdx

- Threads in a grid are organized into **a two-level hierarchy**
- In Figure
 - each thread block is organized into a 4x2x2 three-dimensional array of threads
 - this gives Grid 1 a total of $4 \times 16 = 64$ threads
- Each thread uses indices to decide what data to work on
 - **blockIdx**: 1D, 2D, or 3D (CUDA 4.0)
 - **threadIdx**: 1D, 2D, or 3D



CUDA Thread Organization

- When a thread executes the kernel function, references to the **blockIdx** and **threadIdx** variables return the **coordinates of the thread**
- Additional built-in variables, **gridDim** and **blockDim**, provide the **dimension of the grid** and the **dimension of each block**
- **$\text{threadID} = \text{blockIdx.x} * \text{blockDim.x} + \text{threadIdx}$** identifies the part of the input data to read from and the part of the output data structure to write to
 - **Example** Thread 3 of Block 0 has a threadID value of $0 * M + 3 = 3$
 - **Example** Thread 3 of Block 5 has a threadID value of $5 * M + 3$

CUDA threads, blocks and grids

- Nvidia use the **Compute Capability** specification to encode what each generation of GPU chips is capable of
- The Compute Capability (CC) of a GPU can be discovered by running the **deviceQuery** utility

	Compute Capability			
Item	1.x	2.x	3.x	5.x
Max. number of grid dimensions	2	3		
Grid maximum x-dimension	$2^{16} - 1$		$2^{31} - 1$	
Grid maximum y/z-dimension	$2^{16} - 1$			
Max. number of block dimensions	3			
Block max. x/y-dimension	512	1024		
Block max. z-dimension	64			
Max. threads per block	512	1024		
GPU example (GTX family chips)	8800	480	780	980

CUDA Thread Organization

- The exact organization of a grid is determined by the **execution configuration** provided at kernel launch
 - The first parameter specifies the dimensions of the grid as **# blocks**
 - The second specifies the dimensions of each block as **# threads**
 - Each such parameter is a **dim3** type, a C struct with three unsigned integer fields: x, y, and z
 - Example

```
dim3 dimGrid(128, 1, 1);  
dim3 dimBlock(32, 1, 1);  
vecAddKernel<<<dimGrid, dimBlock>>>(. . .);
```

oppure

```
dim3 cat(128, 1, 1);  
dim3 dog(32, 1, 1);  
KernelFunction<<<cat, dog>>>(. . .);
```

Execution Configuration Examples

Assuming we have

```
dim3 b(3,3,3);  
dim3 g(20,100);
```

Different grid-block combination are possible

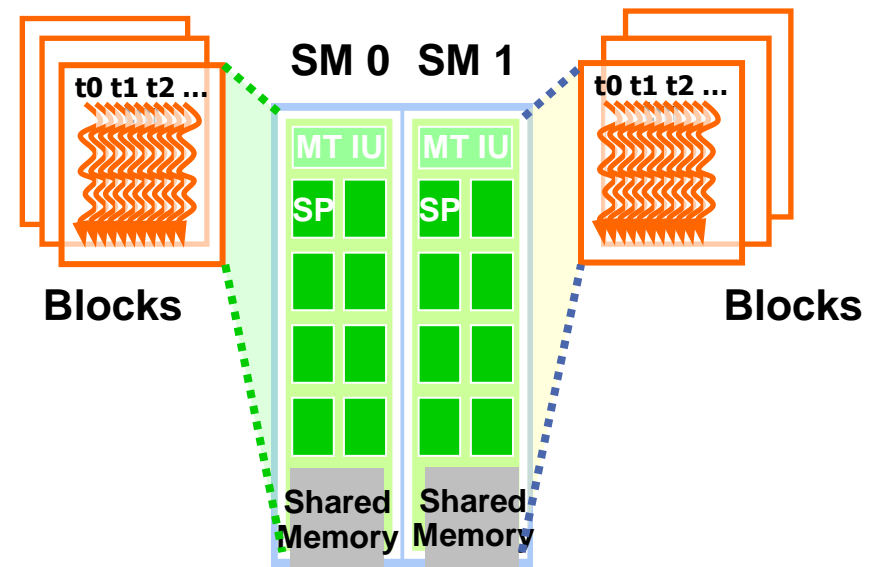
- `foo<<<g,b>>>()`; // Run a 20x100 grid made of 3x3x3 blocks
- `foo<<<10,b>>>()`; // Run a 10-block grid, each block made by 3x3x3 threads
- `foo<<<g,256>>>()`; // Run a 20x100 grid, made of 256 threads
- `foo<<<g,2048>>>()`; // An invalid example: maximum block size is 1024 threads even for compute capability 5.x
- `foo<<<5,g>>>()`; // Another invalid example, that specifies a block size of 20x100=2000 threads
- `foo<<<10,256>>>`; // simplified configuration for a 1D grid of 1D blocks

Synchronization

- CUDA allows threads in the same block to coordinate their activities using a barrier synchronization function, **__syncthreads ()**
 - the thread that executes the function call will be held at the calling location until every thread in the block reaches the location
- A **__syncthreads ()** statement must be executed by all threads in a block of the kernel before any moves on to the next phase

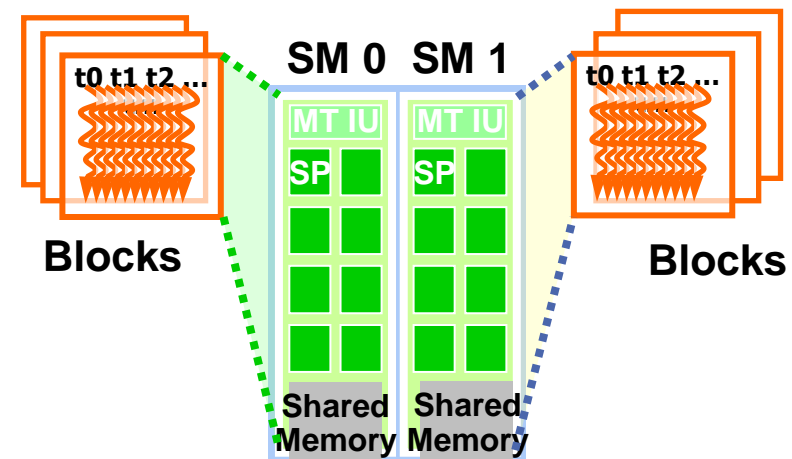
Thread and Block Assignment

- Once a **kernel is launched**, the CUDA runtime system generates the corresponding **grid of threads**
 - threads are assigned to execution resources on a block-by-block basis
- The execution resources are organized into streaming multiprocessors (SMs)
- Each device has a limit on the number of block that can be assigned to each SM



Thread and Block Assignment

- When an *insufficient* amount of any one or more types of resources needed for the simultaneous execution of blocks, the *CUDA runtime automatically reduces the number of blocks assigned to each SM*
- The runtime system maintains a list of blocks that need to execute and assigns new blocks to SMs as they complete the execution of blocks previously assigned to them

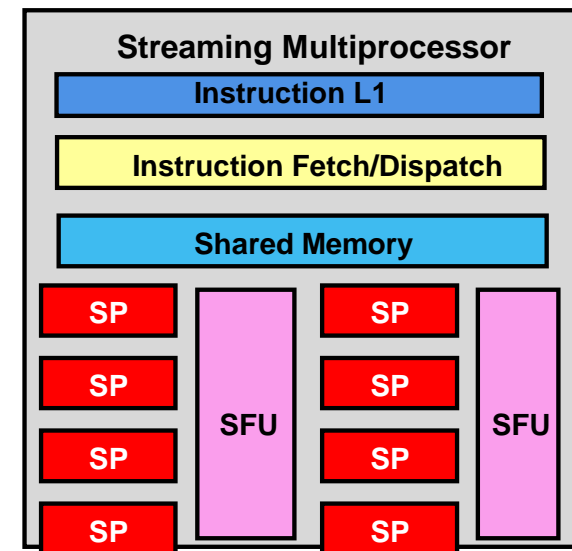
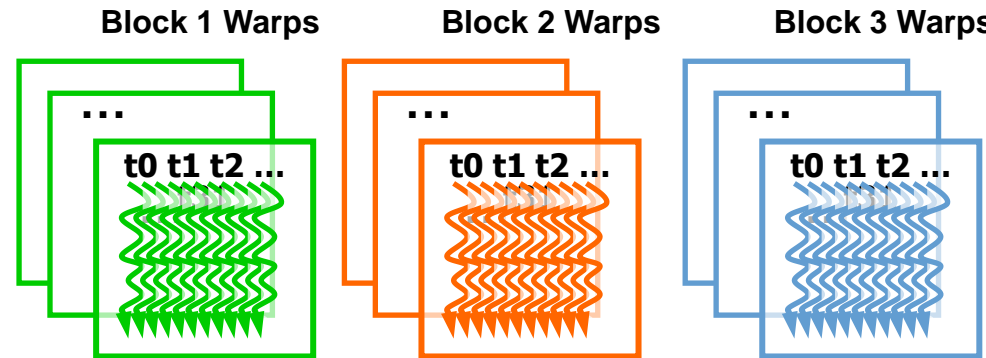


Thread Scheduling

- Once a block is assigned to a streaming multiprocessor, it is further divided into 32-thread units called **warps**
- The warp is the **unit of thread scheduling** in SMs
- Each warp consists of 32 threads of **consecutive threadIdx values**:
 - threads 0 through 31 form the first warp
 - threads 32 through 63 the second warp, and so on
- We can calculate the **number of warps that reside in an SM** for a given block size and a given number of blocks assigned to each SM

Thread Scheduling

- Each Block is executed as 32-thread Warps
 - Warps are scheduling units in SM
- **Example** If 3 blocks are assigned to an SM and each block has 256 threads, how many **warps** are there in an SM?
 - 3 blocks, each block 256 threads
 - each block has $256/32 = 8$ warps
 - having 3 blocks in each SM, we have $8 \times 3 = 24$ warps in each SM



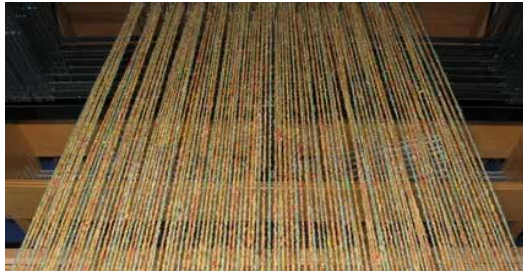
Thread Scheduling

- **Why** do we need to have so many warps in an SM if there are only 8 SPs in an SM?
 - The answer is for **efficiently executing long-latency operations** such as global memory accesses
 - When an instruction executed by the threads in a warp needs to wait for the result of a previously initiated long-latency operation, the **warp is not selected for execution**
 - **Another resident warp** (that is no waiting for results) is selected for execution
 - If more than one warp is ready for execution, a **priority mechanism** is used to select one for execution
 - This mechanism of filling the latency of expensive operations with work from other threads is often referred to as **latency hiding**

