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Algoritmi per le Reti

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# The "Real" Approximation Factor of the MST Heuristic for the Minimum Energy Broadcasting

**Michele Iovino**

- Introduction
- Definition and Notation
- Augmenting Algorithm
- Experimental Results
- High Density Case
- Conclusions

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# The Problem

**Minimum Energy Broadcast Routing (MEBR)**

Assign the transmission range of each station so as to induce a broadcast communication from the source with a minimum overall power consumption.

# The Problem

When a node *s* transmits with power $P_s$, a node *r* can receive its message iff:

$$\frac{P_s}{\|s,r\|^2} > 1$$

# Where do we start?

- The problem is NP–hard
- Approximate solution using Minimum Spanning Tree (MST)
- Lower bound of the approximation equal to 6

# What do we want to improve?

- Classify some specific family of instances
- Propose a new method to produce instances
- Provide theoretical results that lead to an almost tight

  $4-$approximation ratio for high density instances

- Introduction
- **Definition and Notation**
- Augmenting Algorithm
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# Definition of the Problem

Minimum Energy Broadcast Routing (MEBR) problem in the 2–dimensional space:

- **S**: set of points
- **$G_2$(S)**: complete weighted graph
- **r(x)**: range of node *x*
- **cost(r) = $\sum_{x \in S} r(x)^2$**: the total cost

# Definition of the Problem

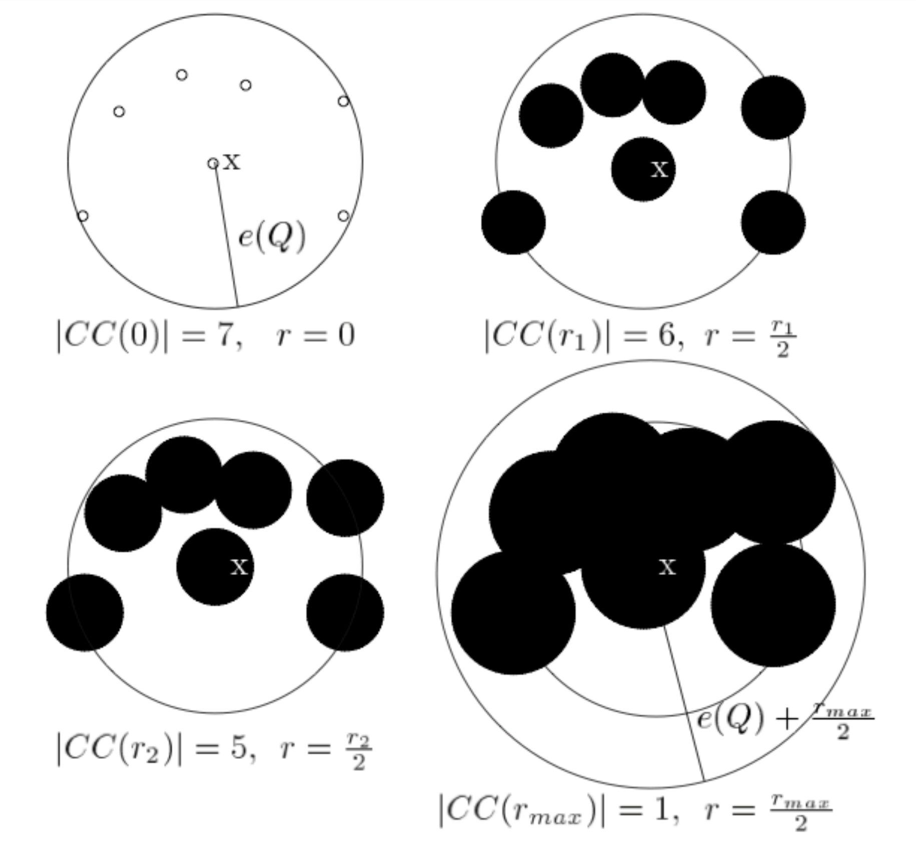
How do we solve the problem:

- **Calculate $T_2(S)$**: Minimum Spanning Tree of $G_2(S)$
- **$MST(G_2(S))$**: cost of the MST where:
  - $T_2(S)$ rooted at s
  - edges toward leaves
  - Range r(x) of the maximum edge from x to its children

# 8–approximation

Considering $C_1$, a circle of radius one centred in $s$ and covering the area of interest, an 8–approximation ration can be proven by assigning a growing circle to each node till all the circle create a connected component.

