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Distance-1 Constrained Channel Assignment in Single Radio Wireless Mesh Networks

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Abstract— **This paper addresses channel assignment and random medium access design for single-radio multi-channel mesh networks. Two prior approaches include: (i) designing MAC protocols that dynamically select channels based on local information and (ii) partitioning the mesh into subnetworks with different channels and using 802.11 as the medium access protocol. Both of these approaches suffer from limited throughput improvement; the first approach due to wrong or incomplete channel state information that inherently arises in a multi-hop wireless environment, while the second approach due to high interference within each subnetwork. In this paper, we first introduce D1C-CA, Distance-1 Constrained Channel Assignment. D1C-CA statically assigns channels to a set of links as a function of physical connectivity, contention, and the unique gateway functionality of mesh networks, i.e, all internet (non-local) traffic has a gateway node as its source or destination. To design D1C-CA, we model the channel assignment problem as a new form of graph edge coloring in which edges at distance one are constrained. We prove that the problem is NP-complete and design an efficient heuristic solution for mesh networks. Second, we design an asynchronous control-channel-based MAC protocol that solves multi-channel coordination problems and employs the proposed channel assignment algorithm. Finally, we investigate the performance of our approach through extensive simulations and show considerable performance improvements compared to alternate schemes.**

I. Introduction

Use of multiple channels has the potential to increase aggregate throughput in congested mesh networks. Signifant literature exists on both multi- and single-radio architectures as described in Section VII. Here, we focus on the single-radio multi-channel case in which a single transceiver dynamically switches channels. Because of the lower deployment cost as compared to the multi-transceiver approach, the single-radio architecture was considered in the 802.11s mesh standard [12].

A key challenge with the single-transceiver approach is channel assignment: *Which of the available channels should a node use to transmit at any point in time?* The common approach to channel assignment is to let each transceiver dynamically select the channel for its own data transmissions based on local inference and control packets exchanged with neighbors: we refer to this class of schemes as Transceiver Based Channel Selection (TBCS) [26], [25], [12], [31].

Unfortunately, TBCS protocols are inherently greedy due to making decisions based purely on local information. Consequently, they suffer from two inherent limitations that hinder their performance. First, they are vulnerable to inaccurate channel availability state: Since transceivers choose channels for transmission based on local inference of channel availability state, the performance of any TBCS protocol is dependent on the accuracy of the channel availability state. Such inaccuracies can be due to (i) loss of reception of control packets when a transceiver is tuned to a different channel and (ii) corrupted reception of control messages due to low reception power, channel switching and collisions. Second, even under accurate channel availability state, local decision making in a multi-hop topology does not ensure an optimal allocation. (See [2] for a detailed discussion.)

In this paper, we introduce a new approach to channel assignment and medium access in single-radio multi-channel mesh networks. Our key technique is to employ *static link based* channel assignment that minimizes the number of interfering links with each active link, thereby significantly reducing contention.[1] We exploit two properties of mesh networks to achieve this: First, the gateway node acts as a centralization point to compute the channel assignment based on the gateway's knowledge of topology.[2] Second, because all traffic originating from or destined for the Internet traverses a gateway, links closest to the gateway are the most congested. Consequently, our channel assignment favors these links. In addition to channel assignment, we present a MAC protocol that transmits data on channels determined by our channel assignment scheme while employing a single transceiver, asynchronous random access, and a common control channel for medium arbitration.

In particular, our contributions are as follows. First, we present Distance-1 Edge Coloring (D1EC), a new form of graph coloring. If a graph has a D1EC, it has a sufficient number of channels such that the assignment is contention free, i.e., all data links that have the potential to be active simultaneously (because they do not share a common node) can be active simultaneously. We show that for arbitrary graphs, the D1EC problem is NP complete. Moreover, for structured graphs (geometric and grid), we bound the needed number of channels to have D1EC as a function of the node degree, thereby providing an upper bound on the minimum number of channels to achieve maximum throughput, i.e., a further increase in the number of channels does not lead to an additional increase in throughput. We show that many such structured topologies require a small number of channels for maximum throughput that is feasible in standards such as IEEE 802.11a.

Second, we develop the Distance-1 Constrained Channel Assignment (D1C-CA) algorithm with the objective of minimizing interference among active links in order to increase the

[1] In practice, our approach is more precisely *quasi-static*, with the assignment recomputed under major network changes such as node failure.

[2] In fact, in Cisco's Aironet architecture, *all* network management functions are performed at gateway nodes.

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network's aggregate throughput. Thus, if a valid D1EC exists, the outcome of the algorithm is a heuristic mechanism to find it. If a valid D1EC does not exist, the algorithm minimizes the number of interfering links per channel. In other words, with D1C-CA, some links will be assigned an interference free channel which will allow them to transmit whenever the transmitter-receiver pair is available. The remaining links will be given information regarding the interference of the channel assigned to them. This information will help the MAC protocol to contend for the channel more efficiently.

Once the D1C-CA is completed, a medium access protocol is required to ensure that both transmitter and receiver are available, notify the receiver about the transmitter's intention to transmit a packet and schedule the actual transmission. A broad class of MAC protocols can satisfy this requirement and we develop an example MAC protocol. With this protocol, we corroborate our theoretical results via an extensive set of ns-2 simulations. We show that our scheme yields significant improvement in both aggregate and per-flow throughput as compared to both IEEE 802.11 and a recently proposed multiple channel MAC protocol employing the TBCS approach, irrespective of the number of available channels.

The remainder of this paper is organized as follows. In section II we present our network model. We mathematically formulate the channel assignment problem in section III. An efficient channel assignment algorithm and the underlying MAC protocol are described in sections IV and V. Simulation results are presented in section VI. Related work are reviewed in section VII. We conclude our paper in section VIII. All proofs are placed in the Appendix.