Thread Scheduling

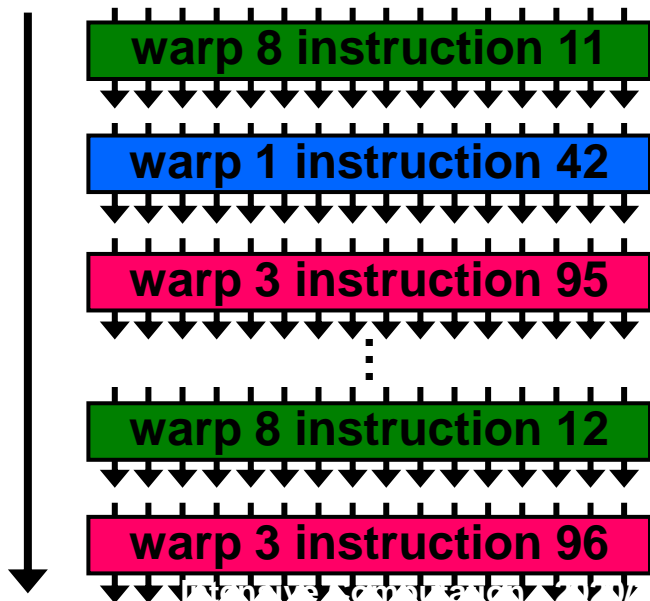
- Note that **warp scheduling** is also used for tolerating other types of long latency operations such as *pipelined floating-point arithmetic* and *branch instructions*
- With enough warps around
 - the hardware will likely find a warp to execute at any point in time
 - full use of the execution hardware in spite of long-latency operations
- The selection of ready warps for execution
 - does not introduce any idle time into the execution timeline
 - **zero-overhead thread scheduling**
- With warp scheduling, the long waiting time of warp instructions is hidden by executing instructions from other warps

SM Warp Scheduling



SM multithreaded
Warp scheduler

time



- SM hardware implements zero-overhead Warp scheduling
 - Warps whose next instruction has its operands ready for consumption are eligible for execution
 - **Eligible Warps** are selected for execution on a prioritized scheduling policy
 - **All threads in a Warp execute the same instruction** when selected
- 4 clock cycles needed to dispatch the same instruction for all threads in a Warp in G80
 - If one global memory access is needed for every 4 instructions
 - A minimum of 13 Warps are needed to fully tolerate 200-cycle memory latency

Thread Scheduling

List of GPU chips and their SM capability

	compute capability					
Item	1.0, 1.1	1.2, 1.3	2.x	3.0	3.5	5.0
Concurrent kernels/device	1		16		32	
Max. resident blocks/SM	8			16		32
Max. resident warps/SM	24	32	48	64		
Max. resident threads/SM	768	1024	1536	2048		
32-bit registers/SM	8k	16k	32k	64k		
Max. registers/thread	128		63		255	

Exercise

Simple exercise (register and shared memory not considered)

- Assume a CUDA device allowing 8 blocks, 1024 threads per SM and 512 thread in each block
- For matrix multiplication, should we use 8x8, 16x16, or 32x32 thread blocks?
- Analyze the pros and cons of each choice:
 - If we use **8x8 blocks**, each block would have only 64 threads, and we will need $1024/64 = 12$ blocks to fully occupy an SM
 - We are limited to 8 blocks in each SM, we will end up with only $64 \times 8 = 512$ threads in each SM
 - Then the SM execution resources will likely be underutilized because there will be fewer warps to schedule around long-latency operations

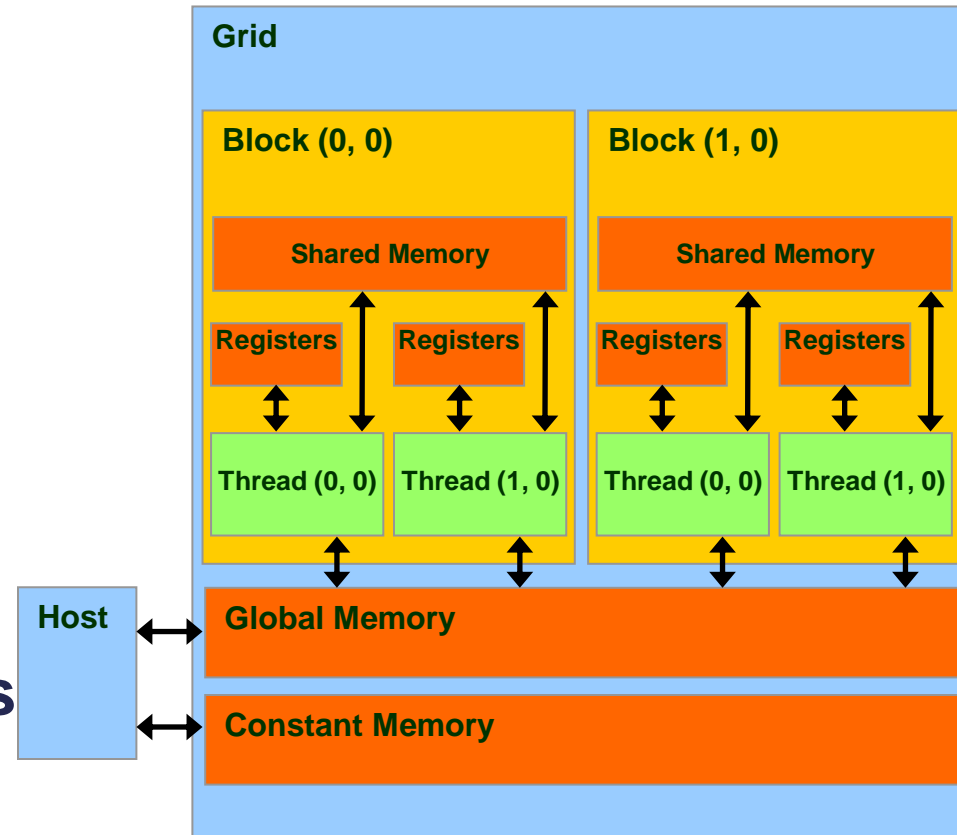
Exercise

Simple exercise (register and shared memory not considered)

- Assume a CUDA device allowing 8 blocks, 1024 threads per SM and 512 thread in each block
- For matrix multiplication, should we use 8x8, 16x16, or 32x32 thread blocks?
 - The **16x16** blocks give 256 threads per block.
 - This means that each SM can take $1024/256 = 4$ blocks.
 - This is within the 8-block limitation.
 - **Good configuration:**
 - full thread capacity in each SM and the
 - maximal number of warps for scheduling around the long-latency oper.
- The **32x32** blocks exceed the limitation of up to 512 threads per block

Programmer View of CUDA Memories

- At the bottom of the figure, we see global memory and constant memory
- These types of memory can be written (W) and read (R) by the host by calling API functions
- The **constant memory** supports short-latency, high-bandwidth, **read-only access by the device** when all threads simultaneously access the same location



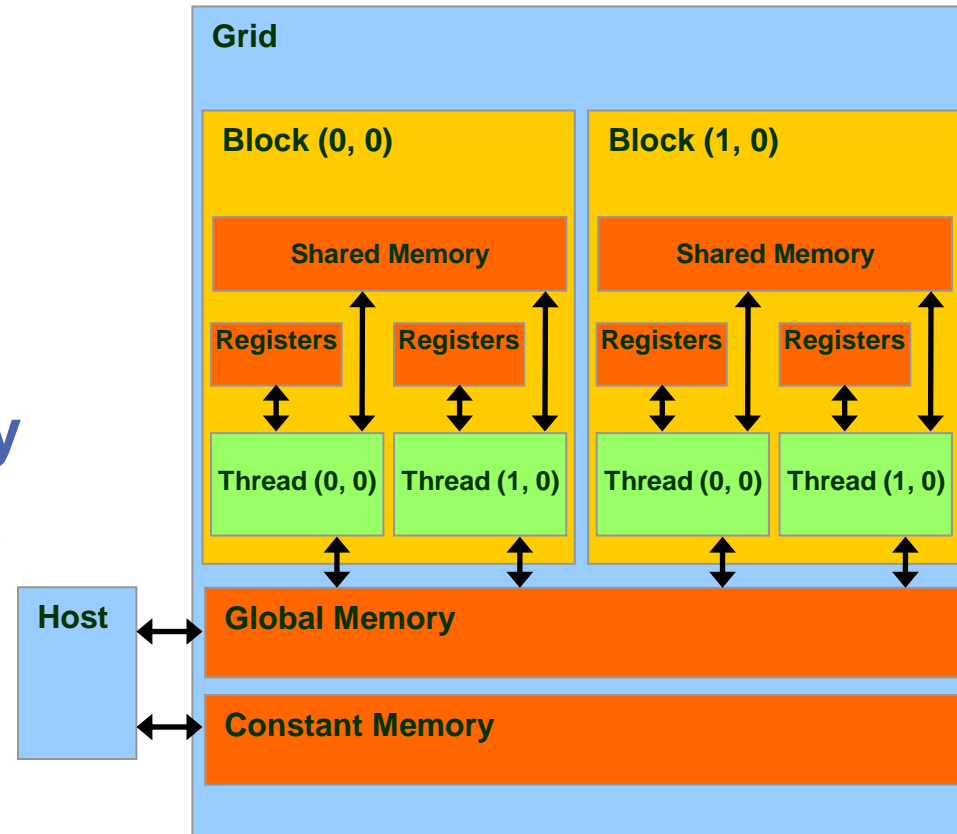
Programmer View of CUDA Memories

Device code can:

- R/W per-thread **registers**
- R/W per-thread **local memory**
- R/W per-block **shared memory**
- R/W per-grid **global memory**
- Read-only per-grid **constant memory**

