HETEROGENEOUS SENSORS

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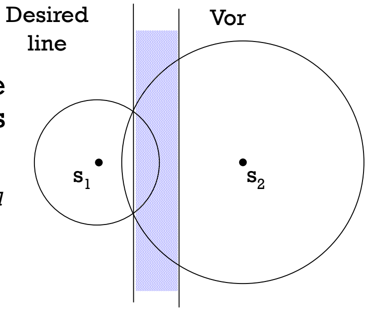
- Sensors are not necessarily all equal. We speak about a heterogeneous sensor network if:
  - the devices are different
  - The sensing and communicating ability of the sensors depend on their position (not smooth terrain, obstacles, …)

- The previously described approaches (based on virtual forces and on Voronoi cells) do not work well with heterogeneous sensors:
  - Virtual forces: forces depend on the distance
  - Voronoi: cells do not take into account the coverage capability

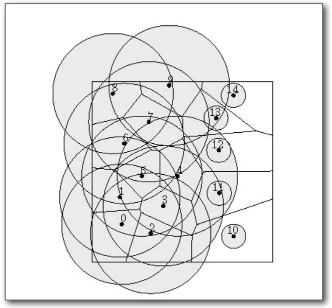
LIMITATIONS OF THE PROTOCOLS BASED ON VORONOI CELLS (1)

- The protocol based on the construction of Voronoi cells would assign:
  - The left halfplane to s_1 (included the blue zone)
  - The right halfplane to s_2

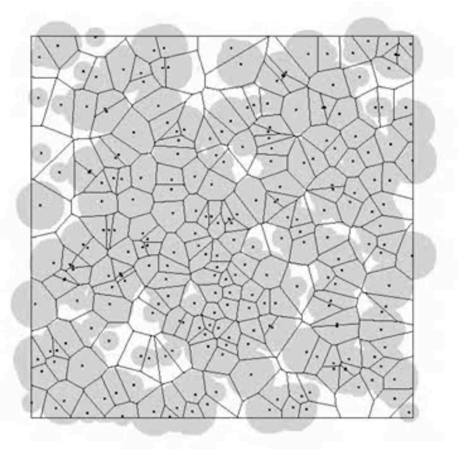


LIMITATIONS OF THE PROTOCOLS BASED ON VORONOI CELLS (2)

- Stale situation:
  - the sensors on the left (big circles) do not move since they completely cover their cells
  - the sensors on the right (small circles) do not move since their circles are completely used to cover a portion of their cell (in other words, their coverage capacity is maximized).



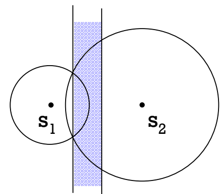
LIMITATIONS OF THE PROTOCOLS BASED ON VORONOI CELLS (3)



A NEW NOTION OF DISTANCE

- In the known algorithms, the heterogeneity is ignored
- We introduce a new notion of distance

  keeping into account:
  - The Euclidean distance
  - The heterogeneity of the devices



- There are many possibilities, but we aim at having:
  - Diagrams with straigh edges (convex polygons)
  - a distance whose set of points equally distant from two sensors contains the intersection of their sensing circles

LAGUERRE DISTANCE (1)

[W. Blaschke. Vorlesungen uber Differentialgeometrie III. Springer Berlin. 1929]

- Defined in $\mathcal{R}^3$
- Given two points $P=(x,y,z)$ and $Q=(x',y',z')$, their Laguerre distance is:
  - $d_L^2(P,Q)=(x-x')^2+(y-y')^2-(z-z')^2$
- P can be seen as the (oriented) circle centered at (x,y) and having radius $|z|$

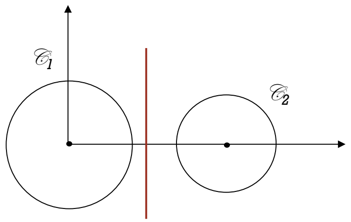
LAGUERRE DISTANCE (2)

- Given two circles $\mathcal{C}_1$ and $\mathcal{C}_2$, centered at C_1 and C_2 respectively, and with radii r_1 and r_2, their Laguerre distance is:
  - $d_L^2(\mathcal{C}_1, \mathcal{C}_2)= d_E^2(C_1, C_2) - (r_1 - r_2)^2$
- The Laguerre distance between a point $P=(x,y)$ and a circle $\mathcal{C}=(x',y',r)$ is:
  - $d_L^2(P,C)=(x-x')^2+(y-y')^2-r^2$

LAGUERRE DISTANCE (3)