II. System Model

We assume a stationary wireless mesh network where each router is equipped with a *single half-duplex transceiver* for back-haul access. There are K non-overlapping frequency channels in the system and each node can listen, receive or transmit to only one of these channels at a time. All nodes in the network use the same fixed transmission power, i.e, there is a fixed transmission range (r $\geq$ 0) and a fixed interference range (R $\geq$ r) associated with every node. The physical graph of the network is modeled as an *undirected* graph *G = (V, E)*. (This assumption is used to ensure that both data and acknowledgement packets are feasible on the same link.) Here, V is a set of vertices denoting the transceivers comprising the wireless network and E is a set of undirected edges between vertices representing inter-node link characteristics. There is an undirected edge $(v_i, v_j) \in$ E connecting vertices v_i and v_j *iff* $\parallel v_i - v_j \parallel \leq R$.

At least one of the routers within the mesh is designated as a *gateway* which provides connectivity to an external network and has information about the mesh network's physical connectivity.

We also assume a separate routing protocol that determines the routes to and from the gateways. We refer to the links that are selected by the routing protocol to forward traffic as *active* links and denote the set of all active links by A. Note that an active link *does not always have a packet to forward, but that it is present in the routing table of a node for packet forwarding*.

III. Channel Assignment Problem Formulation

The objective of channel assignment is to, if possible, assign different channels to any two active links that can be active at the same time *iff* their transmissions occur on two different channels. We denote a channel assignment that can realize this objective as a valid "distance one channel assignment." Note that two cases need not be considered in distance-1 channel assignment. First, non-interfering links need not be considered as they can *always* be active simultaneously even with the same channel. Second, links that share a mutual node should also not be considered as they can *never* transmit simultaneously due to the single radio and half duplex system.

On the other hand, due to the limited number of channels, there might not exist a channel assignment that can realize a valid distance one channel assignment. In this case distance one links with same channel will interfere with each other, however the amount of interference on each active link, and hence the load between channels, can be balanced by minimizing the number of interfering links with each active link.

We now formally define these objectives. Let the distance between two nodes u_1 and u_2 in a graph G, denoted by $d(u_1, u_2)_G$, be the minimum number of hops in G from u_1 to u_2. Accordingly the distance between two links $l_{u_1u_2}$ and $l_{v_1v_2}$, is defined as:

$$d(l_{u_1u_2}, l_{v_1v_2})_G = min\ (d(u_i, v_j)_G)\ i, j \in \{1, 2\} \qquad (1)$$

Note that distance zero between two links defines two links that share a mutual node; distance one between two links defines two links that are within interference range of each other and do not share a mutual node; and distance greater than one between two links defines two links that are out of interference range of each other. Note that according to the channel allocation objective defined above, only links that are at distance one should be assigned different channels. By equating channels to colors, we model this channel assignment problem as a graph edge coloring problem.

Definition: (D1EC problem) Given a physical graph G and a selected subgraph $A \in G$, the ***Distance-1 Edge Coloring (D1EC)*** *problem seeks a mapping of colors to links in A such that any two links in A that are at distance one with respect to G are assigned different colors.*

Note that the D1EC problem is a variation of the classical edge coloring problem where edges at distance zero have to be assigned different channels and is known to be NP-complete [6].

Observe that if a valid D1EC is realized, any selected set of links in A that do not share a node in common can be active at the same time and hence in terms of channel assignment, *maximum throughput* is achieved.

Next we define the *Distance-1 chromatic index* that describes the number of colors needed to have a valid D1EC.

Definition: The ***distance-1 chromatic index****, k_{D1EC}, of a subgraph $A \in G$, is the minimum number of colors to have a valid D1EC of links in A.*

A key problem in channel assignment is to find the Distance-1 chromatic index, i.e., if there exists a valid distance-1 channel assignment for a given topology and number of channels. However, the next theorem proves that finding the distance-1 chromatic index is an NP-complete problem. Later we will provide upper bounds to the *Distance-1 chromatic index* problem.

***Theorem** 1: The decision problem whether k colors are sufficient to have a valid D1EC is NP-complete for k $\geq$ 3.*

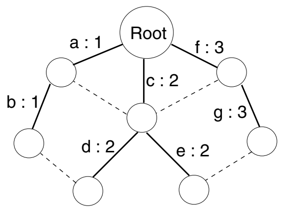

Fig. 1. D1EC of an example mesh topology.

Figure 1 shows an example of D1EC. Here, thick edges represent the set of active links, i.e., $A \in G$. The colors assigned to links: $\{a, b, c, d, e, f, g\}$ are $\{1, 1, 2, 2, 2, 3, 3\}$, respectively. Note that any set of links that do not share a node in common can be active at the same time *iff* at least 3 channels are available, e.g., $\{a, e, g\}$. On the other hand, if the number of available channels is smaller than 3 the D1EC problem cannot be solved, which means that the Distance-1 chromatic index is 3.

We now provide upper bounds on K_{D1EC} to determine the needed number of channels that guarantee a solution for D1EC problem. We assume the selected subgraph A is equal to G. In a real network, only a subset of links are selected by the routing protocol and need to be assigned channels such that A is not equal to G and the needed number of channels can only be smaller. Using the Brook and Vizing theorem [6], the following bounds on k_{D1EC} can be derived for any graph with *maximum degree* Δ:

$$k_{D1EC} \leq min\{|V|, 2 \times (\Delta - 1)^2 + 1\} \qquad (2)$$

The above inequality gives a bound on k_{D1EC} that is of square degree of Δ. However, geometric properties of wireless networks can be used to provide linear bounds.

Our next theorem provides an upper bound on k_{D1EC} for the Unit Disk Graph (UDG) model [18] and random placement of nodes. In the UDG model, all nodes have the same transmission and interference range and this value is the same for all nodes.