# 8–approximation



# 8–approximation

The "black" area is denoted by $a(S, r_{max}/2)$ and its relation with $MST(G_2(S))$ is:

$$MST(G_2(S)) = 2\int_0^{r_{max}} (n(S,r) - 1)r\, dr$$

# 8–approximation

The following bounds are then derived:

$$\frac{\pi}{4}MST(S) + \frac{\pi}{4}r_{max}^2 \leq a(S, \frac{r_{max}}{2}) \leq \pi(1 + \frac{r_{max}}{2})^2$$

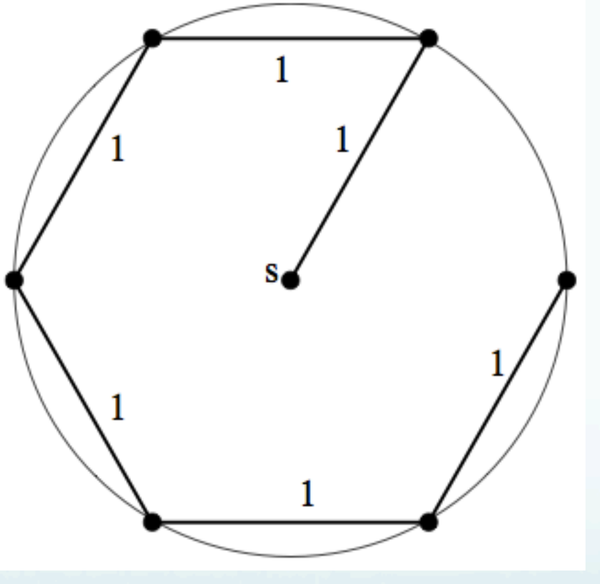
Obtaining:

$$MST(S) \leq 4(1 + r_{max})$$

- Introduction
- Definition and Notation
- **Augmenting Algorithm**
- Experimental Results
- High Density Case
- Conclusions

# The Idea of the Algorithm

The lower bound for the MST heuristic is given by the hexagonal shape:



# The Idea of the Algorithm

The idea is, starting from a random instance, to try to maximize the cost of the MST performing movements and deletions of nodes.

