

INTRODUCTION TO MATLAB

Intensive Computation

2017-2018

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Introduction

- MATLAB stands for MATrix LABoratory
- MATLAB is a **high-level interpreted language** and **interactive environment** for numerical computation, data analysis, visualisation and algorithm development
- MATLAB enables you to perform computationally intensive tasks faster than with traditional programming languages such as C, C++ and Fortran

Introduction

- MATLAB started its life in the late 1970s as an interactive calculator built on top of LINPACK and EISPACK, which were then **state-of-the-art Fortran subroutine libraries for matrix computation**
- In the **80s Cleve Moler** write the first version of MATLAB to give his students at the University of New Mexico easy access to these libraries without writing Fortran
- Matlab has many specialized **toolboxes**

Matlab Screen

• Current Directory

- View folders and m-files

• Command History

- view past commands
- save a whole session using diary

• Command Window

- Type commands

• Workspace

- View program variables
- Double click on a variable to see it in the Array Editor

The screenshot displays the MATLAB R2017a interface. The top menu bar includes options like HOME, PLOTS, APPS, VARIABLE, and VIEW. Below the menu is a toolbar with icons for file operations (New, Open, Save), workspace management (New Variable, Open Variable), and code execution (Run and Time, Clear Commands). The main workspace is divided into several panels:

- Current Directory:** Shows a file explorer view of the current directory, listing folders like 'anaconda3', 'backup', 'Desktop', etc.
- Editor:** Displays a script named 'network_20170622.m' with a table of data. The table has columns numbered 1 to 13 and rows numbered 13 to 33. The data includes numerical values and 'NaN' entries.
- Variables:** Shows a list of variables in the workspace, including 'num' (224830x5 double) and 'txt' (224830x5 cell).
- Command Window:** Shows the MATLAB prompt 'R>>'.
- Workspace:** Shows a list of variables in the workspace, including 'num' (224830x5 double) and 'txt' (224830x5 cell).

Arrows from the text blocks point to specific elements in the screenshot:

- An arrow from 'Current Directory' points to the file explorer panel.
- An arrow from 'Command Window' points to the MATLAB prompt.
- An arrow from 'Workspace' points to the workspace panel.

Helpful commands

- **help** lists all the help topic – *the most important function to learn Matlab*
 - **help *name*** the help text for the functionality specified by *name*, such as a function, method, class, or toolbox
- **who/whos** show the current variables in the workspace
- **dir** list files in the current directory
- **clear all** delete all the variables present in the workspace
 - **clear *var1 var2*** clear variables *var1* and *var2*
- **lookfor** search for keyword in all help entries
 - **lookfor *topic***

Variables and expressions

- In the Command window, the command **prompt** is " >> " Examples:
 - Two types of statement:
 - **evaluation of an expression**
" >> *expression*"
 - **assignment** " >> *variable = expression*"
 - The evaluation of an expression generates a **matrix** assigned to the specified **variable**
 - If you do not specify the name of the *variable* associated to the result, the system "**ans**" is used
- >> 8+2
ans =
10
 - >> a = 5*ans
a =
50
 - >> 6.9
ans =
6.9000

Variables and expressions

- If an expression ends with symbol “;” its value is not displayed on the screen
- MATLAB names are **case-sensitive**
- **No need to declare variables**
- **No need for types**
- Built-in variables. Don't use these names!
 - **i** and **j** can be used to indicate complex numbers
 - **pi** has the value 3.1415926...
 - **ans** stores the last unassigned value (like on a calculator)
 - **Inf** and **-Inf** are positive and negative infinity
 - **NaN** represents 'Not a Number'

Examples:

```
» b = 6+a;
```

```
» b
```

```
b =
```

```
56
```

Variables and expressions

- All variables are created with **double precision**
- The **variables are 1x1 matrices** with double precision
- Double precision values consist of **8 bytes**

- The default display format for variables is 5-digit scaled, fixed-point values
- We can ask for different display formats with command **format**
- The **format** function affects only how numbers display in the Command Window, **not how MATLAB computes or saves them**

The command `FORMAT`

Command `format` changes the display format to the specified *style*

Let us consider $x = 4/3$

- `format short` 1.3333 0.0000 - 5-digit scaled, fixed-point *default*
- `format long` 1.333333333333333 - 15-digit fixed point
- `format short e` 1.3333e+000 - 5-digit floating point
- `format long e` 1.333333333333333e+000 - 15-digit floating point
- `format short g` 1.3333 – best between fixed point and floating point
- `format long g` 1.333333333333333 – best between fixed and floating pt
- `format bank` 1.33 – currency format (dollar or euro)
- `format rat` 4/3 - ratio of small integers
- `format hex` 3ff5555555555555 - hexadecimal (double-precision)

Double precision values

- Only a number of double precision values can be represented
- There is always a **small gap** between two consecutive values
- The command **eps** provides the floating-point relative accuracy
- **eps** returns the distance from 1.0 to the next largest double-precision number, that is $\text{eps} = 2^{-52}$
- **eps(x)** is the positive distance from $\text{abs}(X)$ to the next larger in magnitude floating point number of the same precision as X
- **realmin** returns the smallest positive normalized floating-point number in IEEE double precision about **2.2251e-308** that is 2^{-1022}
- **realmax** returns the largest finite floating-point number in IEEE double precision, about **1.7976e+308** that is 2^{1023}

Matrices

- The simplest way to create a matrix is to use the matrix constructor operator `[]`
- Create a row in the matrix by entering elements within the brackets
- Separate **row elements** with a **comma** or **space**
- For a **new row**, terminate the current row with a **semicolon** or **return**

```
» A = [7 8; 8.9 7; 9 8]
```

```
A =
```

```
7.0000    8.0000
8.9000    7.0000
9.0000    8.0000
```

```
» B = [1 2 3
4 5 6]
```

```
B =
```

```
1 2 3
4 5 6
```

Matrices

- Examples of **functions** for creating **different kinds of matrices**
 - **zeros(n,m)** matrix $n \times m$ of all zeros
 - **ones(n,m)** matrix $n \times m$ of all ones
 - **eye(n,m)** matrix with ones on the diagonal (zeros elsewhere)
 - **rand(n,m)** matrix of uniformly distributed random numbers
 - **diag([a11, a22, a33, ..., aNN])** diagonal matrix
 -

Matrices

- Increase matrices by adding a row or a column having the correct size
- **Column**
 - Given $A = [1 \ 2; 3 \ 4; 5 \ 6]$;
 - Add the column of elements 7 8 9

$A = [A [7; 8; 9]]$ oppure $A = [A [7 \ 8 \ 9]']$

1 2		1 2 7
3 4	→	3 4 8
5 6		5 6 9

Matrices

To access elements of a matrix → matrices' name followed by round brackets containing a reference to the row and column number

- » `A = [7 8; 8.9 7; 9 8]`

`A =`

```
7.0000    8.0000
8.9000    7.0000
9.0000    8.0000
```

- `A(n,m)` access **element** (n,m) of matrix A

» `A(1,2)`

`ans =`

`8`

Note that elements of the matrix are displayed as 5-digit values

Matrices

The colon operator

- The colon operator (**first:last**) generates a 1-by-n matrix (or *vector*) of sequential numbers from the first value to the last

- The default sequence is made up of values **incrementing by 1**

A = 10:15 → **A = 10 11 12 13 14 15**

- The numeric sequence can include negative and fractional numbers

A = -2.5:2.5 → **A = -2.5000 -1.5000 -0.5000 0.5000 1.5000 2.5000**

Matrices

The colon operator

- You can also specify a **step** value with the colon operator in between the starting and ending value (**first:step:last**).