***Theorem** 2: K_{D1EC} for a geometric graph with maximum degree Δ is upper bounded by 18$\times$($\Delta + 1$).*

The proof is based on the geometrical properties of the UDG model in which the number of links that are at distance-1 from any given link can be bounded.

Since this paper is mainly concerned with mesh networks where nodes are not placed randomly over space, we next consider structured node placements and upper bounds on k_{D1EC} for these topologies. Let $G_\Delta, \Delta = 3, 4, 6, 8$ denote the hexagonal, squared, triangular and octagonal grids respectively. Portions of these grids are shown in Figure 6. The next theorem provides an upper bound on k_{D1EC}, for these regular grid topologies:

***Theorem** 3: K_{D1EC} of regular grid topologies $G_\Delta, \Delta = 3, 4, 6, 8$, is sequentially upper bounded by 3,4,7,10.*

While Theorem 3 guarantees that the number of channels available in standards such as IEEE 802.11a is sufficient to have a valid D1EC for many grid topologies, Theorem 2 shows that in a random deployment of nodes, the needed number of channels can become very large and hence there may not be enough channels for the D1EC problem to have a solution. Only in this case, the channel assignment algorithm should allow two links at distance one to be assigned the same channel. For a colored link, the set of links that have the same channel and are at distance one will contend for channel access and hence will have to share the bandwidth. We define the set of links that will contend for channel access with a specific link e as follows.

Definition: Suppose A is a subset of the network graph G, and a channel assignment C' to the links of A is given. The contention degree of a link $e \in A, Co(e)$, *is the **maximum cardinality matching** of a set M with the following properties: M is a subgraph of A containing e and the following set* $\{l \in A |\ Color(l) = Color(e), d(l, e)_G = 1\}$.

Co(e) is the maximum number of contenders a link can have at a given time and on the same channel. Hence the bandwidth provided for a link is dependent on the objective of channel assignment. For example a channel assignment scheme that minimizes the sum of contention degree can result in a high total throughput but starvation of some links, while a channel assignment scheme that minimizes the maximum contention degree has better fairness properties at the cost of lower total throughput. In this paper, we target the following channel assignment objectives:

- *Approximating the D1EC problem*: If a valid D1EC is achieved, each link will have a channel without a need to share its bandwidth such that the maximum possible throughput is achieved. On the other hand as we proved in Theorem 1, the D1EC problem is NP-complete so that we must rely on a heuristic solution.
- *Providing low maximum contention degree after channel assignment*: If the number of channels is not sufficient to solve the D1EC problem, some links which are at distance one have to be assigned the same channel and hence the bandwidth is shared among them. Note that because mesh network traffic aggregates at the gateway, links closer to the gateway should have higher channel assignment priority. Hence it is desirable for channel assignment to have zero contention degree for links connected to the gateway and low maximum contention for other links.

IV. Channel Assignment Protocol

A. Overview

The complete channel assignment protocol comprises of three separate mechanisms. In the first mechanism the Network Control Center (NCC), which is co-located with one of the gateways and is responsible for channel assignment, constructs the physical physical graph G and the set A comprise of the active links which are expected to forward data. *This procedure is run only once during network setup and is updated based on deployment of new nodes or node failures.*

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Algorithm 1 D1C-CA: Distance-1 Constrained Channel Assignment Algorithm

Input:
$G = (V, E)$: Physical Graph Model.
$A = (V, E_A)$: Subgraph of G selected by the routing protocol.
Output:
a) Channels assigned to links present in A.
b) Contention degree of each colored link.

1: Let r_i = i'th root of the mesh for i = 1 to N
2: Let h = max(min($d(v_j, r_i)_A$)) $\forall$ $v_j \in V_i$ and $\forall$ $i \in 1\ to\ N$
3: Let $AvailChan$ = List of available channels
4: **for** all edges $e \in E_A$ **do**
5: $Color(e) \leftarrow 0$
6: $Co(e) \leftarrow 0$
7: **while** *counter* $\neq h + 1$ **do**
8: **for** i = 1 to N **do**
9: $Q = \{v_j \in V | \ d(v_j, r_i)_A = counter\}$ for some $i \in 1\ to\ N$
10: AssignLabel (A, Q)
11: AssignColor (A, Q, G)

Procedure AssignLabel (G1 = (V1, E1), F)
1: delete all colored edges and labels in G1 and let l = 1
2: *WhilenotallVerticesLabeled(F)* **do**
3: pick unlabeled vertex $u \in F$ of minimum neighbors in G1
4: **if** degree(u) = 0
5: label(u) $\leftarrow$ 0
6: **continue**
7: label(u)$\leftarrow$ l
8: increase(l)
9: delete all edges incident on u from $G1$

Procedure AssignColor (G1 = (V1, E1), F, G2 = (V2, E2))
1: **for** j from largest label of $\{v \in F\}$ to 1 **do**
2: let u $\leftarrow$ vertex with label j
3: **if** $\exists$ valid $c \in AvailChan$ for AllUncoloredEdgesOf(u)
4: ColorAllEdgesOf(u)
5: **continue**
6: **for** i = 1 to $\|$uncolored edges connected to u in G1$\|$ **do**
7: Let l = random uncolored edge connected to u
8: c_l = the least indexed color not used by links at distance-1 with respect to G2
9: **if** such color does not exist
10: Let Conflict(l) = {channels taken by root links at distance-1 from l in G2}
11: **if** $\|$AvailChan - ConflictChan(l)$\|$ > 1
12: AvailChan(l)={$c \in AvailChan - ConflictChan(l)$}
13: Let AffectedLinks(l) = $\{e \in G2 | \ d(e, l)_{G2} = 1$ and $Color(e) \in \ AvailChan(l)\}$
14: $\forall$ c $\in AvailChan(l) \Rightarrow$ Contention(c) = Max($Co(e)$) {$\forall$ e $\in$ AffectedLinks(l), Color(e) = c }
15: Let LeastLoaded = {c $\in AvailChan(l)$, Contention(c) is minimum}
16: Assign the highest indexed channel from LeastLoaded
17: update($Co(e)$) for l and AffectedLinks(l)

The second mechanism is the D1C-CA algorithm, described in this section. In particular, based on the physical topology and the forwarding topology, the NCC runs the D1C-CA algorithm which allocates channels to links in A in a way that satisfies the objectives described in section III. After completing the algorithm, the NCC distributes to each node a vector of entries, one entry for each active link connected to it. Each entry is comprised of two elements: the channels assigned to the link and the number of links in its interference neighborhood that are assigned the same channel, i.e., the contention degree of the link.