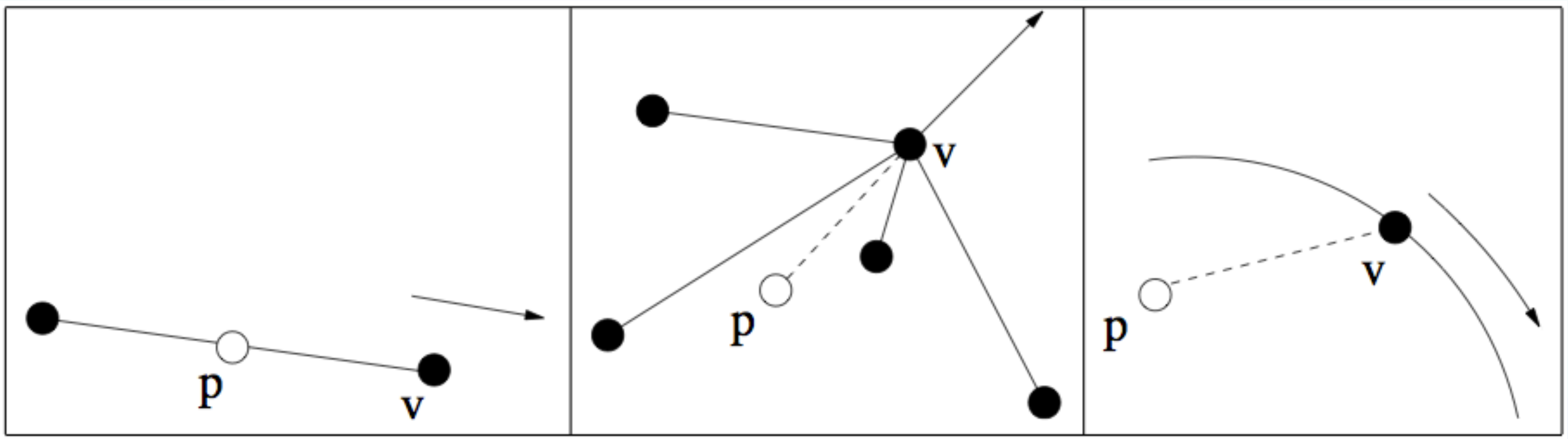
# The Movements

Let $N_v = \{v_1, v_2, \ldots, v_k\}$ be the set of neighbors of v in the MST.

Calculate a median point p(x,y):

$$x = \frac{1}{k}\sum_{i=1}^{k} x_{v_i}, \qquad y = \frac{1}{k}\sum_{i=1}^{k} y_{v_i}.$$

# Augmenting the Edge



# Deleting a Node

When no movements can be performed delete a node.

Which node to delete?

The highest density node!

# Augmenting Algorithm: Initialization

$flag1 = 1;$ * *flag1 determines if there is an allowed movement anymore.*

$flag2 = 1;$ * *flag2 determines if there is an allowed deletion anymore.*

$N = |V| - 1;$ * *Number of available nodes for the augmenting methods.*

$i = 1;$

$j = 1;$

# Augmenting Algorithm: MST

Compute the MST over the complete weighted graph $G$ induced by the set of nodes $V$ in which each edge $\{x, y\}$ has weight $\|x, y\|^2$; save its cost in $cost1$;

# Augmenting Algorithm: Movement While

**while** $flag2 \leq N$ **do**

  **while** $flag1 \leq N$ **do**

    Consider the node $v_i = \{x_i, y_i\}$ and its $k_i$ neighbors,
    $x = \frac{1}{k_i} \sum_{i=1}^{k_i} x_{v_i}$; $y = \frac{1}{k_i} \sum_{i=1}^{k_i} y_{v_i}$;  * *Coordinates of the median point p.*

    Let $rand$ be a random number in $[0, 1]$;

    **if** $v_i$ is not on the circumference **then**

      Let $v_i'$ be a point inside $C_1$ on the line passing through $v_i$ and $p$ in such a way that $\|v_i, p\| < \|v_i', p\| \leq (1 + \epsilon \cdot rand)\|v_i, p\|$;

    **else**

      Let $v_i'$ be a point on the circumference further from $p$ with respect to $v_i$ such that the arc joining $v_i$ and $v_i'$ has length $\epsilon \cdot rand$;

    **end if**

    Compute the MST over the complete weighted graph induced by the set of nodes $(V \setminus v_i) \cup v_i'$; save its cost in $cost2$;

    **if** $cost2 > cost1$ **then**

      $V = (V \setminus v_i) \cup v_i'$;

      $cost1 = cost2$;

      $flag1 = 1$;

    **else**

      $flag1 = flag1 + 1$;  * *The movement is not valid.*

    **end if**

    $i = (i + 1) mod\ N$;

  **end while**

# Augmenting Algorithm: Deletion While

Let $v_j$ be the $j$-th highest degree node of the current MST, compute the MST over the complete weighted graph induced by the set of nodes $V \setminus v_j$; save its cost in $cost2$.