- To generate a series of numbers from 10 to 50 incrementing by 5:

A = 10:5:50 **→** **A = 10 15 20 25 30 35 40 45 50**

- You can increment by **noninteger** values

A = 3:0.2:3.8 **→** **A = 3.0000 3.2000 3.4000 3.6000 3.8000**

- You can **decrement**, specifying a negative step value:

A = 9:-1:1 **→** **A = 9 8 7 6 5 4 3 2 1**

Matrices

Accessing matrix rows or matrix columns

- $A(n,:)$ extracts **row n** of matrix A

» $A(2,:)$

ans =

8.9000 7.0000

- $A(:,m)$ extracts **column m** of matrix A

» $A(:,1)$

ans =

7.0000

8.9000

9.0000

The colon notation “:” allows to specify a sequence of values

The whole row (column) is extracted because the interval is not specified

Matrices

`diag(A)`

- If A is a square matrix, `diag(A)` returns the main diagonal of A

```
» A=[5 6 ; 7 8]
```

```
A =
```

```
5 6
```

```
7 8
```

```
» diag(A)
```

```
ans =
```

```
5
```

```
8
```

- If A is a vector with n components, returns an n-by-n diagonal matrix having A as its main diagonal

```
» diag(ans)
```

```
ans =
```

```
5 0
```

```
0 8
```

Matrices

- **sum(A)**
- If A is a **vector**, then sum(A) returns the sum of the elements
 - » **sum(A)**
 - ans =**
 - 36**
- If A is a **matrix**, then sum(A) treats the columns of A as vectors and returns a row vector whose elements are the sums of each column

```
» A=[0 1 2 ;3 4 5 ;6 7 8 ]
```

```
A =
```

```
0 1 2  
3 4 5  
6 7 8
```

```
» B=sum(A)
```

```
B =
```

```
9 12 15
```

Vectors

- A matrix with only one row or column (that is, a **1-by-n** or **n-by-1** array) is a **vector**, such as:

$C = [1, 2, 3]$ row vector

$D = [10; 20; 30]$ column vector

- An array can be created with the colon operator

$x = 1:6$ \rightarrow $x = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$x = 0.5:0.1:0.7$ \rightarrow $x = 0.5000 \quad 0.6000 \quad 0.7000$

Vectors

- A vector can be created by using **linspace(a,b)** or **linspace(a,b,N)** that generates vectors of (N) points linearly spaced between and including a and b

x = linspace(-1,1) → -1 0 1

x = linspace(-1,1,4) → -1.0000 -0.3333 0.3333 1.0000

- The logspace functions – **logspace(a,b)** or **logspace(a,b,N)** – generate *logarithmically spaced vectors*
- The logspace function is useful for creating frequency vectors
- It is a logarithmic equivalent of linspace and the ":" or colon operator

Vector Indexing

- **IMPORTANT:** MATLAB indexing starts with **1, not 0**
- The **index argument can be a vector**
- In this case, each element is looked up individually, and returned as a vector of the same size as the index vector

```
»x=[12 13 5 8];
```

```
»a=x(2:3);           →      a=[13 5];
```

```
»b=x(1:end-1);      →      b=[12 13 5];
```

Matrix Indexing

- Matrices can be indexed in two ways
 - using subscripts(row and column)
 - using linear indices(as if matrix is a vector)
- Matrix indexing: **subscripts** or **linear indices**

$$\begin{array}{l}
 b(1,1) \rightarrow \begin{bmatrix} 14 & 32 \\ 11 & 81 \end{bmatrix} \leftarrow b(1,2) \\
 b(2,1) \rightarrow \begin{bmatrix} 14 & 32 \\ 11 & 81 \end{bmatrix} \leftarrow b(2,2)
 \end{array}$$

$$\begin{array}{l}
 b(1) \rightarrow \begin{bmatrix} 14 & 32 \\ 11 & 81 \end{bmatrix} \leftarrow b(3) \\
 b(2) \rightarrow \begin{bmatrix} 14 & 32 \\ 11 & 81 \end{bmatrix} \leftarrow b(4)
 \end{array}$$

Picking submatrices

»A = rand(5)

»A(1:3,1:2)

»A([1 5 3], [1 4])

% shorthand for 5x5 matrix

% specify contiguous submatrix

% specify rows and columns 143398

Matrix Indexing

- MATLAB contains functions to help you find desired values within a vector or matrix
 - » **vec = [5 3 1 9 7]**
- To get the minimum value and its index:
 - » **[minVal,minInd] = min(vec);**
- Max works the same way

- To find any the indices of specific values or ranges
 - » **ind = find(vec == 9);**
 - » **ind = find(vec > 2 & vec < 6);**
- To convert between subscripts and indices, use **ind2sub** and **sub2ind**

Scalar operators and functions

- Mathematical operators on scalars
add +, subtract -, divide /, multiply *, power ^
- Trigonometric function
 - sin, cos
 - tan
 - asin, acos
 - atan

The list of ***elementary math functions***

- help **elfun**: trigonometric, exponential, complex, rounding and remainder

The list of ***specialized math functions***

- help **specfun**: specialized, number theoretic, coordinate transforms

Scalar operators and functions

- Some mathematical operators on scalars:
 - **abs** Absolute value and complex magnitude
 - **conj** Complex conjugate
 - **real, imag** Real and Imaginary part of complex number
 - **exp** Exponential
 - **log, log10** Natural and base 10 logarithm
 - **sqrt** Square root
 - **ceil** Round toward positive infinity
 - **floor** Round toward negative infinity
 - **round** Round to nearest integer
- Variables **i** and **j** are both functions denoting the **imaginary unit** and are the square-root of -1

Matrix operations

Matrix operations:

- + **addition** of vectors or matrices (element-by-element)
- - **subtraction** of vectors or matrices (element-by-element)
- * **multiplication** of vectors or matrices (row-by-column)

Note that:

- **addition / subtraction**: matrices with the same number of rows and columns
- **addition / subtraction** with a **scalar**: the scalar is added/subtracted to each element of the matrix
- **multiplication**: the number of columns in the first matrix must be the same as the number of rows in the second matrix

Matrix operations

Matlab has a set of **dot operators**, a dot and a normal algebraic operator, performing element-wise algebraic operations on a matrix

- `.*` element-wise product
- `./` element-wise division
- `.^` element-wise power

\ and / operators for the **solution of linear systems**:

- $x = B/A$ is the solution of the equation $x*A = B$
- $x = A\B$ denote the solution to the equation $A*x = B$

Systems of Linear Equations

- Given a system of linear equations

$$x+2y-3z=5$$

$$-3x-y+z=-8$$

$$x-y+z=0$$

- Construct matrices so the system is described by $Ax=b$

$$\gg A=[1 \ 2 \ -3; -3 \ -1 \ 1; 1 \ -1 \ 1];$$

$$\gg b=[5; -8; 0];$$

- And solve with a single line of code!

$$\gg x=A\b;$$

- x is a 3×1 vector containing the values of x , y , and z
- The **** will work with square or rectangular systems
 - Gives least squares solution for rectangular systems

Matrix functions

- **Matrix functions:**
 - Transpose matrix **A'**
 - Inverse matrix **inv(A)**
 - Matrix determinant **det(A)**
 - Eigenvalues **eig(A)**
 - Rank of matrix **rank(A)**
 - Dimensions **size(A)**

The list of elementary matrices and matrix manipulation

- help **elmat**: elementary matrices, basic array information, matrix manipulation, special variables e costants, specialized matrices, ...