The third mechanism is medium access: As the channel assignment algorithm assigns different channels to links connected to a node, a mechanism is needed to coordinate between each sender and receiver to schedule their transmission. This is the function of the MAC protocol described in Section V.

B. Algorithm Description

The algorithm assumes that there are N gateways present in the mesh (Line 1) and uses hop count metric between nodes and nearest gateway to visit the nodes and assign colors to links connected to them. In each iteration, nodes that are at a specific distance are selected (Line 10), assigned labels (Line 11) and finally edges connected to them are colored (Line 12).

In our coloring procedure, we select the node with highest label (Line 2) and first try to assign the same color to all of the links connected to it (Line 3). If such color does not exist, we randomly select one of uncolored edges connected to it, l (Line 7). If a valid color is found, then the color with lowest index is assigned to l. If a valid color is not found, we *reserve the colors of links connected to gateways* by eliminating channels selected by them from the set of available channels for l (Line 10-12). Next, a color with minimum contention degree is greedily assigned to l (Line 13-15).

As this assignment has impact on the contention degree of links that are at distance one from l and have the same color as l, their contention degree is updated by using Edmond's algorithm which finds maximum cardinality matching for each affected link [6].

We now capture the performance of our proposed heuristic in approximating the D1EC problem, i.e., worst cast ratio between the required number of channels to have a valid D1EC and the number of channels used by our heuristic. We assume the UDG model and that the forwarding subgraph A, is equal to G.

***Theorem** 4:* Algorithm D1C-CA needs at most $C_1.OPT(K_{D1EC})$ channels to have a valid D1EC.

Our proof is based on the unique properties of the geometric graphs and the distance one chromatic index of cliques.

V. Common Channel Reference MAC (CCR-MAC)

In this section we discuss the basic principles of CCR-MAC. CCR-MAC is an asynchronous control channel based MAC protocol that uses reservation packets (similar to RTS/CTS exchange in 802.11) prior to each data packet transmission.

In CCR-MAC the channel allocation algorithm provides the MAC with a channel to use and the contention degree for each of its outgoing links. Two modes of operation exist based on the contention degree: (i) If the channel assigned to a link has a contention degree equal to zero, it is always safe to transmit on this channel without any risk of data-channel collision. Hence, whenever the sender and receiver are available, they can coordinate an immediate data transmission after control message exchange. Note that as the channel assignment algorithm favors the first hop links, which are expected to be the mesh network bottleneck, these links are free of collisions on the data channels.

(ii) In case of a contention degree greater than zero, sending a data packet may result in collisions on data channels. However, since the channel allocation algorithm minimizes the contention degree on each data channel , i.e., minimizes the number of contending transceivers per channel, the risk of data channel collision is small. The details of CCR-MAC protocol is provided in [2]

VI. Performance Evaluation

In this section, we evaluate the performance of our scheme through simulations. We compare our scheme with one-channel-per-gateway IEEE 802.11 and AMCP, a multi-channel MAC protocol in the class of TBCS protocols [25]. AMCP uses a separate channel for control packet exchange and the rest of channels for data packet transmission. Our experiments use the same MAC layer parameters as [25].

A. Simulation Setup

For simulations, we use the ns-2 simulator with CMU wireless extensions. Simulations are performed for two different backbone topologies. In the first topology, the mesh coordinates correspond to a deployed mesh network. This topology uses three gateways and is plotted in figure 2(a). For the second topology, we choose an ideal placement of nodes in a rectangular 5x10 grid topology. In this topology, each node is at a distance of $200m$ from its immediate neighbors and there are two gateways in the middle of 5x5 square grids inside the original 5x10 rectangular grid topology. Each node in these topologies has transmission range of $250m$ and interference range of $550m$ ([22]).

In all simulations, each node initially calculates the shortest path route to the nearest gateway and uses this static route for gateway access. In our scheme, this information is also provided for the NCC which runs the D1C-CA algorithm and allocates channels to all such links selected by the routing protocol. For 802.11 simulations, each gateway and its associated subgraph has a dedicated and separate channel to avoid interference within subnetworks ([29]) and the 802.11 MAC is used with RTS/CTS enabled.

In each of the topologies, 25 nodes are randomly selected as source/sink in upload/download gateway-traffic patterns. We use a packet size of 1000 bytes and set the channel capacity to 2 Mbps. The channel switching delay is set to 80 μsec ([3]) and each source node generates and transmits constant bit rate traffic via UDP. We drive the network to saturation as follows. For a particular scheme and number of channels, we run a series of simulations, increasing the offered load of all the flows proportionally, starting from a low value. We stop when the throughput does not increase any further with a further increase in the offered load.

B. Simulation Results

Aggregate channel utilization: Figures 2(b), 2(c), 3(a) and 3(b) correspond to aggregate upload/download simulation results for each of the topologies. With a single radio constraint at each node, the maximum aggregate throughput of a single link in isolation is limited to 184 pkt/s. Our total throughput simulation results show an average of 150pkt/s achieved at each of the gateways with sufficient number of channels to have a valid D1EC. This shows that our approach can indeed utilize the increase in the number of channels and deliver close to the maximum throughput in topologies with high contention and collisions on the control channel.