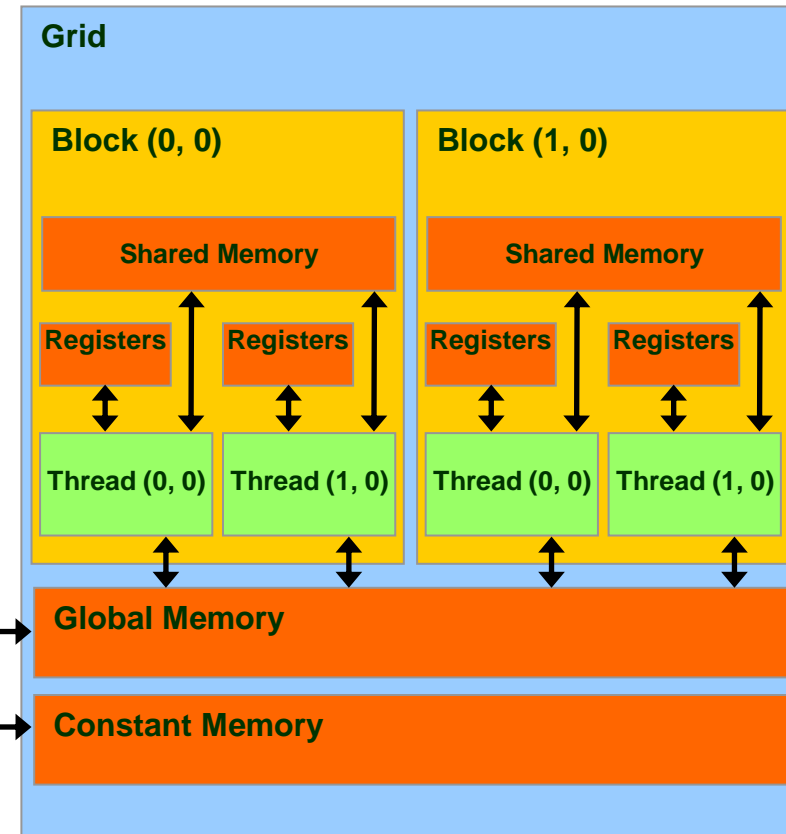
Host code can:

- Transfer data to/from per-grid global and constant memories



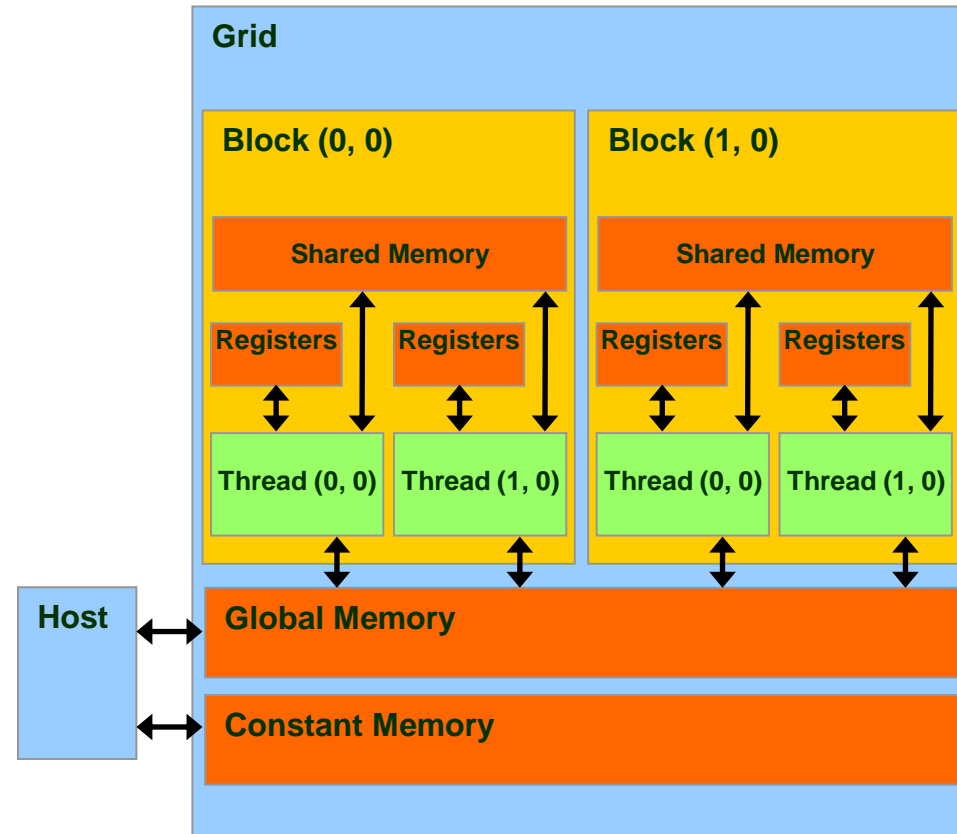
Programmer View of CUDA Memories

- **Registers** and **shared memory** are on-chip memories
- Variables on these memories can be **accessed at very high speed** in a highly parallel manner
- **Registers** are allocated to individual threads and each thread can only access its own registers
- A kernel function uses registers to hold frequently accessed variables private to each thread



Programmer View of CUDA Memories

- **Registers** and **shared memory** are on-chip memories
- **Shared memory** is allocated to thread blocks;
- **all threads in a block** can access variables in the **shared memory** locations allocated to the block
- Shared memory is used by threads to cooperate by sharing their input data and the intermediate results



Variables

- Table presents the CUDA syntax for declaring program variables into the various types of device memory
- Each declaration gives to CUDA variable:
 - A **scope** identifies the range of threads that can access the variable: single thread only, all threads of a block, or all threads of all grids
 - A **lifetime** specifies the portion of the program's execution duration when the variable is available for use: either within a kernel's invocation or throughout the entire application

Variable declaration	Memory	Scope	Lifetime
Automatic Variables	register	thread	kernel
<code>__device__ __shared__ int SharedVar;</code>	shared	block	kernel
<code>__device__ int GlobalVar;</code>	global	grid	application
<code>__device__ __constant__ int ConstantVar;</code>	constant	grid	application

A motivating example

- Lets assume that:
 - We have a kernel that requires 48 registers per thread
 - Target platform is a GTX 580 card (CC 2.0, 16SMs, 32k registers/SM)
 - Execution configuration is a grid of 4x5x3 blocks, each 100 threads
- Each block requires $100 * 48 = 4800$ registers
- The grid is made of $4 * 5 * 3 = 60$ blocks that need to be distributed to the 16 SMs of the card

A motivating example

- There will be 12 SMs that will receive 4 blocks and 4 SMs that will receive 3 blocks → Inefficient
- Additionally, each of the 100-thread blocks would be split into $\lceil \frac{100}{\text{warpSize}} \rceil = \lceil \frac{100}{32} \rceil = 4$ warps
- The first three warps would have 32 threads and the last would have 4 threads!
- So during the execution of the last warp of each block

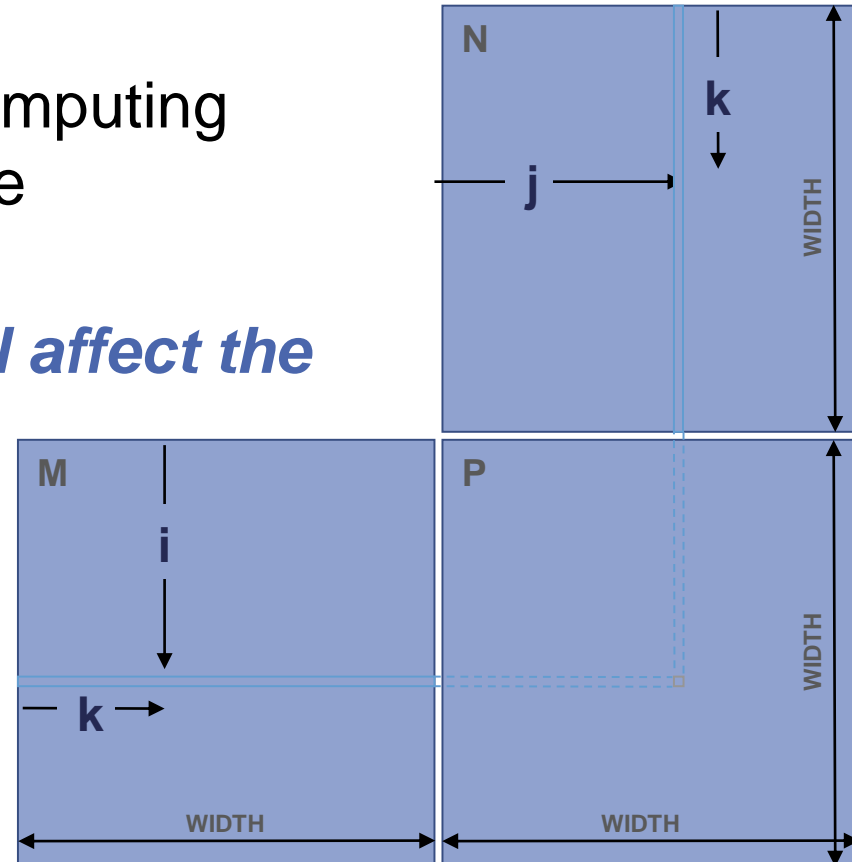
$$\frac{32 - 4}{32} = 87.5\%$$

of the SPs will be idle

MATRIX MULTIPLICATION EXAMPLE

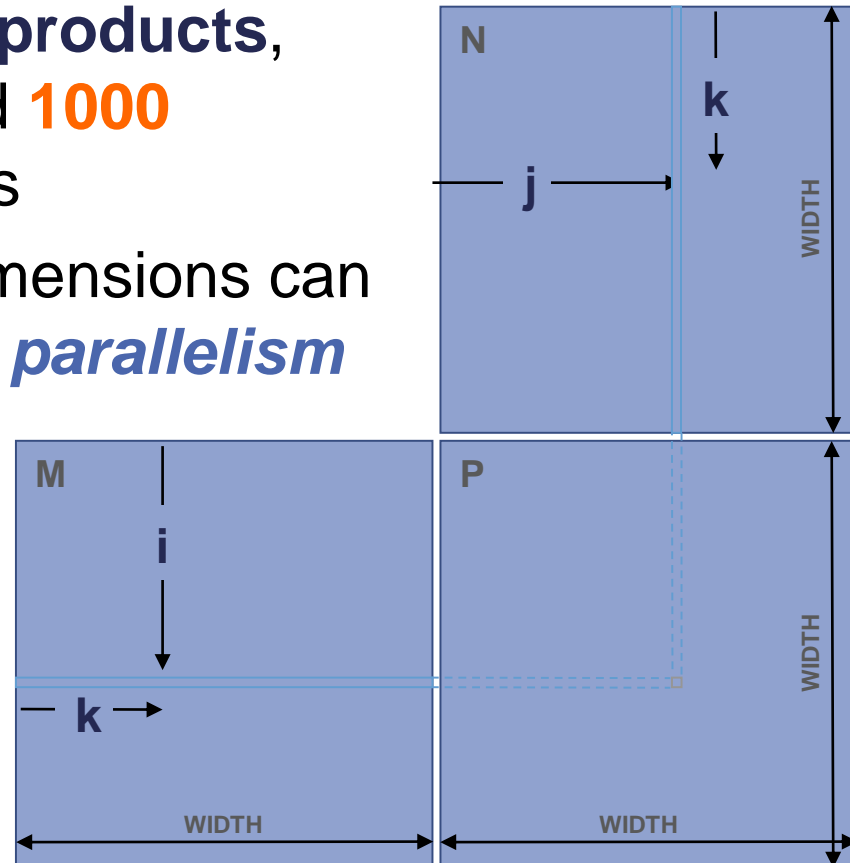
Matrix multiplication

- Each element of the product matrix P is generated by performing a **dot product** between a row of input matrix M and a column of input matrix N : $P = M \times N$
- The dot product operations for computing different matrix P elements can be **simultaneously** performed
- *None of these dot products will affect the results of each other*