**if** $cost2 > cost1$ **then**

  $V = V \setminus v_j$;

  $N = N - 1$;

  $cost1 = cost2$;

  $flag1 = 1$;

  $flag2 = 1$;

**else**

  $flag2 = flag2 + 1$; * *The deletion is not valid.*

**end if**

$j = (j + 1) mod\ N$;

**end while**

# Observations

- The algorithm converge
- Speed up modifying the algorithm in line 17 and 27 with

  cost2 > cost1 + c

- Introduction
- Definition and Notation
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- High Density Case
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# The Results

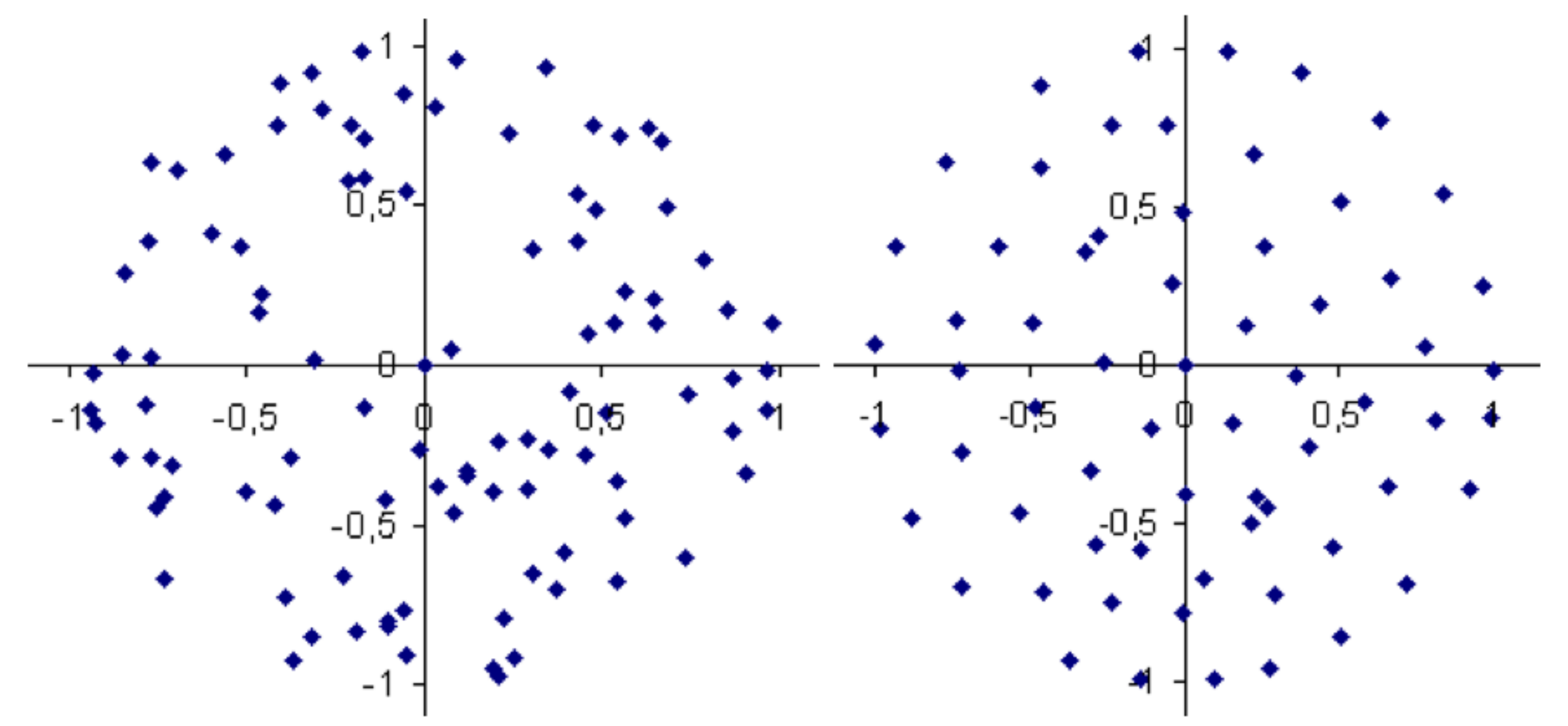
| $n$ | Random | | Augmented, $\epsilon = .5$ | | Augmented, $\epsilon = .1$ | |
|---|---|---|---|---|---|---|
| | Average | Max | Average | Max | Average | Max |
| 5 | 1.301 | 2.8752 | 3.6456 | 4 | 3.6276 | 4 |
| 7 | 1.4799 | 2.4793 | 4.5454 | 5.7386 | 4.5606 | 5.8797 |
| 10 | 1.8019 | 3.1231 | 5.2848 | 5.7851 | 5.353 | 5.9187 |
| 15 | 1.8875 | 2.6691 | 4.8648 | 5.4803 | 4.777 | 5.7728 |
| 20 | 1.854 | 2.6187 | 4.2817 | 5.0906 | 4.1316 | 5.1222 |
| 30 | 1.8252 | 2.2328 | 4.137 | 4.45 | 3.991 | 4.1819 |
| 50 | 1.812 | 1,9718 | 3.7319 | 3,8901 | 3.6331 | 3,7598 |
| 100 | 1.6833 | 1.8829 | 3.5673 | 3.7223 | 3.4898 | 3.812 |