In contrast for AMCP, corrupted and inaccurate channel availability state leads to frequent data packet collisions or unnecessary waiting on the control channel while data channels are free. Consequently, AMCP does not efficiently utilize increase in the number of channels and saturates with small number of available channels.

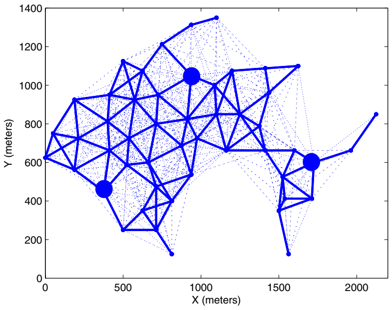
The same trend of performance is present with number of channels smaller than k_{D1EC}. Indeed as D1C-CA attempts to assign channels with zero contention degree to links connected to the gateway, high channel utilization is expected with only one additional channel as links connected to the gateway will no longer be a bottleneck. This observation is shown through the high increase in total throughput with only two available channels in figures 3(a) and 3(b). In contrast, for the topology of figure 2(a), one additional channel is not sufficient to color all links connected to each gateway as gateways interfere with each other. This is the main reason for low throughput improvement in figures 2(c) and 2(b) with 2 channels. With further increase in the number of channels, D1C-CA efficiently splits contention among different links. Moreover, by lowering the contention degree, data channel collision probability is minimized. This contrasts to TBCS protocols and in particular AMCP in which high collision probability is present on each of the channels.

Finally, the one-channel-per-gateway 802.11 simulation results show limited throughput improvement compared to link based channel assignment approaches. This is because of high interference that is present within each subnetwork compared to amount of interference between different subnetworks.

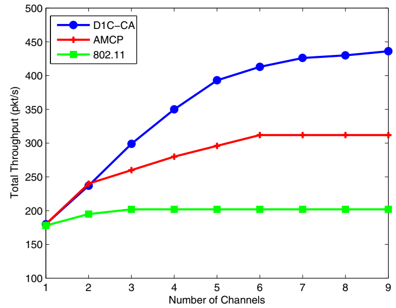
Effect of traffic load: We now evaluate the effect of offered load. Figure 2(d) depicts the aggregate throughput variation in uplink gateway traffic scenario when 9 channels are available. Until 5 pkt/s, the load is too small to exploit the channels and hence all approaches yield the same performance. After that point, channelization becomes effective and multiple channels are exploited to increase aggregate throughput.

Additional increase in the offered load results in degradation of throughput in all approaches. However, this throughput degradation is more severe for 802.11 based approach and especially in the grid topology which is omitted here due to page limitations but provided in [2]. In the grid topology, more hidden terminals are present around the gateway nodes and data packet collisions are present even with RTS/CTS mechanism enabled. This is due to nodes that are within interference range of the gateway node and packet transmission duration that is large compared to the EIFS time. In contrast, other approaches show smooth throughput degradation with increase in the offered load. This is indeed due to channelization and the fact that data packets are transmitted on different channels than RTS/CTS packets.

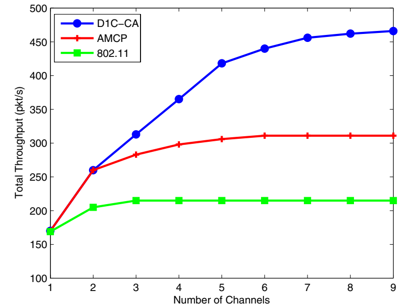
Fairness among flows: With the aggregate throughput metric, some flows might capture the medium completely and result in starvation of other flows. Hence we study per-flow



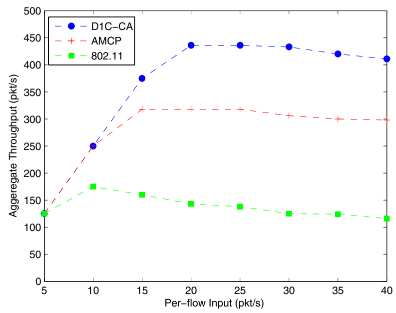
(a) Topology



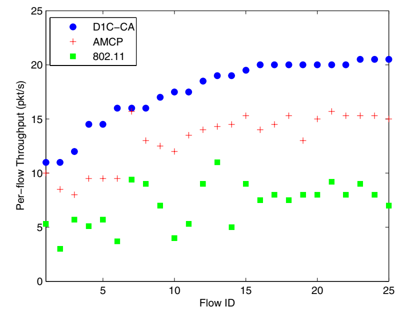
(b) Total upload throughput comparison



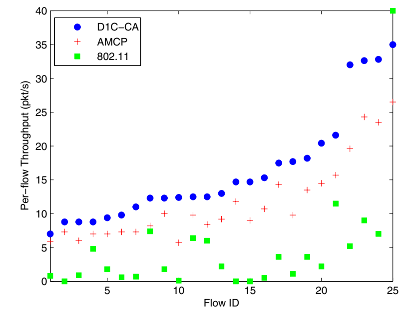
(c) Total download throughput comparison