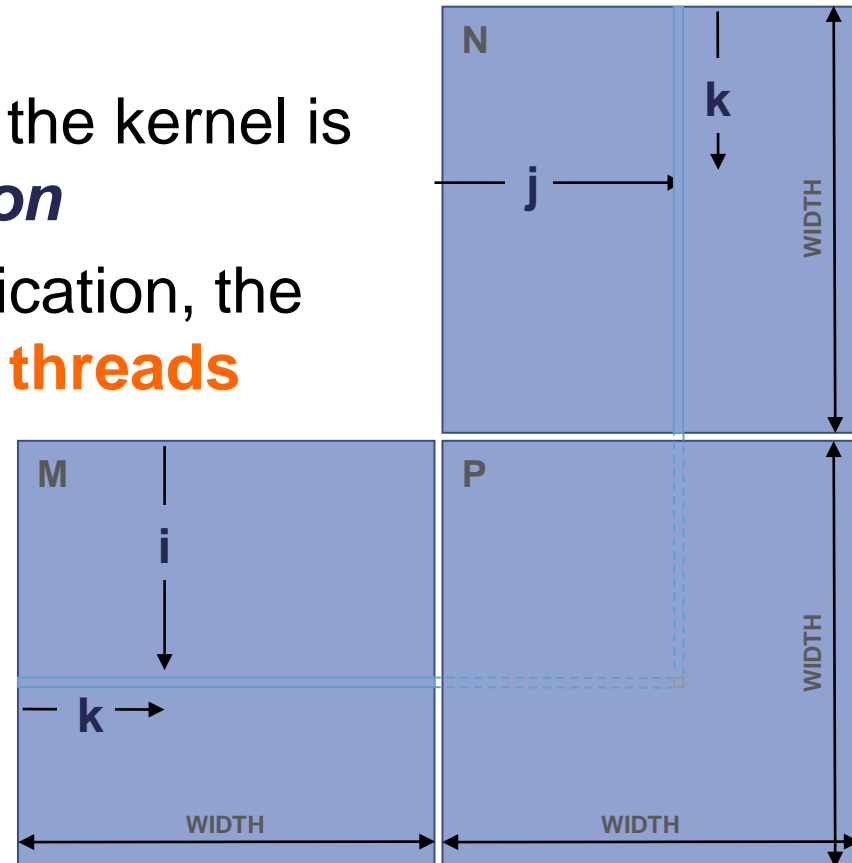
Matrix multiplication

- For large matrices, the number of dot products can be very large
- Example, a **1000 x 1000 matrix multiplication** has **1,000,000 independent dot products**, each involving **1000 multiply** and **1000 accumulate** arithmetic operations
- *Matrix multiplication* of large dimensions can have *very large amount of data parallelism*



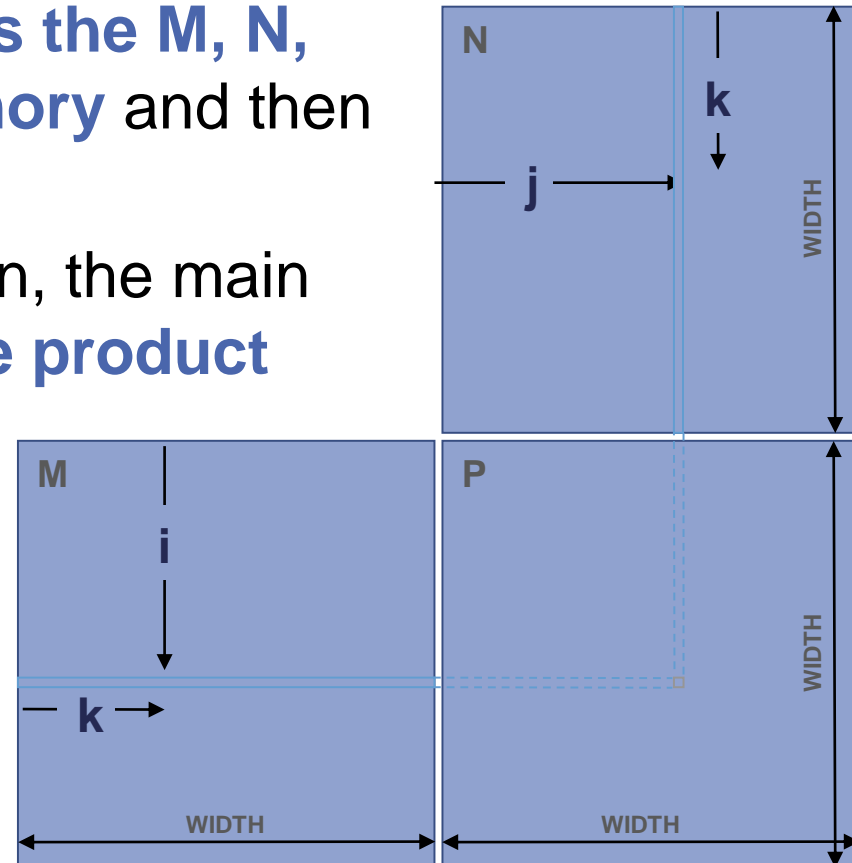
Matrix multiplication

- The entire **matrix multiplication computation** can be implemented as a **kernel**
- **Each thread** is used to compute **one element** of **output matrix P**
- The *number of threads* used by the kernel is a function of the *matrix dimension*
- For a **1000 x 1000 matrix** multiplication, the kernel would generate **1,000,000 threads** when it is invoked



Matrix multiplication

- Assume that the **matrices are square** in shape, and the dimension of each matrix is specified by the parameter **WIDTH**
- The **main program** first **allocates the M, N, and P matrices** in the **host memory** and then performs **I/O to read in M and N**
- **After** completing the multiplication, the main function performs **I/O to write the product matrix P** and to free memory



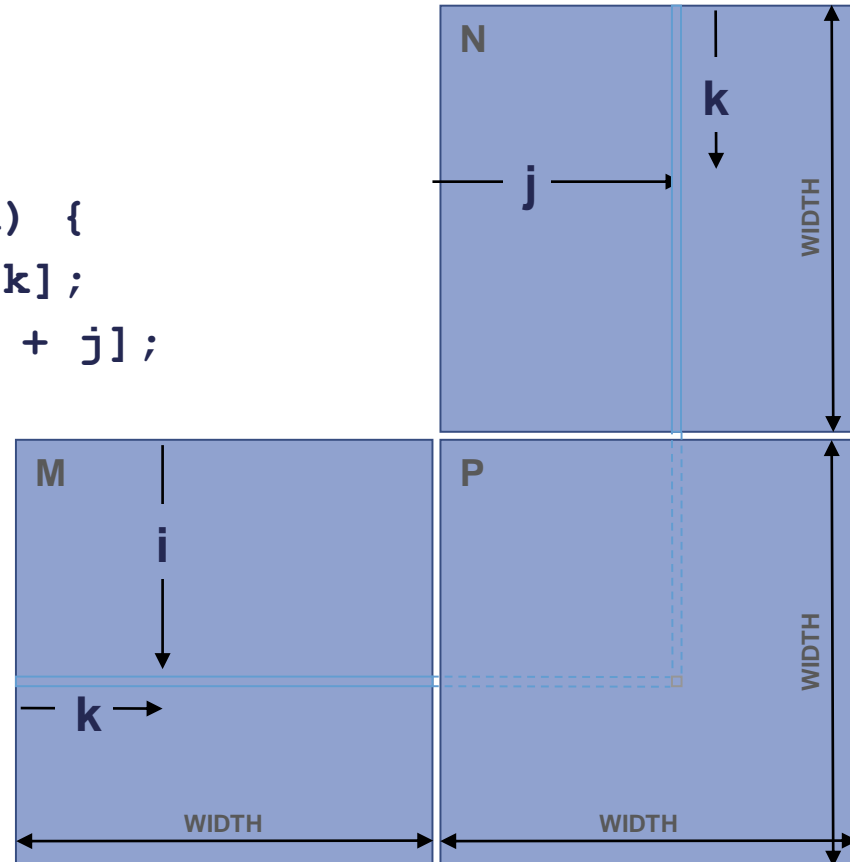
Matrix multiplication

CPU-only matrix multiplication function

```

• void MatrixMulOnHost(float* M, float* N,
  float* P, int Width)
{
for (int i = 0; i < Width; ++i)
  for (int j = 0; j < Width; ++j) {
    float sum = 0;
    for (int k = 0; k < Width; ++k) {
      float a = M[i * width + k];
      float b = N[k * width + j];
      sum += a * b;
    }
    P[i * Width + j] = sum;
  }
}

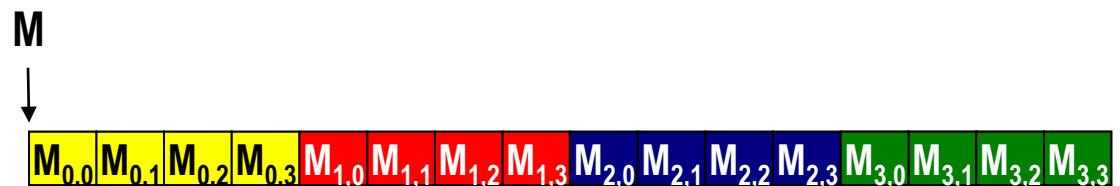
```



Matrix multiplication

- The index used for accessing the M matrix in the innermost loop is $i * \text{Width} + k$
- The M matrix elements are placed into the system memory according to the row-major convention:
 - All elements of a row are placed into **consecutive memory locations**
 - The rows are then placed one after another

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	$M_{1,3}$
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	$M_{2,3}$
$M_{3,0}$	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$