# An Interesting “Side Effect”

With instances with less than 15 nodes the method modifies the distribution of the nodes giving the hexagonal shape.

With instances with higher number of nodes the experiment modifies the node and organise them with an evident regularity of the topology.

# An Interesting "Side Effect"

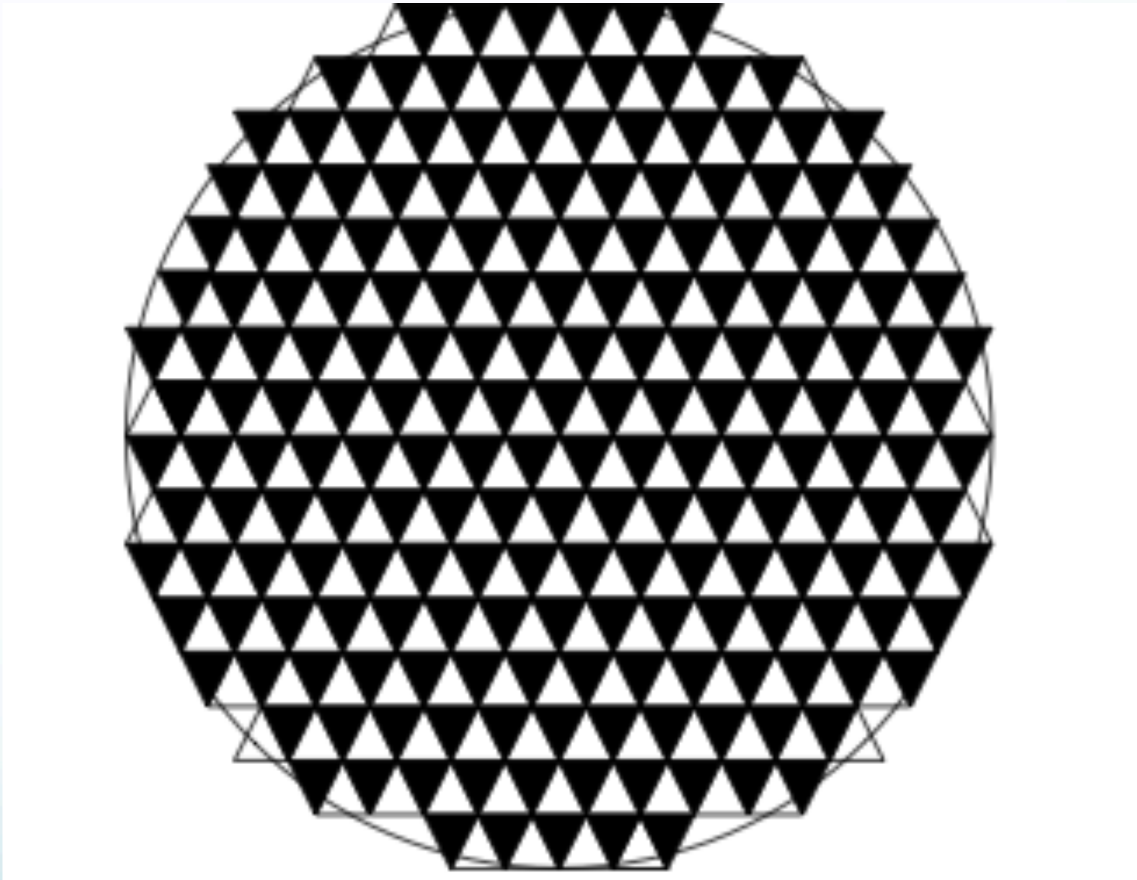


- Introduction
- Definition and Notation
- Augmenting Algorithm
- Experimental Results
- **High Density Case**
- Conclusions