(d) Total throughput variation with offered load

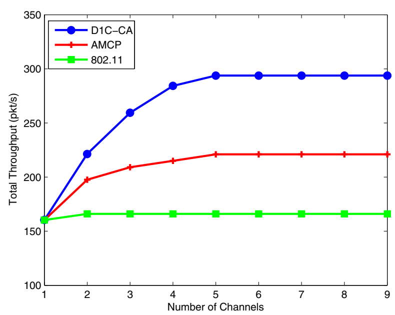


(e) Upload per flow throughput in saturation

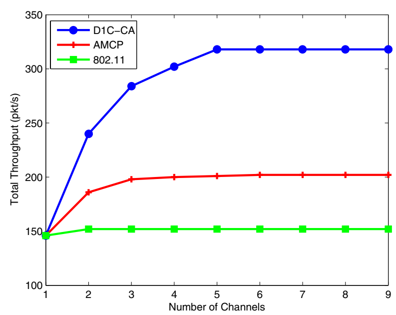


(f) Upload Per flow throughput with backlogged flows

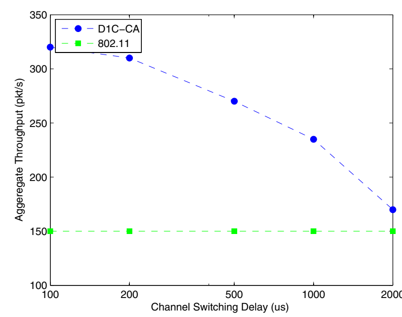
Fig. 2. Deployed mesh simulation results



(a) Total upload throughput comparison



(b) Total download throughput comparison



(c) Impact of channel switching delay

Fig. 3. Grid topology simulation results

throughput results to evaluate fairness among flows. In our simulations we observed fair throughput division among flows for download scenarios irrespective of the approach and the offered load. Hence, we only provide upload simulation results with 9 available channels for the topology of figure 2(a). The same trend of performance was observed with different number of channels and for grid topology.

Figure 2(e) depicts per flow throughput results in the saturation region. As expected, both our scheme and AMCP achieve higher per-flow throughput results compared to 802.11. Furthermore, 23 out of 25 flows have further increased their per-flow throughput in our approach.

Figure 2(f) depicts the fairness characteristics under high traffic load conditions where each flow is fully backlogged and always has a packet to transmit. In this scenario, flows that are closer to the gateway or have fewer hidden terminals and contenders have a higher chance to transmit their data packets. This leads to starvation of many flows in 802.11 where 10 out of 25 flows have reached almost zero throughput. In contrast, fairness properties are better in both AMCP and our approach in which interfering links transmit their data packets on different channels and hence each flow receives a minimum amount of bandwidth. However as expected per-flow throughput results are different from those of figure 2(e), as links closer to the gateway have a higher chance to win the control channel and send their packets to the gateway node.

Effect of channel switching delay: As in our proposed scheme nodes switch between control and data channels at packet level, the channel switching delay overhead can become the bottleneck. In figure 3(c) we have plotted such impact on aggregate achieved throughput. As observed from this figure, below 200 μsec the impact of channel switching is negligible. This is because of packet transmission time which is large compared to the switching delay. However channel switching delay of 2 $msec$ or higher decreases the throughput to the same level as 802.11. This overhead can be simply addressed by reserving a channel for multiple packet transmissions in our MAC protocol.

VII. Related Work

We divide related work into two broad categories: prior use of graph coloring algorithms in wireless networking, and alternative approaches to exploiting frequency diversity.

A. Graph Theoretic Techniques

Graph Theory-Based Coloring. Relevant and widely applied graph theoretic techniques include *list coloring* and *labeling problem*. See [7], [15], [8] for surveys. A list coloring of a graph is an assignment of colors to each vertex from the list of available choices, such that two nodes that are connected with an edge get different colors. The $L(h, k)$ labeling of a graph G, is an assignment of non-negative integers to the nodes of G, such that adjacent nodes are labeled with at least h apart and nodes that share a common neighbor node are at least k apart.

Graph Theory-Based Channel Assignment. Several applications of such theory to networking problems include mapping resource assignment problems for resources such as time, frequency, codes, radios, and routes to different edge and vertex coloring problems [21], [5], [9], [17], [14], [1], [4], [24]. For example in the TDMA link scheduling problem with node exclusive interference model, links that share a node in common (i.e., according to our definition are at distance-0) should be given different time slots. Similarly, with the RTS/CTS model, links that either share a node in common or interfere with each other (i.e., are at distance-0 or 1) should be given different channels. In other applications, channel assignment is used to increase spatial reuse for cellular networks by assigning the same channels to cells which are sufficiently apart.

D1EC Problem. In contrast to all such work, we are the first to formulate and investigate the D1EC problem, a specific form of graph edge coloring in which only edges at distance-1 are constrained. Furthermore, while most graph coloring problems assume sufficient number of colors (time slots, CDMA codes or frequencies), we consider that the number of available channels is constrained without any guarantee that the solution to the D1EC problem exists. This is important because many standards specify a fixed number of available channels, e.g., 12 channels in IEEE 802.11a.

B. Protocols to Exploit Frequency Diversity

A significant body of work exists in protocol design for exploitation of multiple orthogonal frequencies and here we summarize a representative sample.

Single-Radio Protocols. Single-radio multi-channel MAC protocols include [25], [26]. In MMAC [26], nodes are synchronized and meet at a common channel periodically to negotiate channels for use in the next phase. In AMCP [25], a separate frequency channel is used for channel negotiation before each data packet transmission. These protocols belong to the TBCS class and have been considered by the IEEE 802.11s working group as potential approaches to support multi-channel capability in single radio mesh networks [12]. As described previously and demonstrated via simulations, such algorithms can be viewed as local and greedy and can yield inefficient allocations. Likewise, reference [30] proposes assigning channels to disjoint network “components” in order to increase capacity. However, in contrast to our work, if applied to a single-gateway mesh network, the component methodology would yield a static single 802.11 channel allocation.

Multi-Radio Protocols. Previous multi-radio multi-channel MAC protocols include dedication of a radio for control messages or control signals [31], [13], [16]. Other work uses unmodified IEEE 802.11 and employs channel assignment for either load balancing [23], [22], topology control [19], [27], [28], and avoidance of external interference [20]. In contrast, our problem requires only that links that are a specific distance to be assigned different channels. More fundamentally, our results are achieved with only a single half-duplex transceiver.

