Intensive Computation

Homework 2 - Direct and iterative methods for linear systems

20th March 2019

Exercise 1

- Write a function that implements the Gaussian elimination method.
- Call the function on a sparse matrix (represented in the standard way, without using a compact representation), having a given sparsity value.
- Use the command **`spy`** to visualize the sparsity pattern at each iteration of the Gaussian elimination algorithm and produce a movie to show how the final upper triangular matrix fills up.
- Compute the sparsity at each iteration and show it on a graph.

Exercise 2

- Write a function for each pivoting technique for the Gaussian elimination method (Gaussian Elimination with Partial Pivoting/ Complete pivoting/ Rook Pivoting), that is functions GEPP, GECP, GERP.
- Call each function on random generated matrices of size n>= 50, and compute the execution time (using the same matrix for the three techniques), averaging on a set of matrices large enough.

Exercise 3

- Write a function that implements the Jacobi iterative method for a sparse matrix represented using a compact format at your choice.
- Write a function that implements the Gauss-Seidel iterative method for a sparse matrix represented using a compact format at your choice.

Use the two following criteria for checking the convergence of the two methods:

- E1: Error obtained by using the exact values (found with Exercise 1 or Matlab function): $\mathrm{E1} = \left\| e^{(k)} \right\| = \left\| x - x^{(k)} \right\| < \varepsilon$
- E2: Difference between two successive iterations: $\mathrm{E2} = \left\| e^{(k)} \right\| = \left\| x^{(k)} - x^{(k-1)} \right\| < \varepsilon$

Show E1 and E2 on a graph (to compare the number of iterations needed to stop).

Sparse matrices collection

Sparse matrices can be downloaded from the *SuiteSparse Matrix Collection* at the link:
https://sparse.tamu.edu/

The SuiteSparse Matrix Collection (formerly known as the University of Florida Sparse Matrix Collection), is a large and actively growing set of sparse matrices that arise in real applications. The Collection is widely used by the numerical linear algebra community for the development and performance evaluation of sparse matrix algorithms.

You can search square matrices dor example filtering by keyword “linear systems” and set the constant vector *b* as a vector of ones (or zeros).