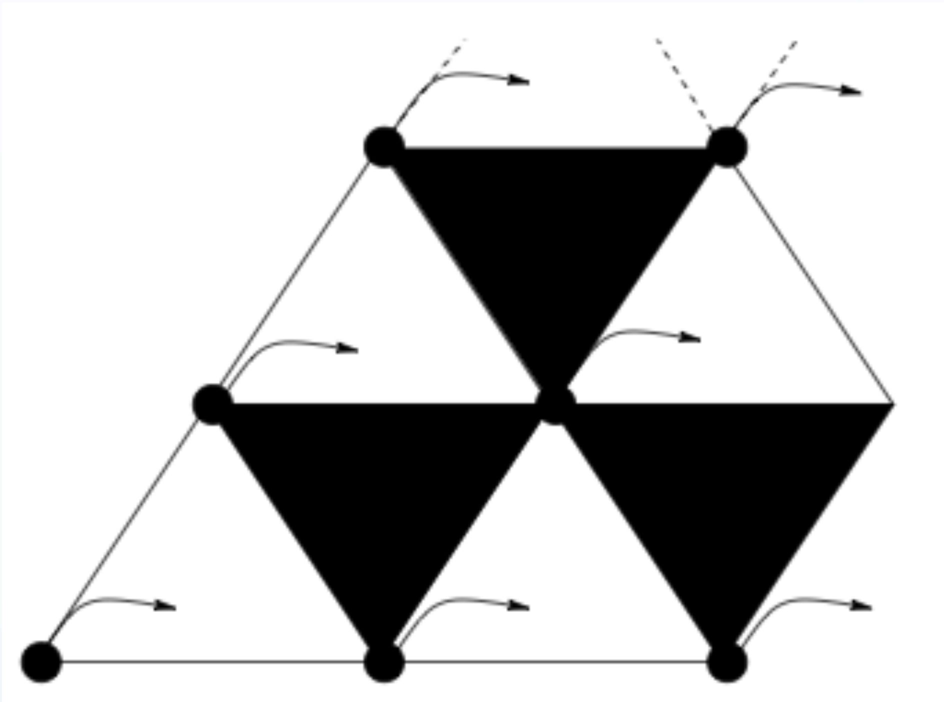
# High–Density Case

Let us assume an high–density uniform distributions of nodes inside $C_1$ and estimate the cost of the MST heuristic.

# Regular Grid



# Associate Triangle to each Node



# Evaluation of MST

The area of an equilateral triangle is $\frac{\sqrt{3}}{4}l^2$.

As the number of nodes of the MST is the number of its edges plus 1, the area of the "white triangles" is:

$$\frac{\sqrt{3}}{4}MST(S) \simeq \frac{\pi 1^2}{2}$$

# Evaluation of MST

Combining them we get:

$$MST(S) \simeq \frac{2\pi}{\sqrt{3}} > 3.62.$$

# The Theorem

In the 2–dimensional Euclidean space, the upper bound on the approximation ratio of the MST heuristic for the Minimum Energy Broadcast Routing problem with high–density distribution of the nodes is between 3.62 and 4.

- Introduction
- Definition and Notation
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- Experimental Results
- High Density Case
- **Conclusions**

# Conclusions

- We examined the MEBR problem by extensive experiments
- We proposed a new method to generate input instances
- We validate the well known lower bound of 6 for the MEBR
- We showed a 4 approximation factor in the high density case

# THANKS FOR YOUR ATTENTION

# Questions?