VIII. Conclusion

In this paper we addressed the channel assignment problem in single radio wireless mesh networks. In our approach, channels are quasi statically assigned to a set of links selected for traffic forwarding by the routing protocol. Moreover, we devised a MAC protocol that supports parallel transmissions using the targeted channel assignment. We modeled channel assignment as a new type of graph coloring problem where edges at distance one are constrained. We proved that the problem is NP-complete and designed an efficient heuristic solution. We further investigated the performance of the assignment algorithm and provided bounds on the minimum number of needed channels such that maximum throughput is achieved. Finally, we provided extensive simulations and demonstrated considerable performance improvements compared to alternate schemes.

IX. Acknowledgement

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References

[1] M. Alicherry, R. Bhatia, and L. E. Li. Joint Channel Assignment and Routing for Throughput Optimization in Multi-Radio Wireless Mesh Networks. In *Proc. ACM MobiCom*, 2005.

[2] Ehsan Aryafar. Distance-1 Constrained Channel Assignment in Single Radio Wireless Mesh Networks. *Master Thesis, http://www.ruf.rice.edu/∼ehsan/*, 2007.

[3] P. Bahl, R. Chandra, and J. Dunagan. SSCH: Slotted Seeded Channel Hopping for Capacity Improvement in IEEE 802.11 Ad Hoc Wireless Networks. In *Proc. ACM MobiCom*, 2004.

[4] H. Balakrishnan, C. L. Barrett, V. S. Anil Kumar, M. V. Marathe, and S. Thite. The Distance-2 Matching Problem and Its Relationship to the MAC-Layer Capacity of Ad Hoc Wireless Networks. In *IEEE Journal on Selected Areas in Communications*, 2004.

[5] A. A. Bertossi and M. A. Bonuccelli. Code Assignment for Hidden Terminal Interference Avoidance in Multihop Packet Radio Networks. *IEEE Transactions on Networking*, 1995.

[6] J. A. Bondy and U. S. R. Murty. *Graph Theory with Applications*. New York: McMillan, 1976.

[7] Tiziana Calamoneri. The L(h, k)-Labelling Problem: A Survey and Annotated Bibliography. *The Computer Journal*, 2006.

[8] D. Fotakis, G. Pantziou, G. Pentaris, and P. Spirakis. Frequency Assignment in Mobile and Radio Networks. 1999.

[9] J. Garcia-Luna-Aceves and J. Raju. Distributed Assignment of Codes for Multihop Packet-Radio Networks. In *Proc. of MILCOM*, 1997.

[10] M. R. Garey and D. S. Johnson. *Computers and Intractability, A Guide to the Theory of NP-completeness*. W. H. Freeman and Company, 1979.

[11] R.L. Graham, B.D. Lubachevsky, K.J. Nurmela, and P.R.J. stergrd. Dense packings of congruent circles in a circle.

[12] IEEE. Draft amendment: ESS mesh networking. *IEEE P802.11s Draft 1.00*, November 2006.

[13] N. Jain and S. Das. A Multichannel CSMA MAC Protocol with Receiver-Based Channel Selection for Multihop Wireless Networks. In *Proc. of the 9th Int. Conf. on Computer Communications and Networks (IC3N)*, 2001.

[14] M. Kodialam and T. Nandagopal. Characterizing the Capacity Region in Multi-Radio and Muli-Channel Mesh Networks. In *Proc. ACM Mobicom*, 2005.

[15] J. Kratochvil, Z. Tuza, and M. Voigt. New Trends in the Theory of Graph Colorings: Choosability and List Coloring. *Contemporary trends in discrete mathematics, DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, 1999.

[16] R. Maheshwari, H. Gupta, and S. R. Das. Multichannel MAC Protocols for Wireless Networks. In *Proc. of SECON*, 2006.

[17] T. Makansi. Transmitter-oriented code assignment for multihop packet radio. *IEEE Transactions on Communications*, 1987.

[18] M. V. Marathe, H. Breu, H. B. Hunt, S. S. Ravi, and D. J. Rosenkrantz. Simple Heuristic for Unit Disk Graphs. *Networks*, 1995.

[19] M. K. Marina and S. R. Das. A Topology Control Approach for Utilizing Multiple Channels in Multi-Radio Wireless Mesh Networks. In *Proc. of BroadNets*, 2005.

[20] K. Ramachandran, E. M. Belding, K. C. Almeroth, and M. M. Buddhikot. Interference-Aware Channel Assignment in Multi-Radio Wireless Mesh Networks. In *Proc. of IEEE Infocom*, 2006.

[21] S. Ramanathan. A Unified Framework and Algorithm for (T/F/C)DMA Channel Assignment in Wireless Networks. In *Proc. of IEEE Infocom*, 1997.

[22] A. Raniwala and T. Chiueh. Architecture and Algorithms for an IEEE 802.11-based Multi-Channel Wireless Mesh Networks. In *Proc. IEEE Infocom*, 2005.

[23] A. Raniwala, K. Gopalan, and T. Chiueh. Centralized Channel Assignment and Routing Algorithms for Multi-Channel Wireless Mesh Networks. *ACM Mobile Computing and Communications Review (MC2R)*, 8, 2004.

[24] G. Sharma, R. R. Mazumdar, and N. B. Shroff. On the Complexity of Scheduling in Wireless Networks. In *Proc. ACM Mobicom*, 2006.

[25] J. Shi, T. Salonidis, and E. Knightly. Starvation Mitigation Through Multi-Channel Coordination in CSMA Multi-hop Wireles Networks. In *Proc. of ACM MobiHoc*, 2006.