- Lemma. Given two circles $\mathscr{C}_1$ and $\mathscr{C}_2$ centered at C_1 and C_2 ($C_1 \neq C_2$) and radii r_1 and r_2, the sets of point equally distant from $\mathscr{C}_1$ and $\mathscr{C}_2$ is a straight line (called radical axis) orthogonal to the segment joining C_1 and C_2 and at distance k from C_1, where

$$k = \frac{d_E(C_1,C_2)}{2} + \frac{r_1^2 - r_2^2}{2d_E(C_1,C_2)}$$



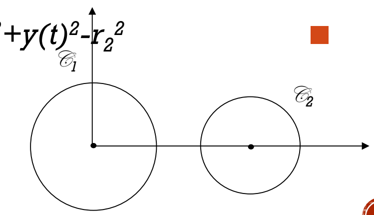
LAGUERRE DISTANCE (4)

Proof. Consider the set of points $P(t)=(x(t), y(t))$ equally distant from $\mathscr{C}_1$ and $\mathscr{C}_2$, i.e. such that

$d_L(P(t), \mathscr{C}_1)=d_L(P(t), \mathscr{C}_2)$.

- If $C_1=C_2$ and $r_1=r_2 \Rightarrow P(t)$ is the whole plane
- If $C_1=C_2$ and $r_1 \neq r_2 \Rightarrow P(t)$ is the empty set
- If $C_1 \neq C_2$:

$x(t)^2+y(t)^2-r_1^2=(d_E(C_1,C_2)-x(t))^2+y(t)^2-r_2^2$ ■



$d_L^2(P,C)=(x-x')^2+(y-y')^2-r^2$

LAGUERRE DISTANCE (5)

- Lemma. Given two circles $\mathscr{C}_1$ and $\mathscr{C}_2$ centered at C_1 and C_2 ($C_1 \neq C_2$) and having radii r_1 and r_2, theri centers lie on the same side w.r.t. the radical axis if and only if

$d_E^2(C_1,C_2) < |r_1^2-r_2^2|$.

Proof. The axis can lie either to the right or to the left.

- Right:

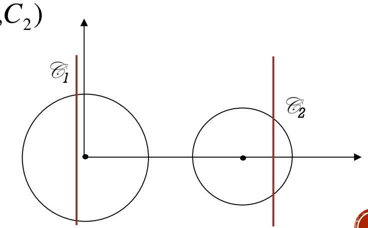
$$k = \frac{d_E(C_1,C_2)}{2} + \frac{r_1^2 - r_2^2}{2d_E(C_1,C_2)} \geq d_E(C_1,C_2)$$

$d_E^2(C_1,C_2) \leq r_1^2-r_2^2 \Rightarrow r_1 \geq r_2$

- Left:

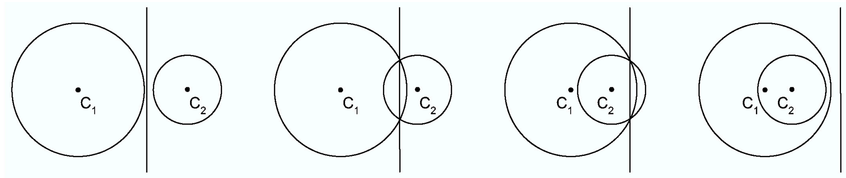
$$\frac{d_E(C_1,C_2)}{2} + \frac{r_1^2 - r_2^2}{d_E(C_1,C_2)} \leq 0$$

$d_E^2(C_1,C_2) \leq r_2^2-r_1^2 \Rightarrow r_2 \geq r_1$ ■



LAGUERRE DISTANCE (6)

Possible positions of the radical axis of two cricles $\mathscr{C}_1$ and $\mathscr{C}_2$

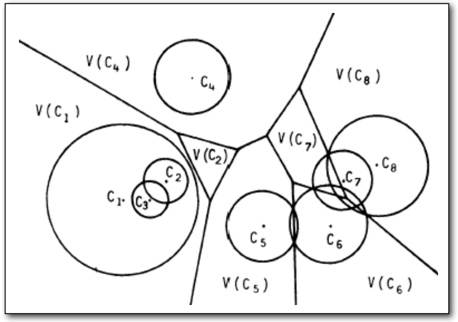


VORONOI-LAGUERRE DIAGRAM (1)

Voronoi-Laguerre diagram of $\mathscr{C}_1, \ldots, \mathscr{C}_n$:

- $V_i = \cap \{p \in \mathscr{R}^2 \mid d_L{}^2(\mathscr{C}_i, P) \leq d_L{}^2(\mathscr{C}_j, P)\}$

[H. Imai, M. Iri, K. Murota. "Voronoi Diagram in the Laguerre Geometry and its Applications". SIAM J. Comput. 14(1), 93-105. 1985]



They have similarities and differences w.r.t. the classical Voronoi diagrams…

VORONOI-LAGUERRE DIAGRAM (2)