MATLAB Programming

Script and Function

- The simplest type of MATLAB program is called a *script*
- A script is a file that contains multiple sequential lines of MATLAB commands and function calls
- You can run a script by typing its name at the command line
- **Script** and **Function** are **M-files** with a **.m extension**
- **Scripts**
 - have no input or output arguments
 - use workspace data
- **Functions**
 - accept input arguments and produce output
 - have their own workspace, separate from the base workspace
 - function variables are local

MATLAB Programming

You can:

- Add **comments** to code using the percent symbol `%`.
- Create **help text** by inserting **comments at the beginning** of your program.
- Help text appears in the Command Window when you use the help function → **help *ProgramName***
- If your program includes a **function**, position the help text immediately below the function definition line (the line with the *function keyword*)

MATLAB Programming

Function - The definition statement is the first executable line

Each function definition includes:

- **function** keyword (*required*) (lowercase characters)
- Output arguments (*optional*)
 - `function output= myfunction(x)`
 - `function [one,two,three] = myfunction(x)`
 - `function myfun(x) or function []=myfunction(x)`
- Function name (*required*)
- Input arguments (*optional*)
 - `function y = myfunction(one,two,three)`

Remark: use the same name for both the file and the function

MATLAB Programming

Example

```
% mean computes the  
% mean of a random  
% values array and the  
% mean among the  
% minimum and maximum  
v=rand(50,1)  
mean=valmean(v)  
meanmm=minmax(v)
```

```
function m=valmean(v)  
  
n=length(v)  
m=sum(v)/n  
  
function mm=minmax(v)  
  
mini=min(v)  
maxi=max(v)  
mm=(mini+maxi)/2
```

Relational and logical operators

The **relational operators** are:

- **<**, **>**, **<=**, **>=**, **==**, and **~=**

Relational operators perform element-by-element comparisons between two arrays

They return a logical array of the same size, with elements set to:

- logical **1** (true) where the relation is true
- logical **0** (false) where the relation is false

The **logical operators** are:

- **&** (and), **|** (or), **~** (not)
- **xor** (xor), **all** (all true), **any** (any true)

Relational and logical operators

- **Examples**

```
>> a=10; b=3; c=25;
```

```
>> a==b
```

```
ans=
```

```
0
```

```
>> a>b
```

```
ans=
```

```
1
```

```
>> a+b > c
```

```
ans=
```

```
0
```

Programming: loop control

With *loop control statements*, you can repeatedly execute a block of code

for statements loop a specific number of times, and keep track of each iteration with an incrementing index variable

- **for index=starting value:increment:final value**
 program statements
end

Remark *indent* the loops for readability, especially when they are nested

Programming: loop control

- **Example**

```
x = ones(1,10);  
for n = 2:10  
    x(n) = 2 * x(n - 1);  
end
```

- **Example**

```
for i=1:m  
    for j=1:n  
        H(i,j)=1/(i+j-1);  
    end  
end
```

Programming: loop control

while repeatedly executes one or more program statements in a loop as long as an expression remains true

```
while expression  
    statements  
end
```

- Expressions can include *relational operators* (such as < or ==) and *logical operators* (such as &&, ||, or ~)
- To programmatically **exit the loop**, use a **break** statement
- To skip the rest of the instructions in the loop and begin the next iteration, use a **continue** statement

Programming: loop control

Examples

- `x = 3.;`
`while x < 25`
 `x = x + 2`
`end`
- Fibonacci
`a(1)=1; a(2)=1; c=15;`
`n=2;`
`while a(n) < c`
 `a(n+1) = a(n) + a(n-1);`
 `n=n+1;`
`end`

Programming: loop control

- ***if expression, statements, end***
evaluates an expression, and executes the statements when the expression is true
- ***elseif*** and ***else*** are optional, and execute statements only when previous expressions in the if block are false
- An *if block* can include multiple ***elseif*** statements

```
if expression  
    statements  
elseif expression  
    statements  
else  
    statements  
end
```

Programming: loop control

Example

```
if x > 0
    y = sqrt(x);
elseif x == 0
    y = 0;
else
    y = NaN;
    disp('y undefined')
end
```

Programming: loop control

switch case otherwise

Switch among several cases based on expression

```
switch switch_expr
case case_expr
    statements
case {case_expr1,case_expr2,case_expr3,...}
    statements
...
otherwise
    statements
end
```

Programming: loop control

Example

```
name='rose';  
switch name  
case 'rose'  
    disp('the flower is a rose')  
case 'tulip'  
    disp('the flower is a tulip')  
case 'daisy'  
    disp('the flower is a daisy')  
otherwise  
    disp('it's a flower')  
end
```

Strings

- **strcat** *Concatenate strings*

`t = strcat(s1, s2, s3, ...)` horizontally concatenates corresponding rows of the character arrays **s1**, **s2**, **s3** etc.

All input arrays must have the same number of rows (or any can be a single string). When the inputs are all character arrays, the output is also a character array

- **strcmp** *Compare strings*

`tf = strcmp(s1, s2)` compares the strings **s1** and **s2** and returns logical 1 (true) if they are identical, and 0 (false) otherwise

- **strfind** *Find one string within another*

`k = strfind(text, pattern)` returns the starting indices of any occurrences of the string **pattern** in the string **text**

Advanced Data Structures

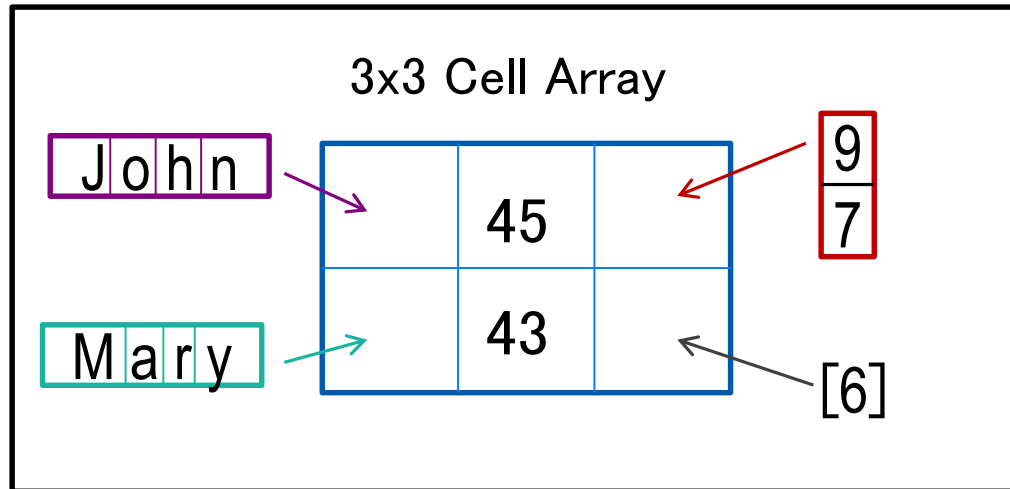
- We have used 2D matrices
 - Can have n-dimensions
 - Every element must be the same type (ex. integers, doubles, characters...)
 - Matrices are space-efficient and convenient for calculation

Sometimes, more complex data structures are more appropriate

- **Cell array**: it's like an array, but elements don't have to be the same type
- **Structs**: can bundle variable names and values into one structure

Cell

- A cell is just like a matrix, but each field can contain anything (even other matrices):



- One cell can contain people's names, ages, and the ages of their children

Cell

- To initialize a cell, specify the size

```
»a=cell(3,10);
```

- a will be a cell with 3 rows and 10 columns

- or do it manually, with curly braces {}

```
»c={'hello world',[1 5 6 2],rand(3,2)};
```

- c is a cell with 1 row and 3 columns

- Each element of a cell can be anything

- To access a cell element, use curly braces {}

```
»a{1,1}=[1 3 4 -10];
```

```
»a{2,1}='hello world 2';
```

```
»a{1,2}=c{3};
```


Structs

- **Structs** allow you to name and bundle relevant variables
 - Like C-structs, which are objects with fields
- To **initialize** an empty struct:
 - » **s=struct;**
 - size(s) will be 1x1
 - initialization is optional but is recommended when using large structs
- To add fields:
 - » **s.name = 'Jack Bauer';**
 - » **s.scores = [95 98 67];**
 - » **s.year = 'G3';**
 - **Fields can be anything:** *matrix*, *cell*, even *struct*
 - Useful for keeping variables together