Matrix multiplication

- To port the matrix multiplication function into CUDA, we can modify the MatrixMultiplication() function to **move the bulk of the calculation to a CUDA device**

```
void MatrixMulOnDevice(float* M, float* N, float* P, int Width)
{
    int size = Width * Width * sizeof(float);
    float* Md, Nd, Pd;
    ...
1.// Allocate device memory for M, N, P and
    // load M, N to device memory
2.// Kernel invocation code to have the device to perform
    // the actual matrix multiplication
3.// copy P from the device
    // Free device matrices
}
```

Matrix multiplication

- To port the matrix multiplication function into CUDA we can modify the MatrixMultiplication() function to offload the calculation to a CUDA device

```
void MatrixMulOnDevice(float* M, float* N, float* P, int MWidth, int NWidth, int PWidth, int PHeight, int dth)
{
    int size = Width * Width * Height;
    float* Md, Nd, Pd;
    ...
1. // Allocate device memory for M, N, P and
    // load M, N to device memory
2. // Kernel invocation code to have the device to perform
    // the actual matrix multiplication
3. // copy P from the device
    // Free device matrices
}
```

Part 1

- **allocates** device (GPU) memory to hold copies of the M, N, and P matrices,
- **copies** these matrices over to the device memory

Matrix multiplication

- To port the matrix multiplication function into CUDA, we can modify the MatrixMultiplication() function to offload the bulk of the calculation to a CUDA device

```
void MatrixMulOnDevice(float* M, float* N, float* P, int Width)
{
    int size = Width * Width * 3;
    float* Md, Nd, Pd;
    ...
1. // Allocate device memory for M, N, P and
    // load M, N to device memory
2. // Kernel invocation code to have the device to perform
    // the actual matrix multiplication
3. // copy P from the device
    // Free device matrices
}
```

Part 2

- **invokes** a kernel that launches parallel execution of the actual matrix multiplication on the device

Matrix multiplication

- To port the matrix multiplication function into CUDA, we can modify the MatrixMultiplication() function to offload the bulk of the calculation to a CUDA device

```
void MatrixMulOnDevice(float* M, float* N, float* P, int Width)
{
    int size = Width * Width * Width;
    float* Md, Nd, Pd;
    ...
1. // Allocate device memory for M, N, P and
    // load M, N to device memory
2. // Kernel invocation code to have the device to perform
    // the actual matrix multiplication
3. // copy P from the device
    // Free device matrices
}
```

Part 3

- **copies** the product matrix P from the device memory back to the host memory

Width)

Matrix multiplication

Assume **M**, **N** and **P** are on the **host** and
Md, **Nd** and **Pd** on **device**

```
void MatrixMulOnDevice(float* M, float* N, float* P, int Width)
{
    int size = Width * Width * sizeof(float);
    float* Md, Nd, Pd;
    ...
1. // Allocate and Load M, N to device memory
    cudaMalloc(&Md, size);
    cudaMemcpy(Md, M, size, cudaMemcpyHostToDevice);
    cudaMalloc(&Nd, size);
    cudaMemcpy(Nd, N, size, cudaMemcpyHostToDevice);
    // Allocate P on the device
    cudaMalloc(&Pd, size);
```

Matrix multiplication

Assume M, N and P are on the **host** and
device

```
void MatrixMulOnDevice(float* M, float* N, float* P, float* Md, Nd, Pd)
{
    int size = Width * Width * sizeof(float);
    float* Md, Nd, Pd;
```

...

```
1. // Allocate and Load M, N to device memory
   cudaMalloc(&Md, size);
   cudaMemcpy(Md, M, size, cudaMemcpyHostToDevice);
   cudaMalloc(&Nd, size);
   cudaMemcpy(Nd, N, size, cudaMemcpyHostToDevice);
   // Allocate P on the device
   cudaMalloc(&Pd, size);
```

The two symbolic constants, `cudaMemcpyHostToDevice` and `cudaMemcpyDeviceToHost` are **predefined constants** of the CUDA programming environment, **recognized by `cudaMemcpy`**

Matrix multiplication

```
2. // Kernel invocation code - to be shown later
...
3. // Read P from the device
   cudaMemcpy(P, Pd, size, cudaMemcpyDeviceToHost);
   // Free device matrices
   cudaFree(Md);
   cudaFree(Nd);
   cudaFree(Pd);
}
```

-The product data is copied from device memory to host memory so the value will be available to main() by a call to the `cudaMemcpy()` function

- Then Md, Nd, and Pd are freed from the device memory by calls to the `cudaFree()` functions

Matrix multiplication

```
// Matrix multiplication kernel - thread specification
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int
    Width)
{
    // 2D Thread ID
    int tx = threadIdx.x;
    int ty = threadIdx.y;
    // Pvalue stores the Pd element that is computed by the thread
    float Pvalue = 0;
    for (int k = 0; k < Width; ++k)
    {
        float Mdelement = Md[ty * Md.width + k];
        float Ndelement = Nd[k * Nd.width + tx];
        Pvalue += Mdelement * Ndelement;
    }
    // Write the matrix to device memory each thread writes one element
    Pd[ty * Width + tx] = Pvalue;
}
```

Matrix multiplication

```
// Matrix multiplication kernel - thread specification
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd,
int Width)
{
// 2D Thread ID
int tx = threadIdx.x;
int ty = threadIdx.y;
// Pvalue stores the Pd element
float Pvalue = 0;
for (int k = 0; k < Width; ++k)
{
float Mdelement = Md[ty * Md.width + k];
float Ndelement = Nd[k * Nd.width + tx];
Pvalue += Mdelement * Ndelement;
}
// Write the matrix to device memory
Pd[ty * Width + tx] = Pvalue;
}
```

The CUDA-specific keyword `__global__` in front of the declaration of `MatrixMulKernel()` indicates that:

- the function is a kernel
- it can be called from a host functions to generate a grid of threads on a device

Matrix multiplication

```
// Matrix multiplication kernel - thread specification
__global__ void MatrixMulKernel(float* Md, float* Nd, float* Pd, int Width)
{
    // 2D Thread ID
    int tx = threadIdx.x;
    int ty = threadIdx.y,
    // Pvalue stores the Pd element
    float Pvalue = 0;
    for (int k = 0; k < Width; ++k)
    {
        float Mdelement = Md[ty * Md.width + k];
        float Ndelement = Nd[k * Nd.width + tx];
        Pvalue += Mdelement * Ndelement;
    }
    // Write the matrix to device memory
    Pd[ty * Width + tx] = Pvalue;
}
```

- The keywords `threadIdx.x` and `threadIdx.y` refer to the thread indices of a thread
- The original loop variables `i` and `j` are now replaced with `threadIdx.x` and `threadIdx.y`
- The CUDA threading hardware generates all of the `threadIdx.x` and `threadIdx.y` values for each thread (instead of the loop increment the values of `i` and `j` for loop iteration)

Matrix multiplication

- Limitation of this simple code is the **size of matrices**: 16x16
- Infact the kernel function does not use **blockIdx**
- Then, we are limited to using **only one block of threads**
- Even if we used more blocks, threads from different blocks would calculate the same Pd element if they have the same threadIdx value
- The code can only calculate a product matrix of up to 512 elements → infact a thread block can have only up to 512 threads and each thread calculates one element of the matrix

Matrix multiplication

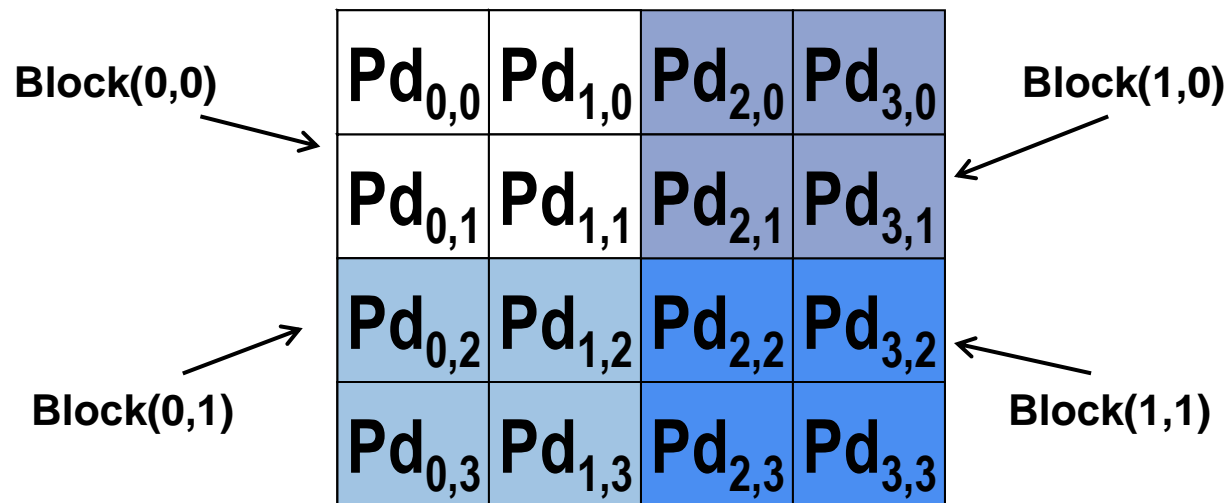
- For square matrices, 32×32 requires more than 512 threads per block \rightarrow then we are limited to 16×16
- This is obviously **not acceptable**
- The product matrix must have millions of elements in order to have a sufficient amount of data parallelism to benefit from execution on a device
- Now we revise the matrix multiplication kernel function using **multiple blocks**

Matrix multiplication

- In order to accommodate larger matrices, we need to use **multiple thread blocks**
- Conceptually, **we break Pd into square tiles**
- All the Pd **elements of a tile** are computed by a **block of threads**
- By keeping the dimensions of these Pd tiles small, we keep the total number of threads in each block under **512**, the **maximal allowable block size**
- In the following, we abbreviate:
 - **threadIdx.x** and **threadIdx.y** as **tx** and **ty**
 - **blockIdx.x** and **blockIdx.y** as **bx** and **by**

Matrix multiplication

- Consider a very small matrix - 4x4 - and a very small TILE_WIDTH value - 2 - and divide the matrix into **4 tiles**
- We create **blocks** organized into **2x2 arrays of threads**
- **Each block calculates 4 Pd elements**