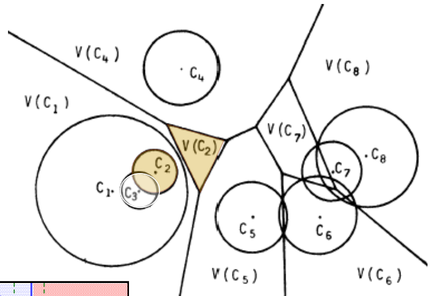
Similarities:

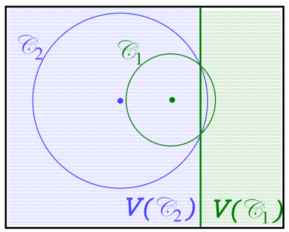
- Voronoi-Laguerre polygons partition the plane
- V_i is always convex because it is the intersection of some halfplanes
- if $r_i=0$ for each $i=1, \ldots, n$, theVoronoi-Laguerre diagram is in fact the classical Voronoi diagram.

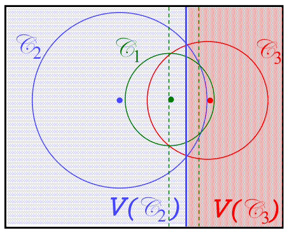
VORONOI-LAGUERRE DIAGRAM (3)

Differences:

- $\mathscr{C}_i$ can be external to V_i (see $\mathscr{C}_2$)
- V_i can be empty (e.g. if $\mathscr{C}_i$ is inside the union of other circles - see $\mathscr{C}_3$)







VORONOI-LAGUERRE DIAGRAM (4)

- Theorem. Given n circles $\mathscr{C}_i$ centered at $C_i=(x_i,y_i)$ and having radii r_i, $i=1, \ldots, n$, let V_i be their Voronoi-Laguerre polygons.

  For each i and j, $V_i \cap \mathscr{C}_j \subseteq \mathscr{C}_i$.

  In other words, the intersection of V_i with a circle $\mathscr{C}_j$ is included in $\mathscr{C}_i$.

VORONOI-LAGUERRE DIAGRAM (5)

Proof. By contradiction, assume that ther exists a point $P\subseteq V_i$ in $\mathcal{C}_j$ but non in $\mathcal{C}_i$, for some $j\neq i$.

- Since $P\subseteq V_i$ it holds $d_L(P, \mathcal{C}_i) < d_L(P, \mathcal{C}_j)$ for each $j\neq i$, i.e.

$$d_E^2(P, \mathcal{C}_i)-r_i^2<d_E^2(c_j,P)-r_j^2$$

- Since P is in $\mathcal{C}_j$ but non in $\mathcal{C}_i$,

$$d_E^2(P, \mathcal{C}_j)\le r_j^2 \text{ and } d_E^2(P, \mathcal{C}_i) \ge r_i^2$$

- Combining: 0<0 Contradiction. ■

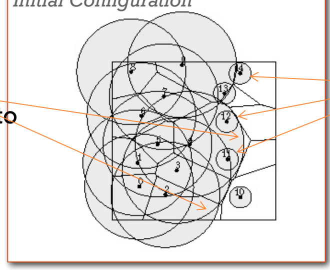
ALGORITHM BASED ON VORONOI-LAGUERRE DIAGRAM (1)

Algorithm executed by each sensor s_i:

- Compute V_i
- If s_i is inside V_i, move toward the minimax (by at most by $d_i^{max}=r_{tx}/2-r_i$ where $r_{tx}=min_i r_i^{tx}$) if the coverage of V_i is increased
- If s_i is outside V_i, move toward the minimax (by at most $d_i^{max}=r_{tx}/2-r_i$)
- if V_i is empty, do nothing.

ALGORITHM BASED ON VORONOI-LAGUERRE DIAGRAM (2)

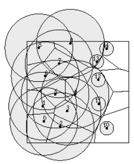
Initial Configuration



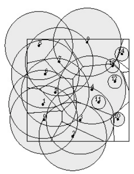
Big Circles
Some of them move to better cover their polygons

Small circles
Many of them move because they are external w.r.t. their polygon

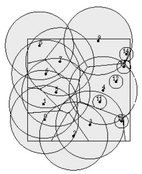
Initial Configuration



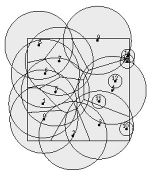
Round 6



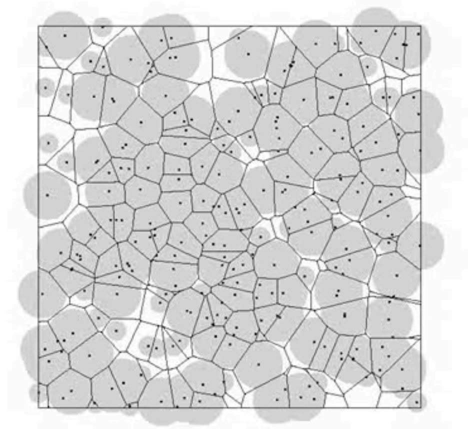
Round 9



Round 12:
The Stale is solved!