Structs

- To initialize a struct array, give field, values pairs

```
» ppl=struct('name',{'John','Mary','Leo'},...
'age',{32,27,18},'childAge',{[2;4],1,[]});
```

- `size(s2)=1x3`
- every cell must have the same size

```
» person=ppl(2);
```

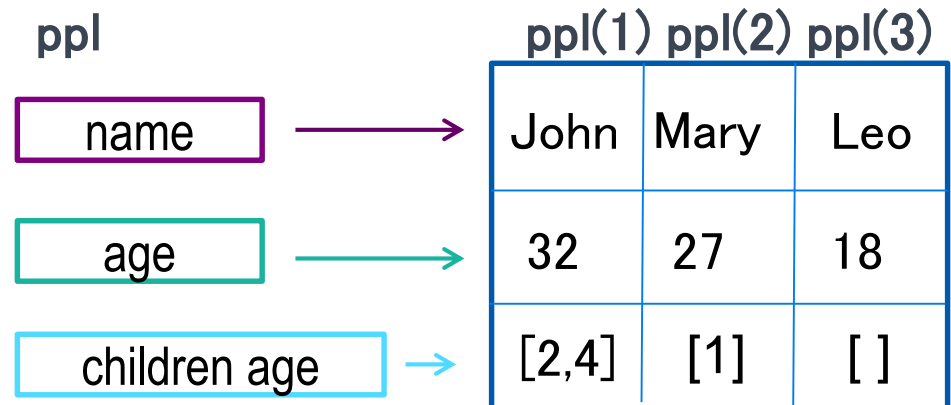
- person is now a struct with fields name, age, children
- the values of the fields are the second index into each cell

```
» person.name
```

- returns 'Mary'

```
» ppl(1).age
```

- returns 32



Structs

- To access 1x1 struct fields, give name of the field

»**stu=s.name;**

»**scor=s.scores;**

- 1x1 structs are useful when passing many variables to a function. put them all in a struct, and pass the struct

- To access nx1 struct arrays, use indices

»**person=ppl(2);**

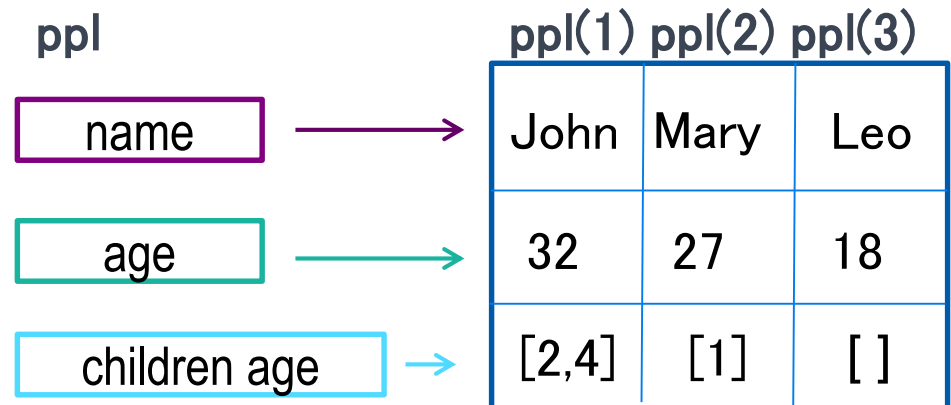
- person is a struct with name, age, and child age

»**personName=ppl(2).name;**

- personName is 'Mary'

a=[ppl.age];

- a is a 1x3 vector of the ages



Polynomial

- A polynomial is represented by an array containing the coefficients of the polynomial in descending powers of the polynomial decreasing order
- The polynomial $3x^3 + 2x + 8$ can be represented as:

```
» pol = [3 0 2 8]
```
- To evaluate a polynomial in x , where x can be a vector, you can use `polyval(p, x)` where p is the polynomial

```
» polyval(pol, 1)
ans =
    13
```

Polynomial

- `roots` computes the roots of the polynomial
- `r=roots (p)` returns a column vector whose elements are the roots of the polynomial `p`
- Row vector `p` contains the coefficients of the polynomial
- Example: the polynomial $x^3 - 6x^2 + 11x - 6$

```
» p= [1 -6 11 -6]; format long;
```

```
» roots (p)
```

```
ans =
```

```
3.0000000000000000
```

```
3.0000000000000000
```

```
3.0000000000000000
```

Polynomial

Remark There are some complications with **multiple roots**

The polynomial r^3+3r^2+3r+1 have just one root $r = -1$, but

```
roots([1 3 3 1])
```

returns three different (though close) values

```
ans =
```

```
-1.00000913968880
```

```
-0.99999543015560 + 0.00000791513186i
```

```
-0.99999543015560 - 0.00000791513186i
```

Even worse for $p(x)=(x+1)^7$ (coefficients [1 7 21 35 35 21 7 1])

Polynomial

Operations with polynomials

- $\mathbf{p} = \text{conv}(\mathbf{u}, \mathbf{v})$ multiplication of the polynomials whose coefficients are the elements of \mathbf{u} and \mathbf{v}
- $[\mathbf{q}, \mathbf{r}] = \text{deconv}(\mathbf{u}, \mathbf{v})$ polynomial division - the quotient is returned in vector \mathbf{q} and the remainder in vector \mathbf{r} such that $\mathbf{v} = \text{conv}(\mathbf{u}, \mathbf{q}) + \mathbf{r}$
- $\mathbf{p} = \text{polyfit}(\mathbf{x}, \mathbf{y}, \mathbf{n})$ finds the coefficients of a polynomial $\mathbf{p}(\mathbf{x})$ of degree \mathbf{n} that fits the data, $\mathbf{p}(\mathbf{x}(\mathbf{i}))$ to $\mathbf{y}(\mathbf{i})$, in a least squares sense. The result \mathbf{p} is a row vector of length $\mathbf{n}+1$ containing the polynomial coefficients in descending powers

Polynomial

- `poly` gives the polynomial with specified roots
- `p=roots(r)` where `r` is a vector, returns a row vector whose elements are the coefficients of the polynomial whose roots are the elements of `r`
- `p=roots(A)` where `A` is an n-by-n matrix, returns an n+1 element row vector whose elements are the coefficients of the characteristic polynomial, $\det(\lambda I - A)$

Remark `poly(A)` generates the characteristic polynomial of `A`, and `roots(poly(A))` finds the roots of that polynomial, which are the **eigenvalues** of `A`

Plotting

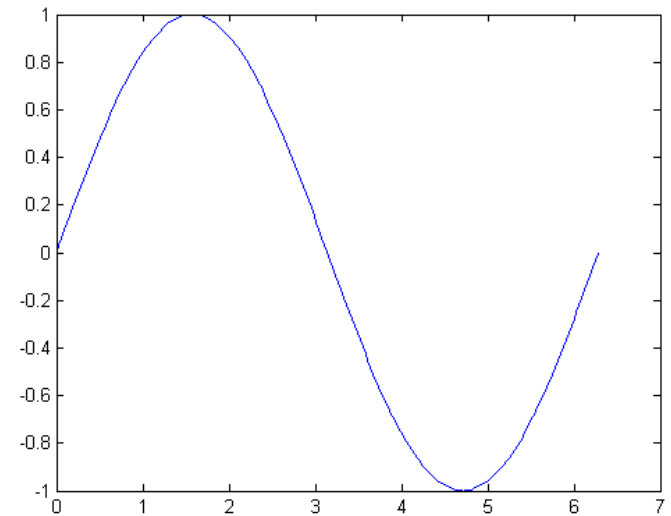
The function `plot` creates a 2D line plot - it can be used in different ways

- **Example**

```
» n = 31  
» x = linspace(0,2*pi,n)  
» y = sin(x)  
» plot(x,y)
```

x is a vector of linearly spaced values between 0 and 2π

y is the vector of values of sine function evaluated at the values in x



Plotting

- Command **plot** is:

- **plot(X, Y, options)**

Where **X** is for abscissas and **Y** is for ordinates

options sets the *line style*, *marker symbol*, and *color*

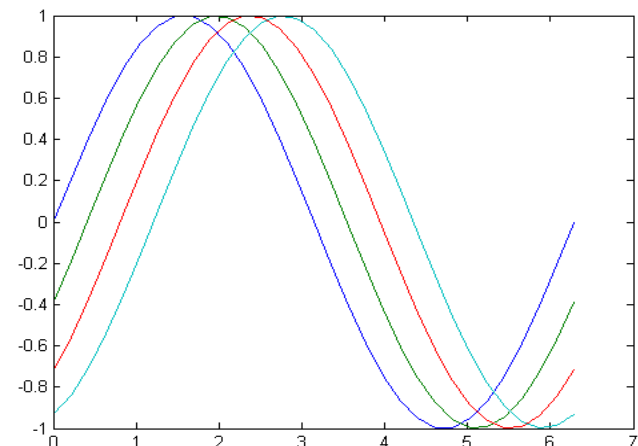
- To plot **multiple lines** in the same windows, we can use two ways:

```
y2 = sin(x - .4) ;
```

```
y3 = sin(x - .8) ;
```

```
y4 = sin(x - 1.2) ;
```

- **plot(x, y, x, y2, x, y3, x, y4)**
- **plot(x, [y; y2; y3; y4])**



Plotting

- Another way to plot **multiple line** in the same window is by using commands **hold on** and **hold off**:

```
» x = linspace(0,2*pi)
```

```
» y1 = cos(x)
```

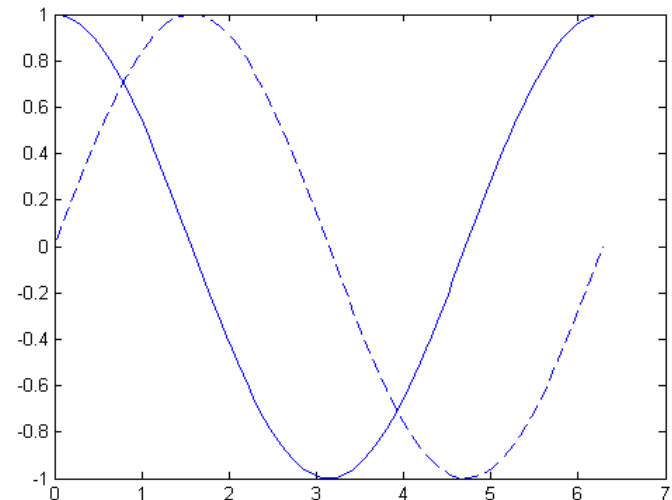
```
» y2 = sin(x)
```

```
» plot(x,y1,'-')
```

```
» hold on
```

```
» plot(x,y2,'--')
```

```
» hold off
```



Plotting

- You can add a **title** and **axis labels** to the graph
 - » `title('title of the graph')`
 - » `xlabel('x axis')`
 - » `ylabel('y axis')`
- **axis** - axis scaling and appearance
- **legend** - graph legend
- **text** - create text object in current axes
 - » `text(x(70)+0.5,r(70),'r = -2x')`
- **grid on** add grid lines for 2D and 3D plots

Plotting

Other functions for graphs are:

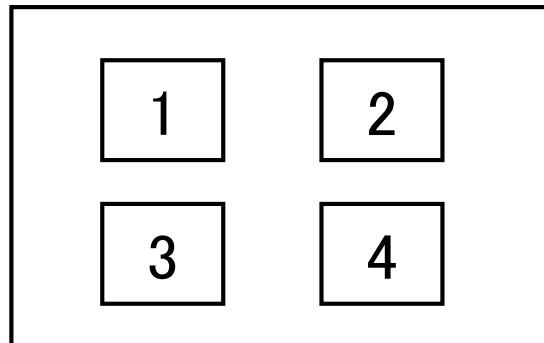
- **loglog** Log-log scale plot
- **semilogx** Semilogarithmic plot (x logarithmic, y linear)
- **semilogy** Semilogarithmic plot (x linear, y logarithmic)
- **errorbar** Plot error bars along curve
- **bar** Bar graph
- **stairs** Stairstep graph
- **scatter** Scatter plot

Plotting

subplot divides the current figure into **grid**, it numbers the cells by rows

» **subplot(m,n,p)**

divides the current figure into an **m-by-n** grid and plots in the **grid position** specified by p



Plotting

`fplot(fun, lims)` plots a function

- **fun**, that must be *a string*
- between the limits specified by **lims**, specifying the *x-axis limits* ([xmin xmax]), or the *x- and y-axes limits*, ([xmin xmax ymin ymax])

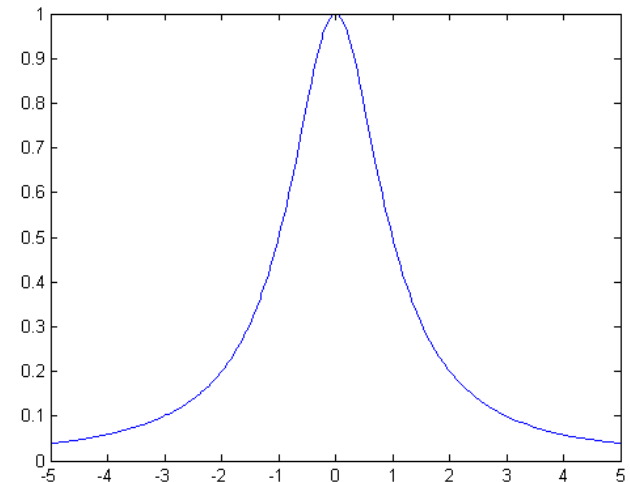
```
» fun = '1/(1+x^2)';
```

```
» lims = [-5, 5];
```

```
» fplot(fun, lims);
```

or the equivalent

```
» fplot('1/(1+x^2)', [-5, 5])
```



Plotting

- `fplot(fun, limits, LineSpec)` plots `fun` using the line specification *LineSpec*

```
fplot(fun, lims, '- -')
```

```
fplot(fun, lims, 'r -')
```

- `fplot` can plot a vector of functions

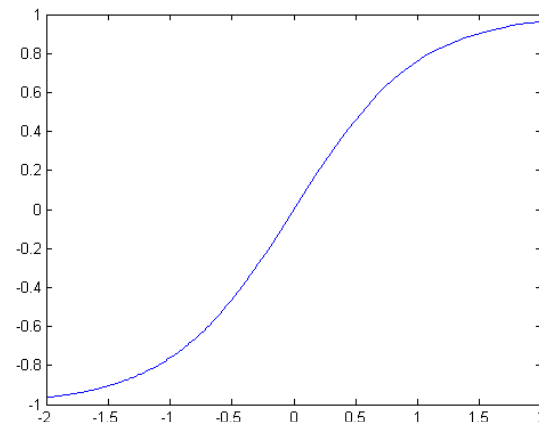
```
fplot('[sin(t), sin(t-.25), sin(t-.5)]', [0, 2*pi])
```


Plotting

- **ezplot** plots the expression $\text{fun}(x)$ over the default domain $-2\pi < x < 2\pi$, where $\text{fun}(x)$ is an explicit function of only x
- **ezplot(fun, [xmin, xmax])** plots $\text{fun}(x)$ over the domain: $x_{\min} < x < x_{\max}$
- Both for **fplot** and **ezplot** **fun** can be a **function handle**

```
fh = @tanh;
```