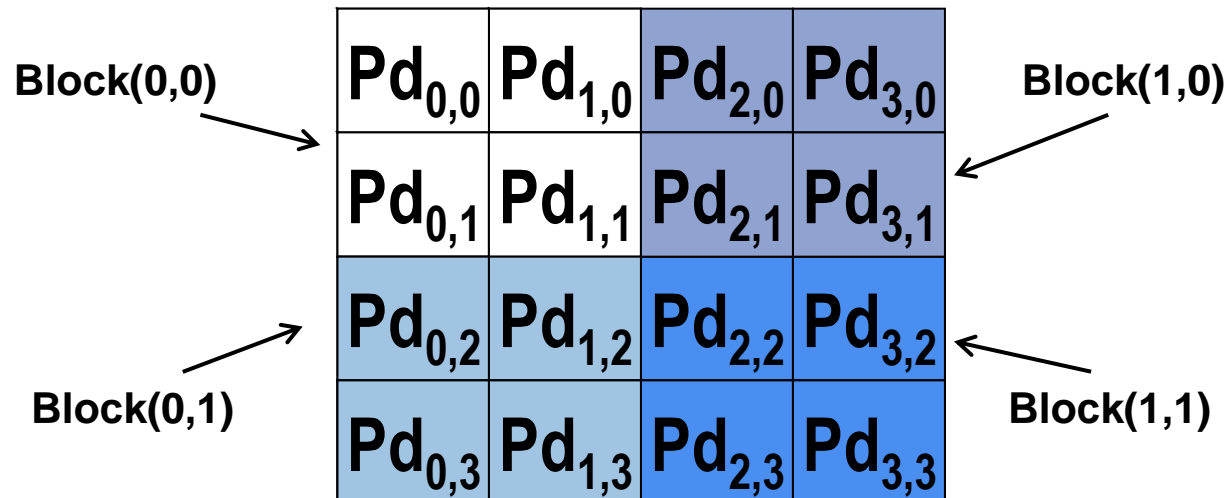


Matrix multiplication

- thread (0, 0) of block (0, 0) calculates $Pd_{0,0}$
- thread (0, 0) of block (1, 0) calculates $Pd_{2,0}$

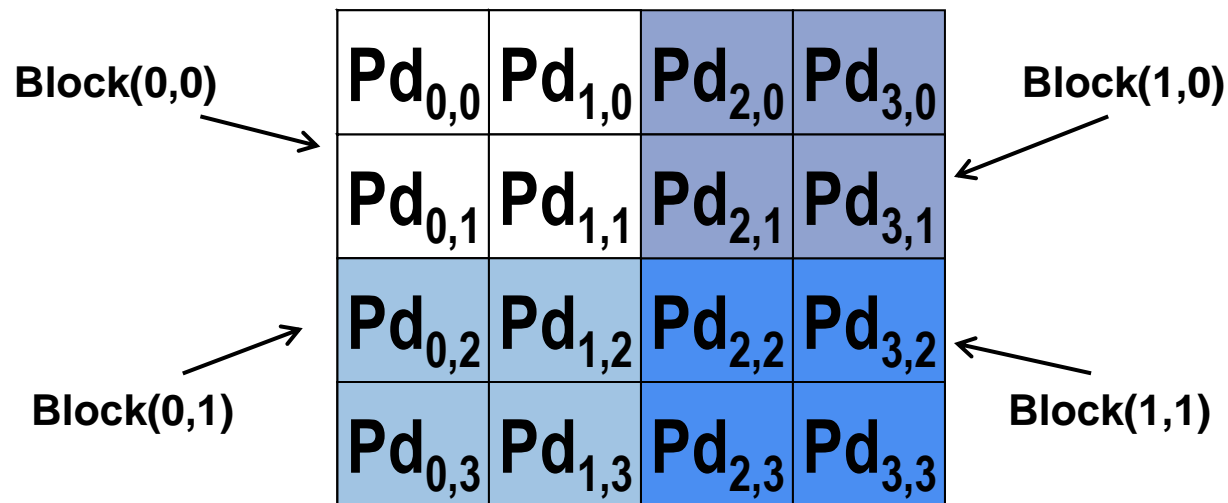
The Pd element calculated by thread (0, 0) of block (1, 0) can be computed by:

$$Pd[bx * TILE_WIDTH + tx] [by * TILE_WIDTH + ty] = Pd[1 * 2 + 0][0 * 2 + 0] = Pd[2][0]$$



Matrix multiplication

- We also need the **row index y** of M_d and the **column index x** of N_d for input values
- The **row index** of M_d used by thread (tx, ty) of block (bx, by) is **$(by * \text{TILE_WIDTH} + ty)$**
- The **column index** of N_d used by the same thread is **$(bx * \text{TILE_WIDTH} + tx)$**

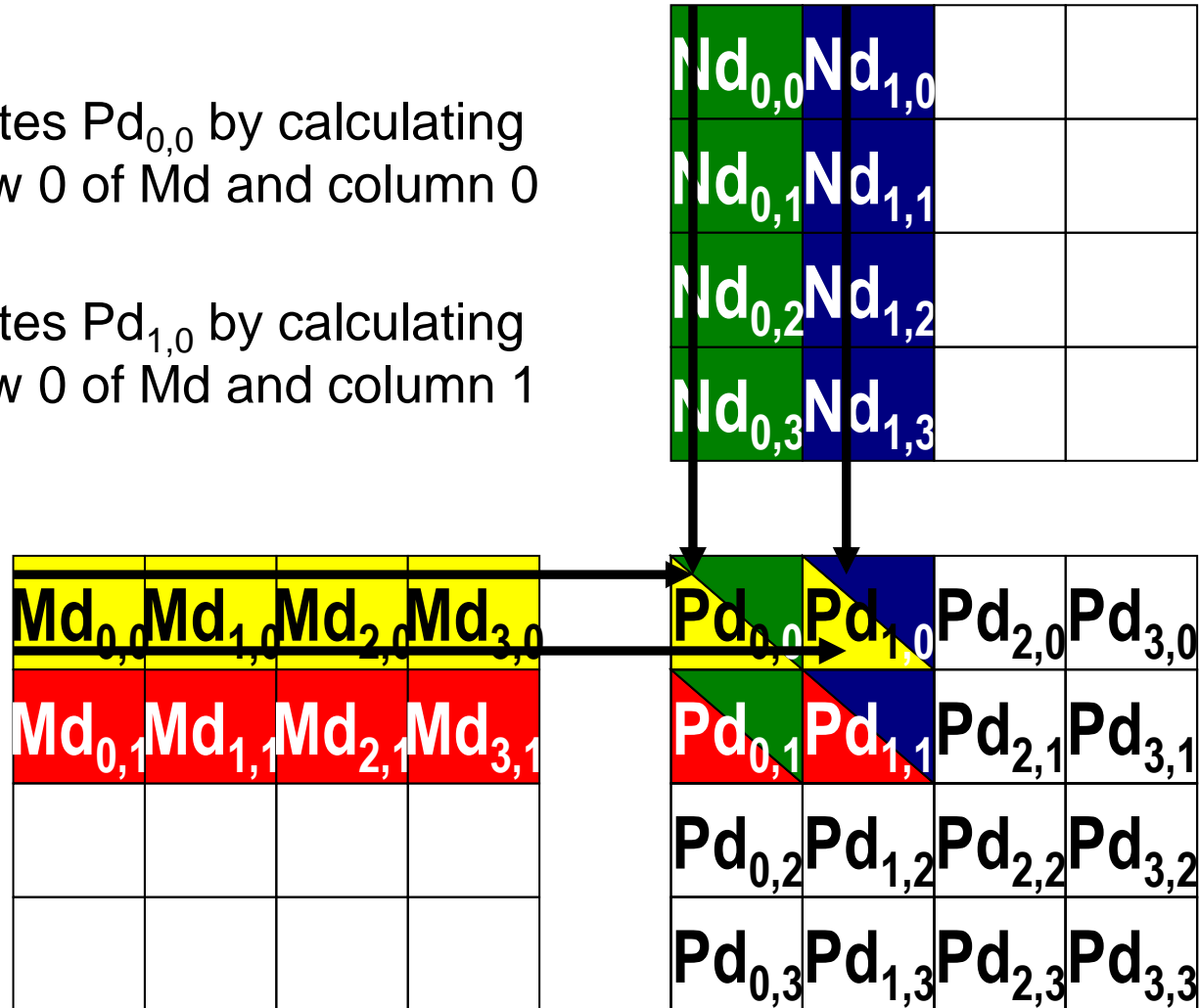


Matrix multiplication

- Threads in block (0, 0) produce four dot products:

- Thread (0, 0) generates $Pd_{0,0}$ by calculating the dot product of row 0 of Md and column 0 of Nd
- Thread (1, 0) generates $Pd_{1,0}$ by calculating the dot product of row 0 of Md and column 1 of Nd
- ...

- The arrows of $Pd_{0,0}$, and $Pd_{1,0}$ shows the row and column used for generating their result value



Matrix multiplication

Revised matrix multiplication kernel function **with blocks**

```
__global__ void MatrixMulKernel(float* Md, float* Nd,
    float* Pd, int Width)
{
    // Calculate the row index of the Pd element and M
    int Row = blockIdx.y*TILE_WIDTH + threadIdx.y;
    // Calculate the column index of Pd and N
    int Col = blockIdx.x*TILE_WIDTH + threadIdx.x;

    float Pvalue = 0;
    // each thread computes one element of the
    // matrix
    for (int k = 0; k < Width; ++k)
        Pvalue += Md[Row*Width+k] * Nd[k*Width+Col];

    Pd[Row*Width+Col] = Pvalue;
}
```

Each thread uses its `blockIdx` and `threadIdx` values to identify the row index - `Row` - and the column index - `Col` - of the `Pd` element

Matrix multiplication

- Revised matrix multiplication kernel function

```
__global__ void MatrixMulKernel(float* Nd, float* Pd, int Width)
{
    // Calculate the row index of the Pd element
    int Row = blockIdx.y*TILE_WIDTH + threadIdx.y;
    // Calculate the column index of Pd element
    int Col = blockIdx.x*TILE_WIDTH + threadIdx.x;

    float Pvalue = 0;
    // each thread computes one element of the
    // sub-matrix
    for (int k = 0; k < Width; ++k)
        Pvalue += Nd[Row*Width+k] * Nd[k*Width+Col];

    Pd[Row*Width+Col] = Pvalue;
}
```

Each thread performs:

- a dot product on the row of Md and column of Nd to generate the value of the Pd element
- eventually writes the Pd value to the appropriate global memory location

Matrix multiplication

- This kernel can handle matrices of up to $16 \times 65,535$ elements in each dimension
- In the situation where matrices larger than this new limit are to be multiplied, one can divide the Pd matrix into submatrices of a size permitted by the kernel
- All blocks can run in parallel with each other and will fully utilize parallel execution resources