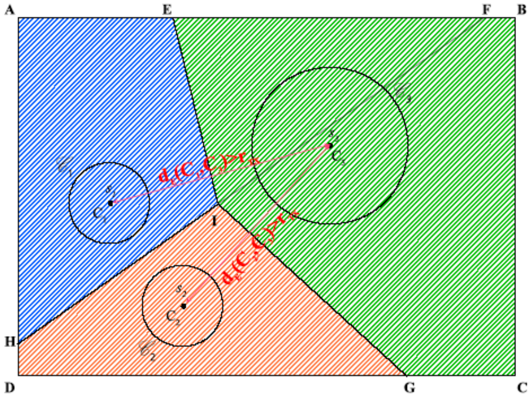
ALGORITHM BASED ON VORONOI-LAGUERRE DIAGRAM (3)



PROPERTIES OF THE ALGORITHM (1)

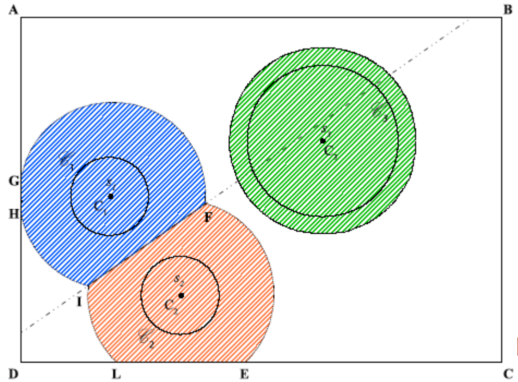
Obs.:

- "local" polygon≠"global" polygon and the set of local polygons do not constitute a partition!



PROPERTIES OF THE ALGORITHM (2)

- We define a curve polygon V'_i generated intersecting the "local" polygon with the circle of radius $d_i^{max}+r_i=r_{tx}/2$.



PROPERTIES OF THE ALGORITHM (3)

- Lemma. $V'_i \cap V'_j = \phi \;\; \forall i \neq j$
- Lemma. $\forall i \neq j,\; V'_i \cap \mathscr{C}_j \subseteq \mathscr{C}_i$

  In other words, each curve polygon can be covered by the sensor generating it better than by any other sensor.

PROPERTIES OF THE ALGORITHM (4)

Th. The algorithm converges.

Proof. Let $V'^{(k)}_i$ be the curve polygon of s_i at round k.

- Let $A_i^{(k)}$ and $A_i^{(k)}(s_i)$ be the areas covered inside $V'^{(k)}_i$ by all the sensors and by the sole sensor s_i at round k, respectively. Let $A_i'^{(k)}$ be the covered area considering the positions of the sensors at round $k+1$.
- Obs. $A_i'^{(k)} \neq A_i^{(k+1)}$
- Let $A^{(k)}_{total}$ be the area covered by the AoI by all the sensors.
- We have to prove that $A^{(k)}_{total} < A^{(k+1)}_{total}$

PROPERTIES OF THE ALGORITHM (5)

Proof. (cntd.)

- $\mathscr{P}^{(k)}=\{V'_1{}^{(k)}, V'_2{}^{(k)}, \dots, AoI \setminus \cup_i V'_i{}^{(k)}\}$ is a partition of the AoI.
- $AoI \setminus \cup_i V'_i{}^{(k)}$ is constituted by points that are uncovered and cannot be covered in a single round; it does not contribute to $A^{(k)}_{total}$.
- $A^{(k)}_{total} = \Sigma_i A_i^{(k)}$
- $A_i^{(k)} = A_i^{(k)}(s_i)$ (by the previous lemma)
- $A_i^{(k)}(s_i) < A_i'^{(k)}(s_i)$ (by the algorithm)
- $A_i'^{(k)}(s_i) \leq A_i'^{(k)}$
- Hence: $A^{(k)}_{total} = \Sigma_i A_i^{(k)} < \Sigma_i A_i'^{(k)}$
- Since the coverage at round $k+1$ does not depends on the partition: $\Sigma_i A_i'^{(k)} = A^{(k+1)}_{total}$ ■

PROPERTIES OF THE ALGORITHM (6)

- Convergence does not imply termination.
- In order to guarantee termination, we introduce a minimum movement threshold ε, so that sensors do not move if they are suppose to do by less than ε.
- Corollary. The algorithm, with the addition of the minimum movement threshold, terminates.

OPEN PROBLEMS

- Obstacles and terrain asperities
  - Anisotropy
  - Movement obstacles
- AOI with complex shape
  - concave regions and corridors
- …