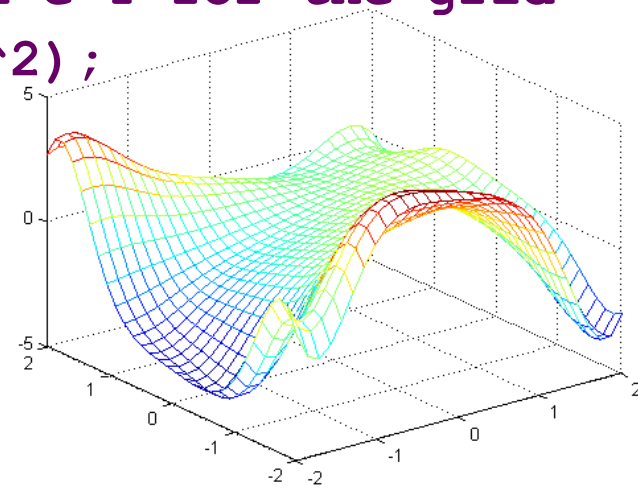
```
fplot(fh, [-2, 2])
```



Plotting

3D plot with **mesh** and **surf**

- **mesh** and **surf** plot a surface
- **mesh** and **surf** create 3D surface plots of matrix data generated by the command **meshgrid**
 - » `n=30; m=n;`
 - » `x=linspace(-2,2,n);`
 - » `y=linspace(-2,2,n);`
 - » `[X,Y]=meshgrid(x,y); % matrices X e Y for the grid`
 - » `Z=(1-Y).*cos(X.^2)+(X-1).*cos(Y.^2);`
 - » `mesh(X,Y,Z);`



Data and file management

You can load variables from file into workspace with **load**

For example if you want analyze data coming from a program, like the following, that are in the file data.dat

```
1      0.2000      -5
2      0.2500      -9
3      0.0740     -23
4      0.0310     -53
5      0.0160    -105
6      0.0090    -185
7      0.0050    -299
8      0.0030    -453
9      0.0020    -653
10     0.0020    -905
```

Data and file management

If you load these data with the function **load**, a matrix is created of size 10x3

```
>> load data.dat
```

```
>> whos
```

```
Name Size Bytes Class
```

```
data 10x3 240 double array
```

```
Grand total is 30 elements using 240 bytes
```

load filename is the command form

load 'filename' is the function form

Data and file management

```
>> M = load('data.dat')  
M =  
1.0000    2.0000   -5.0000  
2.0000    0.2500   -9.0000  
3.0000    0.0740  -23.0000  
4.0000    0.0310  -53.0000  
5.0000    0.0160 -105.0000  
6.0000    0.0090 -185.0000  
7.0000    0.0050 -299.0000  
8.0000    0.0030 -453.0000  
9.0000    0.0020 -653.0000  
10.0000   0.0020 -905.0000
```

Data and file management

save save workspace variables to file

- **save (filename)**

saves all variables from the current workspace in a formatted binary file (MAT-file) called *filename*

if *filename* is not specified the file **Matlab.mat** is created

- **save (filename, variables)**

saves only the variables or fields of a structure array specified by variables

- **save (filename, variables, fmt)**

saves in the file format specified by *fmt* - *variables* is optional

Data and file management

Example

```
% mytable.m
n=input('Insert the number of values n:');
x=linspace(0,pi,n);
s=sin(x);
c=cos(x);
v=(1:n);
save mytable.dat v x s c -ascii
```

Data and file management

Example

To visualize the table saved in the previous example with save we can load the file and display the table

```
% viewtable.m
load mytable.dat
A=mytable;
disp('-----');
fprintf('k\t x(k)\t sin(x(k))\t cos(x(k))\n');
disp('-----');
fprintf('%d\t %3.2f\t %8.5f\t %8.5f\n',A);
```


Data and file management

dir List directory

dir *directory_name* or **dir(' *directory_name* ')** lists the files in a directory -- Pathnames and wildcards may be used

dir *.m lists all the M-files in the current directory

D = dir(' *directory_name* ') returns the results in an M-by-1 **structure** with the fields:

name -- filename

date -- modification date

bytes -- number of bytes allocated to the file

isdir -- 1 if name is a directory and 0 if not

datenum -- modification date as a MATLAB serial date number

Improving performance

Techniques for Improving Performance

- **Preallocating Arrays**

- **for** and **while** loops that incrementally increase the size of a data structure each time through the loop can adversely affect performance and memory use
- resizing arrays often requires MATLAB to spend extra time looking for larger contiguous blocks of memory, and then moving the array into those blocks
- you can improve code execution time by **preallocating** the maximum amount of space required for the array

Improving performance

Techniques for Improving Performance

- **Preallocating a Nondouble Matrix**

- When you preallocate a block of memory to hold a matrix of some type other than double, **avoid** using the method

```
A = int8(zeros(100))
```

- This statement preallocates a 100-by-100 matrix of `int8`, first by creating a full matrix of double values, and then by converts each element to `int8`

- Creating the array as `int8` values saves time and memory

```
A = zeros(100, 'int8')
```

Improving performance

Techniques for Improving Performance

- **Vectorization**
 - MATLAB is optimized for operations involving matrices and vectors
 - The process of revising loop-based, scalar-oriented code to use MATLAB matrix and vector operations is called **vectorization**
- Vectorizing your code is worthwhile for several reasons:
 - *Appearance*: Vectorized mathematical code appears more like the mathematical expressions, making the code easier to understand
 - *Less Error Prone*: Without loops, vectorized code is often shorter, and fewer lines of code mean fewer programming errors
 - *Performance*: Vectorized code often runs much faster

Improving performance

- **Vectorizing Code for General Computing**
 - This code computes the sine of 1,001 values ranging from 0 to 10:

```
i = 0;
for t = 0:.01:10
    i = i + 1;
    y(i) = sin(t);
end
```

- This is a vectorized version of the same code:

```
t = 0:.01:10;
y = sin(t);
```

Improving performance

- **Vectorizing Code for Specific Tasks**

- This code computes the cumulative sum of a vector at every fifth element:

```
x = 1:10000;  
ylength = (length(x) - mod(length(x),5))/5;  
y(1:ylength) = 0;  
for n= 5:5:length(x)  
    y(n/5) = sum(x(1:n));  
end
```

- This code shows one way to accomplish the task:

```
x = 1:10000;  
xsums = cumsum(x);  
y = xsums(5:5:length(x));
```

Improving performance

- **Array Operations**

- Array operators perform the same operation for all elements in the data set

- **Example**

- collect the volume (V) of various cones by recording their diameter (D) and height (H)
- The volume for that single cone: $V = 1/12 * \pi * (D^2) * H$
- Consider 10,000 cones
- The vectors D and H each contain 10,000 elements

```
for n = 1:10000
```

```
    V(n) = 1/12*pi*(D(n)^2)*H(n);
```

```
end
```

- Vectorized Calculation

```
V = 1/12*pi*(D.^2).*H;
```

More examples

Use built-in Matlab functions

- **find** is a very important function
 - Returns indices of nonzero values
 - Can simplify code and help avoid loops
- Basic syntax: `index=find(cond)`

```
»x=rand(1,100);
```

```
»inds = find(x>0.4 & x<0.6);
```

- **Inds will contain the indices at which x has values between 0.4 and 0.6.**
- **This is what happens:**
 - `x>0.4` returns a vector with 1 where true and 0 where false
 - `x<0.6` returns a similar vector
 - The `&` combines the two vectors using an **and**
 - The `find` returns the indices of the 1's

More examples

- Given $x = \sin(\text{linspace}(0, 10 \cdot \pi, 100))$, how many of the entries are positive?

- Using a loop and if/else

```
count=0;
for n=1:length(x)
    if x(n)>0
        count=count+1;
    end
end
```

- Being more clever

```
count=length(find(x>0));
```

- **Avoid loops!** Built-in functions will make it faster to write and execute

IMAGES AND MATLAB

Images

- A digital image can be considered as a large array of discrete dots, each of which has a brightness associated with it
- These dots are called picture elements or more simply **pixels**
- The pixels surrounding a given pixel constitute its **neighborhood**
- A neighborhood can be characterized by its **shape** in the same way as a matrix: 3x3 neighborhood, 5x7 neighborhood...