Matrix multiplication

- Revised host code to be used in the **MatrixMultiplication()** to launch the revised kernel **MatrixMulKernel()** with multiple blocks
- Note that the **dimGrid** is **Width/TILE_WIDTH** for both the x dimension and the y dimension

```
// Set up the execution configuration
dim3 dimGrid(Width/TILE_WIDTH,Width/TILE_WIDTH)
dim3 dimBlock(TILE_WIDTH,TILE_WIDTH)

// launch the device computation thread
MatrixMulKernel<<<dimGrid, dimBlock>>>(Md,Nd,Pd,Width);
```

Matrix multiplication

Memory access: global vs shared

- The table shows the global memory accesses done by all threads in block(0,0)
- The threads are listed in the horizontal direction, with the time of access increasing downward in the vertical direction

	$P_{0,0}$ thread _{0,0}	$P_{1,0}$ thread _{1,0}	$P_{0,1}$ thread _{0,1}	$P_{1,1}$ thread _{1,1}
Access order ↓	$M_{0,0} * N_{0,0}$	$M_{0,0} * N_{1,0}$	$M_{0,1} * N_{0,0}$	$M_{0,1} * N_{1,0}$
	$M_{1,0} * N_{0,1}$	$M_{1,0} * N_{1,1}$	$M_{1,1} * N_{0,1}$	$M_{1,1} * N_{1,1}$
	$M_{2,0} * N_{0,2}$	$M_{2,0} * N_{1,2}$	$M_{2,1} * N_{0,2}$	$M_{2,1} * N_{1,2}$
	$M_{3,0} * N_{0,3}$	$M_{3,0} * N_{1,3}$	$M_{3,1} * N_{0,3}$	$M_{3,1} * N_{1,3}$

Matrix multiplication

- Each thread accesses **4 elements of Md** and **4 elements of Nd** during its execution
- there is a significant **overlap** of the Md and Nd accesses:
 - **thread(0,0)** and **thread(1,0)** both access **Md_{1,0}** as well as the rest of row 0 of Md
 - **thread(1,0)** and **thread(1,1)** both access **Nd_{1,0}** as well as the rest of column 1 of Nd

Access order ↓

	$P_{0,0}$ thread _{0,0}	$P_{1,0}$ thread _{1,0}	$P_{0,1}$ thread _{0,1}	$P_{1,1}$ thread _{1,1}
	$M_{0,0} * N_{0,0}$	$M_{0,0} * N_{1,0}$	$M_{0,1} * N_{0,0}$	$M_{0,1} * N_{1,0}$
	$M_{1,0} * N_{0,1}$	$M_{1,0} * N_{1,1}$	$M_{1,1} * N_{0,1}$	$M_{1,1} * N_{1,1}$
	$M_{2,0} * N_{0,2}$	$M_{2,0} * N_{1,2}$	$M_{2,1} * N_{0,2}$	$M_{2,1} * N_{1,2}$
	$M_{3,0} * N_{0,3}$	$M_{3,0} * N_{1,3}$	$M_{3,1} * N_{0,3}$	$M_{3,1} * N_{1,3}$

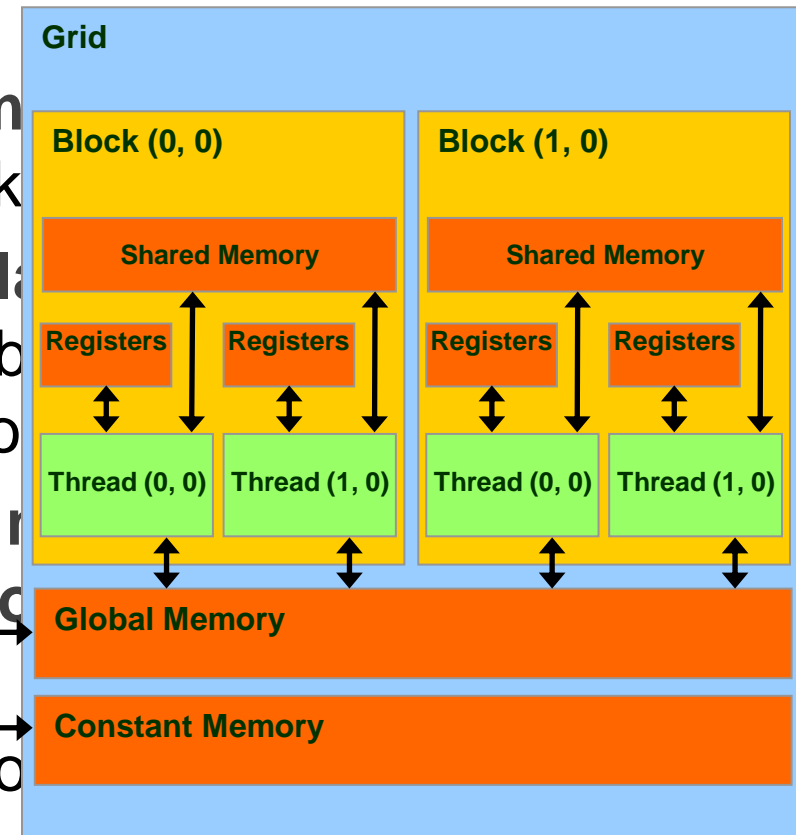
Matrix multiplication

- The kernel is written so both $\text{thread}(0,0)$ and $\text{thread}(1,0)$ access these Md row 0 elements from the **global memory**
- In general, **every Md and Nd element** is accessed exactly **twice** during the execution of $\text{block}(0,0)$
- If $\text{thread}(0,0)$ and $\text{thread}(1,0)$ **collaborate** so that Md elements are only loaded from global memory once, the total number of accesses to the global memory reduced by half
- The potential **reduction in global memory traffic** in the matrix multiplication example is **proportional** to the **dimension of the blocks** used
- $N \times N$ blocks \rightarrow potential reduction of global memory is N

Matrix multiplication

- The kernel is written so both thread(0,0) and thread(1,0) access these Md row 0 elements from the **global memory**

- In general, every Md and Nd element is accessed **twice** during the execution of block
- If thread(0,0) and thread(1,0) collaborate, only $2d$ elements are only loaded from global memory, reducing the total number of accesses to the global memory
- The potential **reduction in global memory accesses** is achieved by using a **dimension of the blocks** smaller than d
- $N \times N$ blocks \rightarrow potential reduction of d

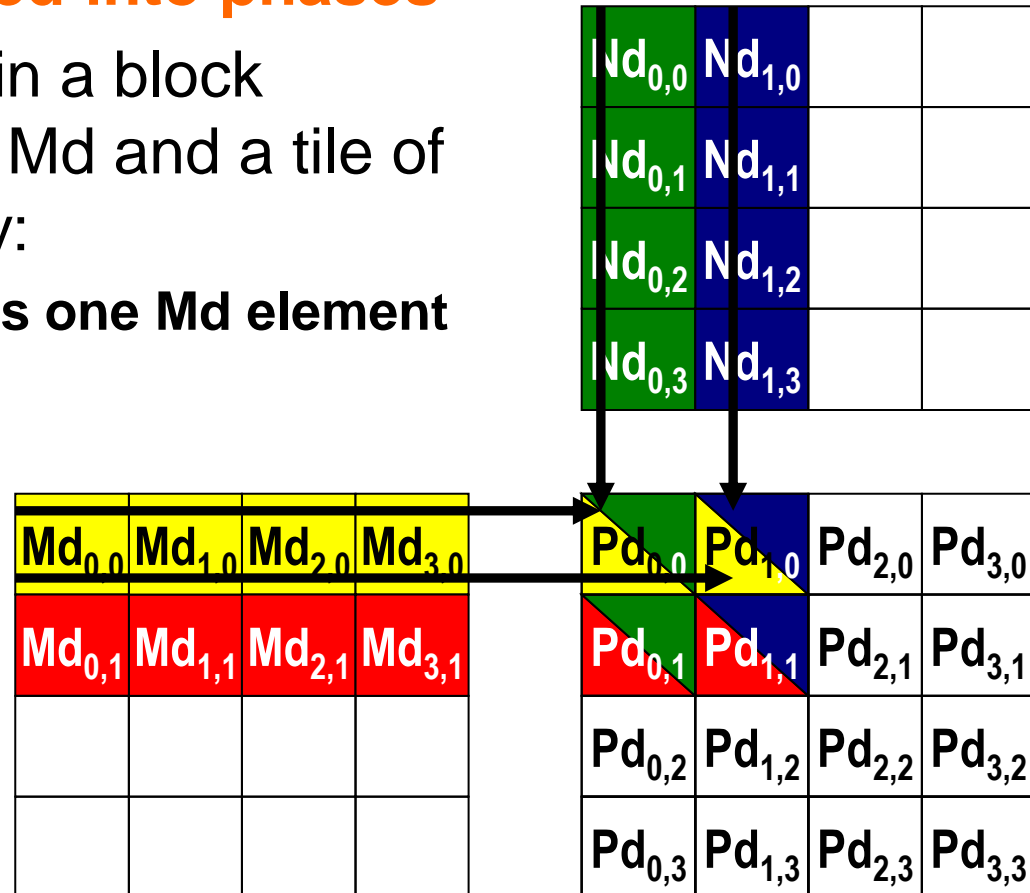


Matrix multiplication

- The multiplication algorithm where threads collaborate to **reduce the traffic to the global memory** exploits the **shared memory**
- Threads collaboratively load M_d and N_d elements into the **shared memory** before they individually use these elements in their dot product calculation
- The size of the **shared memory is quite small**
- To not exceed the capacity of the shared memory when loading M_d and N_d elements we consider M_d and N_d matrices divided into **tiles**