48	219	168	145	244	188	120	58
49	218	87	94	133	35	17	148
174	151	74	179	224	3	252	194
77	127	87	139	44	228	149	135
138	229	136	113	250	51	108	163
38	210	185	177	69	76	131	53
178	164	79	158	64	169	85	97
96	209	214	203	223	73	110	200

Current pixel

3 × 5 neighbourhood

Types of digital image

- **Binary:** Each pixel is just **black** or **white**. Since there are only two possible values for each pixel (0,1), we only need **one bit** per pixel



1	1	0	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	1	1	0
0	0	0	0	0	1

Types of digital image

- **Grayscale:** Each pixel is a shade of gray, normally from **0** (black) to **255** (white), that is each pixel can be represented exactly **one byte**
- Other greyscale ranges can be used, generally power of **2**



230	229	232	234	235	232	148
237	236	236	234	233	234	152
255	255	255	251	230	236	161
99	90	67	37	94	247	130
222	152	255	129	129	246	132
154	199	255	150	189	241	147
216	132	162	163	170	239	122

Types of digital image

- **True Color**, or **RGB**: Each pixel has a particular color, described by the amount of **red**, **green** and **blue**
- Each components has a range 0–255, for a total of **256³** different possible colors
- **Three matrices** representing the **red**, **green** and **blue** values for each pixel



49	55	56	57	52	53
58	60	60	58	55	57
58	58	54	53	55	56
83	78	72	69	68	69
88	91	91	84	83	82
69	76	83	78	76	75
61	69	73	78	76	76

Red

64	76	82	79	78	78
93	93	91	91	86	86
88	82	88	90	88	89
125	119	113	108	111	110
137	136	132	128	126	120
105	108	114	114	118	113
96	103	112	108	111	107

Green

66	80	77	80	87	77
81	93	96	99	86	85
83	83	91	94	92	88
135	128	126	112	107	106
141	129	129	117	115	101
95	99	109	108	112	109
84	93	107	101	105	102

Blue

Image Import and Export

- Read and write images in Matlab

```
img = imread('apple.jpg');  
dim = size(img);  
figure;  
imshow(img);  
imwrite(img, 'output.bmp', 'bmp');
```

- Alternatives to `imshow`

```
imagesc(I)
```

```
imshow(I)
```

```
image(I)
```

Image and Matrices

How to build a matrix (or image)?

Intensity Image:

```
row = 256;  
col = 256;  
img = zeros(row, col);  
img(100:105, :) = 0.5;  
img(:, 100:105) = 1;  
figure;  
imshow(img);
```

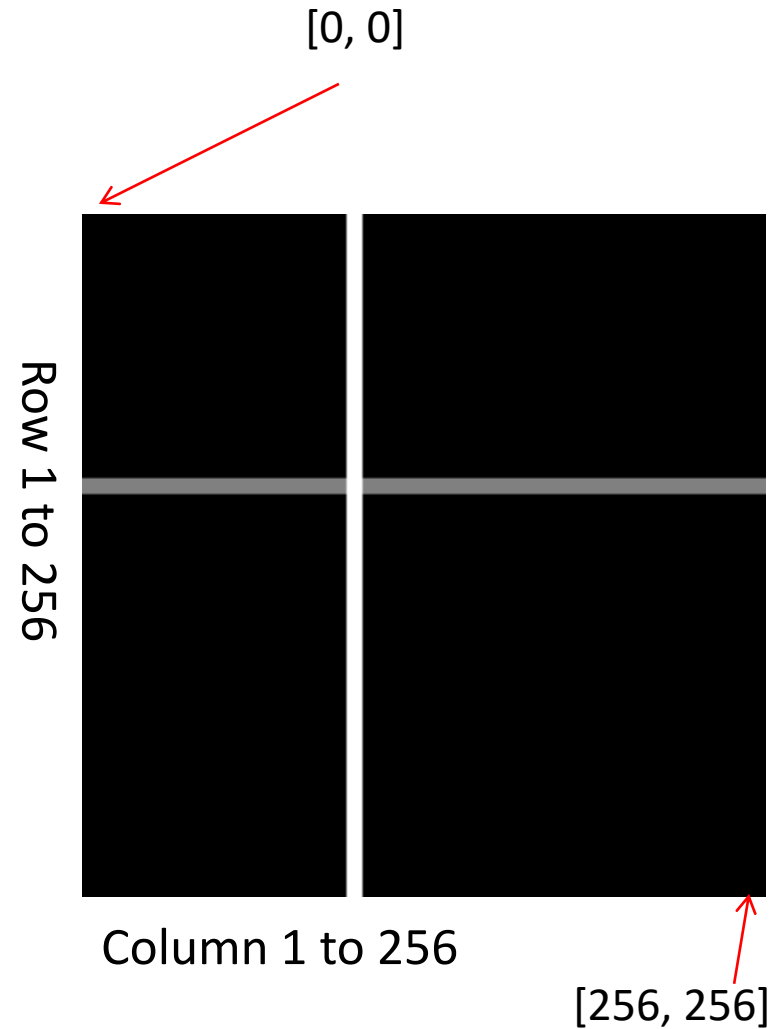


Image and Matrices

Binary Image

```
row = 256;  
col = 256;  
img = rand(row,  
col);  
img = round(img);  
figure;  
imshow(img);
```

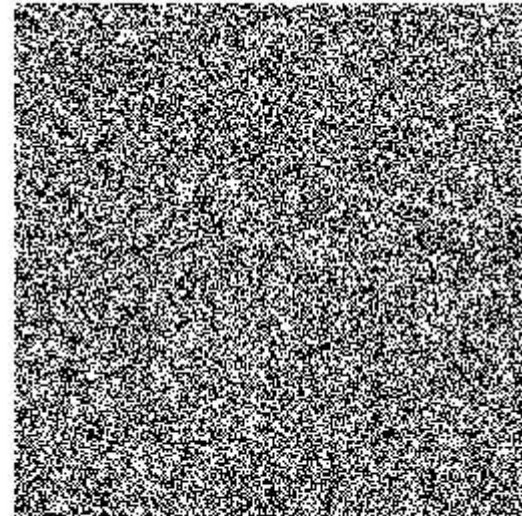


Image display

- `image` - create and display image object
- `imagesc` - scale and display as image
- `imshow` - display image
- `colorbar` - display colorbar
- `getimage` - get image data from axes
- `truesize` - adjust display size of image
- `zoom` - zoom in and zoom out of 2D plot

Image information

iminfo returns information about the image

impixel(i,j) returns the value of the pixel (i,j)

```

Filename: 'aster.tif'
FileModDate: '13-Mar-2008 16:54:26'
FileSize: 17224424.00
Format: 'tif'
FormatVersion: []
Width: 4100.00
Height: 4200.00
BitDepth: 8.00
ColorType: 'grayscale'
FormatSignature: [77.00 77.00 0 42.00]
ByteOrder: 'big-endian'
NewSubFileType: 0
BitsPerSample: 8.00
Compression: 'Uncompressed'
PhotometricInterpretation: 'BlackIsZero'
StripOffsets: [525x1 double]
SamplesPerPixel: 1.00
RowsPerStrip: 8.00
StripByteCounts: [525x1 double]
XResolution: 1.00
YResolution: 1.00
ResolutionUnit: 'None'
Colormap: []
PlanarConfiguration: 'Chunky'
TileWidth: []
TileLength: []
TileOffsets: []
TileByteCounts: []
Orientation: 1.00
FillOrder: 1.00
GrayResponseUnit: 0.01
MaxSampleValue: 255.00
MinSampleValue: 0
Thresholding: 1.00
Software: 'ERDAS IMAGINE '
SampleFormat: 'Unsigned integer'

```

Image conversion

- `gray2ind` - intensity image to index image
- `im2bw` - image to binary
- `im2double` - image to double precision
- `im2uint8` - image to 8-bit unsigned integers
- `im2uint16` - image to 16-bit unsigned integers
- `ind2gray` - indexed image to intensity image
- `mat2gray` - matrix to intensity image
- `rgb2gray` - RGB image to grayscale
- `rgb2ind` - RGB image to indexed image

Point Processing: Arithmetic operations

Arithmetic operations act by applying a simple function $y=f(x)$ to each gray value in the image

- Simple functions include **adding** or **subtract** a constant value to each pixel: $y = x \pm C$ (`imadd`, `imsubtract`)
- **Multiplying** each pixel by a constant: $y = C \cdot x$ (`immultiply`, `imdivide`)
- **Complement**: For a grayscale image is its photographic negative.