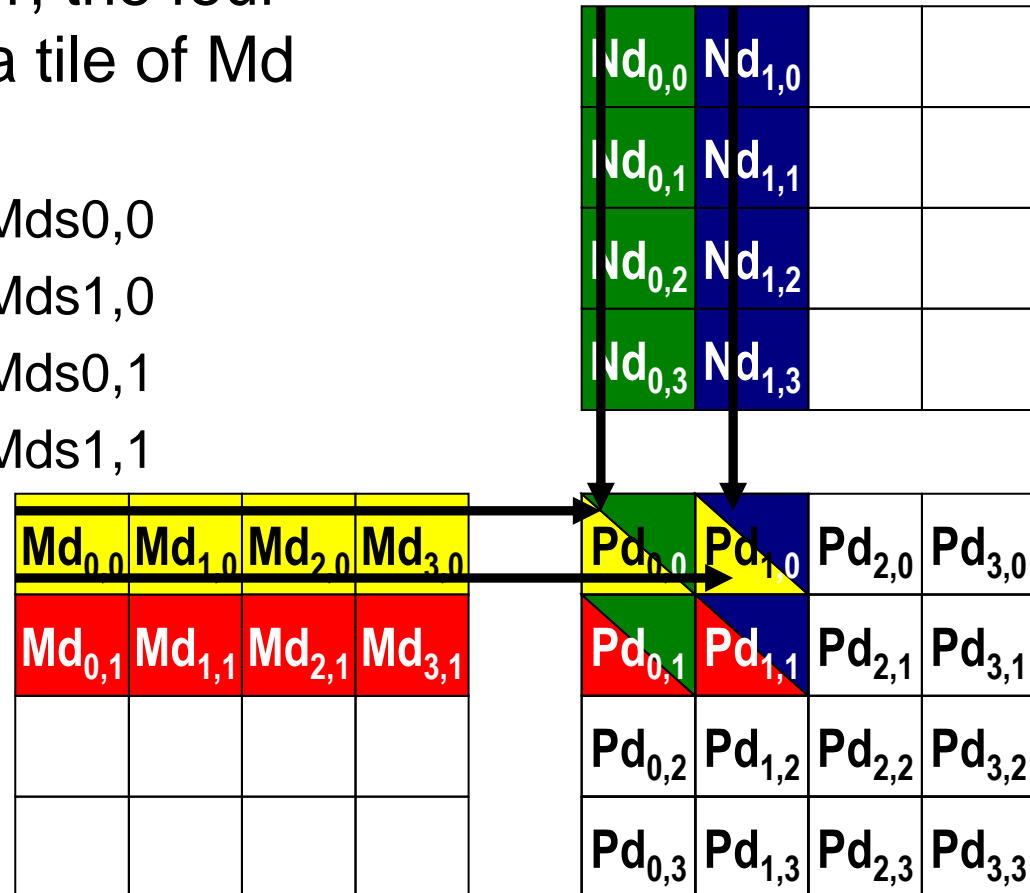
Matrix multiplication

- M_d and N_d divided into 2×2 tiles
- The **dot product calculations** performed by each thread are now **divided into phases**
- In each phase, all threads in a block collaborate to load a tile of M_d and a tile of N_d into the shared memory:
 - every thread in a block loads one M_d element and one N_d element



Matrix multiplication

- Activities of threads in block(0,0) (other blocks are the same)
- At the beginning of Phase 1, the four threads of block(0,0) load a tile of M_d into shared memory
 - thread(0,0) loads $M_{d0,0}$ into $M_{ds0,0}$
 - thread(1,0) loads $M_{d1,0}$ into $M_{ds1,0}$
 - thread(0,1) loads $M_{d0,1}$ into $M_{ds0,1}$
 - thread(1,1) loads $M_{d1,1}$ into $M_{ds1,1}$
- The shared memory array for the M_d elements is **M_{ds}** , and for the N_d elements is **N_{ds}**



Matrix multiplication

- The shared memory array for the **Md** elements is **Mds**, and for the **Nd** elements is **Nds**

	Phase 1			Phase 2		
$T_{0,0}$	Md _{0,0} ↓ Mds _{0,0}	Nd _{0,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{1,0} *Nds _{0,1}	Md _{2,0} ↓ Mds _{0,0}	Nd _{0,2} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{1,0} *Nds _{0,1}
$T_{1,0}$	Md _{1,0} ↓ Mds _{1,0}	Nd _{1,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{0,0} *Nds _{1,0} + Mds _{1,0} *Nds _{1,1}	Md _{3,0} ↓ Mds _{1,0}	Nd _{1,2} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{0,0} *Nds _{1,0} + Mds _{1,0} *Nds _{1,1}
$T_{0,1}$	Md _{0,1} ↓ Mds _{0,1}	Nd _{0,1} ↓ Nds _{0,1}	PdValue _{0,1} += Mds _{0,1} *Nds _{0,0} + Mds _{1,1} *Nds _{0,1}	Md _{2,1} ↓ Mds _{0,1}	Nd _{0,3} ↓ Nds _{0,1}	PdValue _{0,1} += Mds _{0,1} *Nds _{0,0} + Mds _{1,1} *Nds _{0,1}
$T_{1,1}$	Md _{1,1} ↓ Mds _{1,1}	Nd _{1,1} ↓ Nds _{1,1}	PdValue _{1,1} += Mds _{0,1} *Nds _{1,0} + Mds _{1,1} *Nds _{1,1}	Md _{3,1} ↓ Mds _{1,1}	Nd _{1,3} ↓ Nds _{1,1}	PdValue _{1,1} += Mds _{0,1} *Nds _{1,0} + Mds _{1,1} *Nds _{1,1}

time →

Matrix multiplication

	Phase 1		
$T_{0,0}$	Md _{0,0} ↓ Mds _{0,0}	Nd _{0,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{1,0} *Nds _{0,1}
$T_{1,0}$	Md _{1,0} ↓ Mds _{1,0}	Nd _{1,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{0,0} *Nds _{1,0} + Mds _{1,0} *Nds _{1,1}
$T_{0,1}$	Md _{0,1} ↓ Mds _{0,1}	Nd _{0,1} ↓ Nds _{0,1}	PdValue _{0,1} += Mds _{0,1} *Nds _{0,0} + Mds _{1,1} *Nds _{0,1}
$T_{1,1}$	Md _{1,1} ↓ Mds _{1,1}	Nd _{1,1} ↓ Nds _{1,1}	PdValue _{1,1} += Mds _{0,1} *Nds _{1,0} + Mds _{1,1} *Nds _{1,1}

- At the beginning of **Phase 1**, the four threads of block(0,0) load a tile of **Md** into the shared memory and a tile of **Nd**
- These values are used in the calculation of the dot product
- **Note that** each value in the shared memory is used **twice**

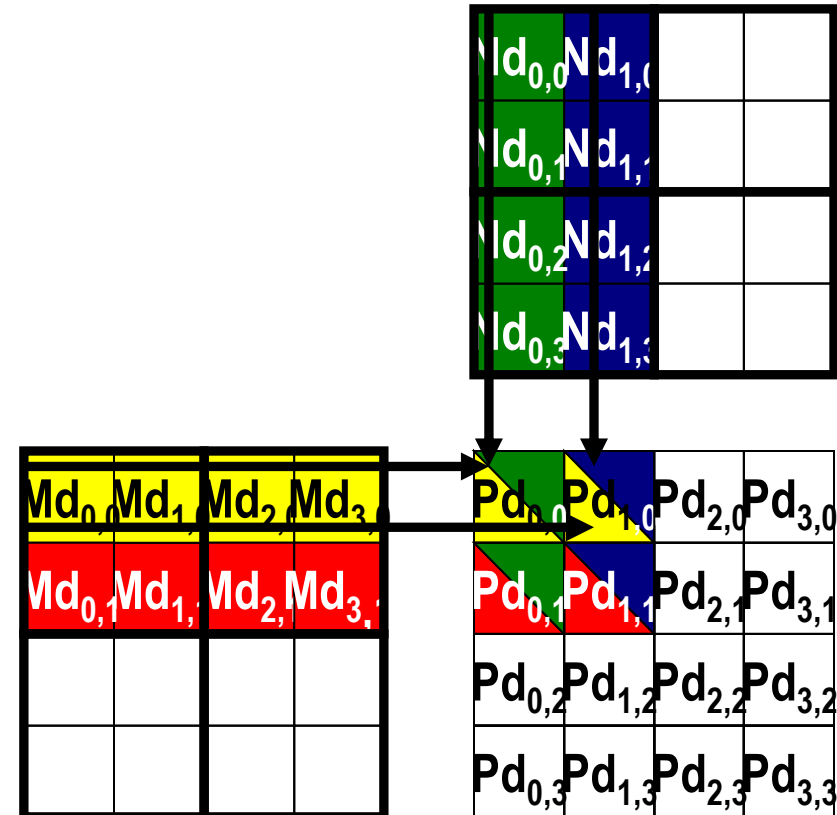
Example

- **Md**_{0,1} is loaded by thread_{0,1} into Mds_{0,1} and is used once by thread_{0,1} and once by thread_{1,1}
- **Nd**_{1,0} is loaded by thread_{1,0} into Nds_{1,0} and is used once by thread_{1,0} and once by thread_{1,1}

Matrix multiplication

	Phase2		
$T_{0,0}$	Md_{2,0} ↓ Mds _{0,0}	Nd_{0,2} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{1,0} *Nds _{0,1}
$T_{1,0}$	Md_{3,0} ↓ Mds _{1,0}	Nd_{1,2} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{0,0} *Nds _{1,0} + Mds _{1,0} *Nds _{1,1}
$T_{0,1}$	Md_{2,1} ↓ Mds _{0,1}	Nd_{0,3} ↓ Nds _{0,1}	PdValue _{0,1} += Mds _{0,1} *Nds _{0,0} + Mds _{1,1} *Nds _{0,1}
$T_{1,1}$	Md_{3,1} ↓ Mds _{1,1}	Nd_{1,3} ↓ Nds _{1,1}	PdValue _{1,1} += Mds _{0,1} *Nds _{1,0} + Mds _{1,1} *Nds _{1,1}

- **Phase 2** is similar and it allow to complete the computation
- **Note that** the two phases use the same Mds e Nds.



Matrix multiplication

Tiled matrix multiplication kernel using shared memories

```
__global__ void MatrixMulKernel(float* Md, float* Nd, float*
    Pd, int Width)
{
1.  __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];
2.  __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];

3.  int bx = blockIdx.x;  int by = blockIdx.y;
4.  int tx = threadIdx.x; int ty = threadIdx.y;

// Identify the row and column of the Pd element to work on
5.  int Row = by * TILE_WIDTH + ty;
6.  int Col = bx * TILE_WIDTH + tx;
```

Matrix multiplication

```
7.     float Pvalue = 0;
// Loop over the Md and Nd tiles required to compute
// the Pd element
8.     for (int m = 0; m < Width/TILE_WIDTH; ++m) {
// Collaborative loading of Md and Nd tiles into shared
// memory
9.         Mds[ty][tx] = Md[Row*Width+(m*TILE_WIDTH + tx)];
10.        Nds[ty][tx] = Nd[Col+(m*TILE_WIDTH + ty)*Width];
11.        __syncthreads();
12.        for (int k = 0; k < TILE_WIDTH; ++k)
13.            Pvalue += Mds[ty][k] * Nds[k][tx];
14.            syncthreads();
        }
15.    Pd[Row*Width+Col] = Pvalue;
}
```