Addition



Image: I



Image: I+50

Subtraction



Image: I



Image: I-80

Multiplication



Image: I



Image: I*3

Division



Image: I



Image: I/2

Complement



Image: I



Image: 255-I

Image filtering

- **Filtering** is used to **enhance** or **attenuate** some characteristic of the image
- **Filtering** modifies the pixels in an image based on some function of a local neighborhood of each pixel



- Filtering generates a new image
- **Linear filtering** (cross-correlation, convolution) replace each pixel by a linear combination of its neighbors

Image filtering

- **Linear filtering** uses a matrix of coefficients **W**
- **Image F** is obtained from image **I** using **W**:

$$F[x, y] = \sum_{s=-a}^a \sum_{t=-b}^b W[s, t] I[x + s, y + t]$$

- Where **W** and the submatrix of **I** are:

$W[-1,-1]$	$W[-1,0]$	$W[-1,1]$
$W[0,-1]$	$W[0,0]$	$W[0,1]$
$W[1,-1]$	$W[1,0]$	$W[1,1]$

$I[x-1,y-1]$	$I[x-1,y]$	$I[x-1,y+1]$
$I[x,y-1]$	$I[x,y]$	$I[x,y+1]$
$I[x+1,y-1]$	$I[x+1,y]$	$I[x+1,y+1]$

Image filtering

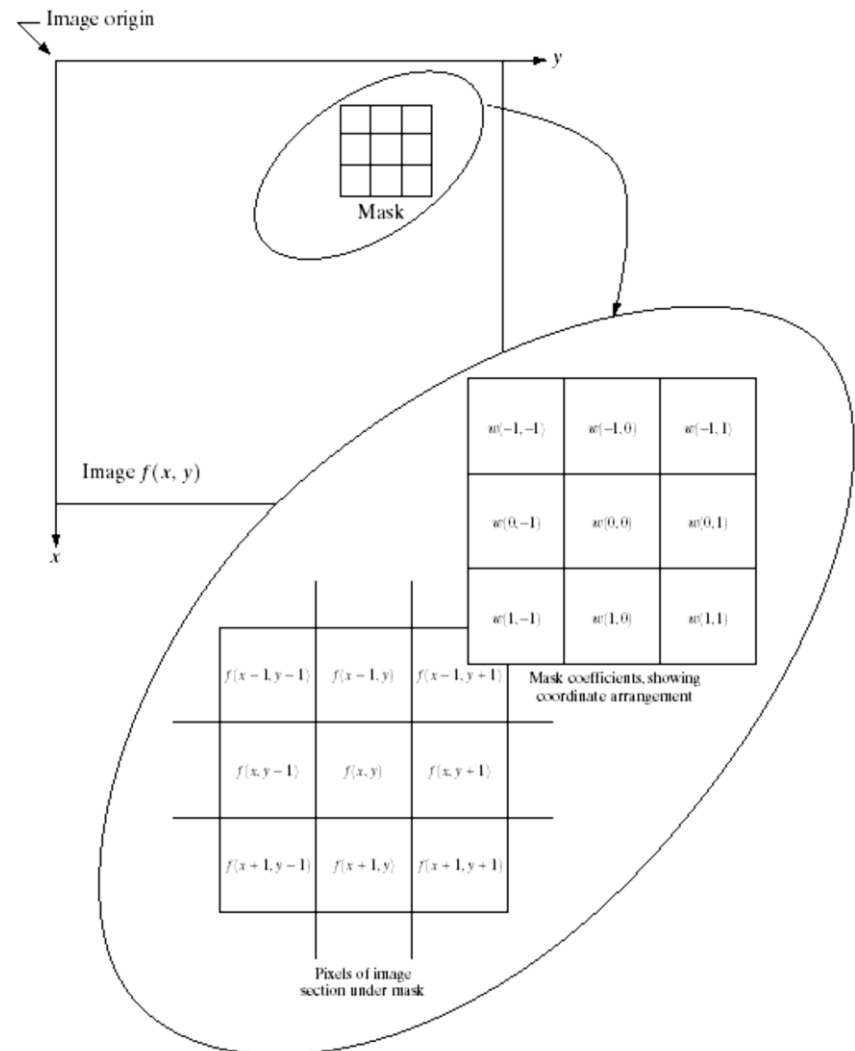
- **Convolution** Same as cross-correlation, except that the kernel is *flipped* (horizontally and vertically)

$$F[x, y] = \sum_{s=-a}^a \sum_{t=-b}^b W[s, t] I[x - s, y - t]$$

- The prescription for the linear combination - W - is called the **kernel** (or **mask**, or **filter**) of the cross-correlation/convolution

Image filtering

- **Smoothing filters:** mean filter, gaussian filter, median filter
- **Sharpening filters**



Smoothing filter

- Mean filter

$$W_{medio} = \frac{1}{a \cdot b} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

Smoothing filter

- **Gaussian filter**: weights of filter follow a gaussian distribution

$$G_{\sigma}(x, y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Example

$$G_{\sigma} = \frac{1}{273} \begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Gaussian filter

- Removes high-frequency components from the image (low-pass filter)



Median filter

The median filter **selects** a sample from the window, does **not** average

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

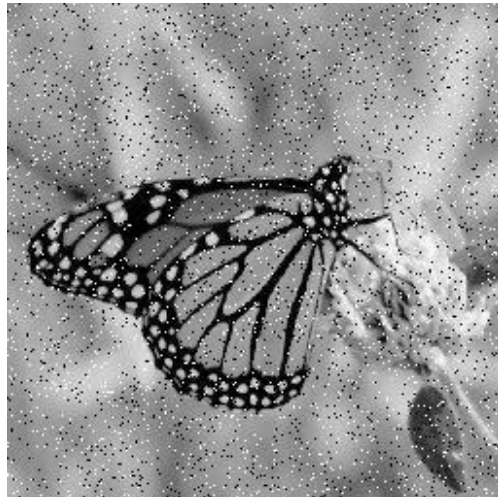
Neighbourhood values:

**115, 119, 120, 123, 124,
125, 126, 127, 150**

Median value: 124

Median filter

Best suited for *salt and pepper noise*



Sharpening filter

- **Sharpening** filters emphasize fine details in the image, exactly the opposite of the low-pass filter such as Gaussian filter → it just uses a different convolution kernel
- A **high-pass filter** can be used to make an image appear sharper.
- Usually the **central pixel is positive**, whereas **adjacent pixels are negative**

-1	-1	-1
-1	8	-1
-1	-1	-1

Sharpening filter

- First, I is modified by using a gaussian filter
- Then I_s is obtained as a linear combination among image I and the Gauss filtered image, with a suitable value of k usually equal to 1

$$\bar{I}[x, y] = I[x, y] - (G_\sigma * I)[x, y]$$

$$I_s[x, y] = I[x, y] + k\bar{I}[x, y]$$