[26] J. So and N. Vaidya. Multi-Channel Mac for Ad Hoc Networks: Handling Multi-Channel Hidden Terminals Using a Single Transceiver. In *Proc. ACM MobiHoc*, 2004.

[27] A. P. Subramanian, H. Gupta, and S. R. Das. Minimum interference channel assignment in multi-radio wireless mesh networks. Technical report, WINGS Lab, SUNY at Stony Brook, 2006.

[28] J. Tang, G. Xue, and W. Zhang. Interference-Aware Topology Control and Qos Routing in Multi-Channel Wireless Mesh Networks. In *Proc. of ACM MobiHoc*, 2005.

[29] Tropos Networks. http://www.tropos.com.

[30] R. Vedantham, S. Kakumanu, S. Lakshmanan, and R. Sivakumar. Component Based Channel Assignment in Single Radio, Multi-channel Ad Hoc Networks. In *Proc. ACM Mobicom*, 2006.

[31] S.-L. Wu, C.-Y. Lin, Y.-C. Tseng, and J.-P. Sheu. A New Multi-Channel Mac Protocol with On-Demand Channel Assignment for Multi-hop Mobile Ad Hoc Networks. In *Proc. International Symposium on Parallel Architectures, Algorithms and Networks (I-SPAN)*, 2000.

APPENDIX

Proof of Theorem 1: It is clear that the problem is in NP. We prove that it is NP-complete by showing a reduction from *GRAPH K-Colorability* to this problem. Without loss of generality we assume A is equal to G. It is proved [10] that *GRAPH K-Colorability* is NP-complete for every fixed $k \geq 3$.

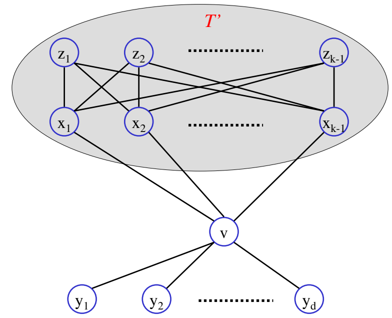

Fig. 4. The graph T

For every graph H, we construct another graph G such that H is K-colorable if and only if G has a D1EC with K colors.

Before constructing G, consider the graph $T_{K,d}$ shown in figure 4. This graph consists of a vertex v adjacent to a set of $K+d-1$ other vertices. The vertices y_1,...,y_d connected to v are called the heads of $T_{K,d}$. The rest of vertices denoted by x_i are respectively connected to another set of K - 1 vertices denoted by z_j. The important property of this graph, which is used throughout the proof, is based on the following lemma:

Lemma 1: In any distance-1 edge coloring of $T_{K,d}$ (figure 4) with k colors, the color of all vy_j edges is the same.

Proof: First, we prove that this graph is K colorable. In order to do this, we color the whole graph by giving K different colors to all x_i and v. Next we color all the edges connected to each x_i by giving them the color given to x_i. The remaining edges are vy_j edges which can be colored by giving them the color used for v. Now we prove that in any other distance-1 coloring of this graph with K colors, the colors used by all vy_j edges is the same. Consider the subgraph of $T_{K,d}$ (T') composed of nodes x_i and z_j and all the edges between them. It is easy to see that this graph contains a set of K - 1 edges which all are mutually at distance-1 from each other. Hence in any distance-1 coloring of $T_{K,d}$ using k colors, this subgraph should be colored with K - 1 different colors. Now, since $T_{K,d}$ is K colorable and each vy_j edge is at distance one from all edges in T', only one color remains for all vy_j.□

Now, we are ready to construct the graph G from the graph H. Corresponding to each vertex v of degree d in H, we put a copy C_v of $T_{K,d}$ in G. Each head of C_v corresponds to one of the edges incident to v. If two vertices u and v in H are joined by an edge e, their corresponding heads in C_u and C_v are connected through e, in the resulting graph G. We claim that G has distance-1 edge edge coloring with K colors if and only if H is K colorable.

Assume that G has a distance-1 edge coloring with K colors. By the property of $T_{K,d}$, we know that for every vertex $v \in V(H)$ the color of all vy_j edges in C_v is the same. Color the vertex v in H with the color of vy_1. Since for any two adjacent vertices u and v in H there exists edges uy_i and vy_j that are at distance one from each other (through an edge e in H), the color of uy_1 and vy_1 in C_u and C_v is not the same and therefore the color of the vertices u and v, can not be the same. Thus, the coloring is a proper vertex coloring of H.

Conversely, assume that H has a proper vertex coloring using K colors. We construct a distance-1 edge coloring of G

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using K colors. For every v, we color **all** the edges connected to v of C_v in G with the color of v in H. These set of edges include vy_j and vx_i. The remaining edges of graph G are divided to two sets. The first set includes The edges in T' associated with each C_v. These edges can be colored with $k-1$ colors as we showed in lemma 1. The remaining set of edges includes original edges in graph H between any two vertices u and $v \in H$ that are still present in G through corresponding heads in C_u and C_v. It is easy to check that for any such sample edge e between two heads C_u and C_v, the set of edges at distance-1 is a subset of uy_i and vy_j. Since all these edges are colored with 2 colors, there are k - 2 colors yet available which we randomly assign one to each such e. □

Proof of Theorem 2: Our proof is based on division of the physical graph into hexagonal cells of diameter R (figure 5), where R is the transmission range of nodes in the network.

Within each cell of diameter R, at most $\Delta+1$ nodes can exist as all nodes inside a cell are connected and maximum degree of graph is Δ. Now we propose an algorithm for an arbitrary node placement, which uses number of channels equal to the upper bound. Our algorithm is based on a coloring of *nodes* in the physical graph. Assume a coloring to each node is given. We color the edges among different nodes as follows: a) For an edge inside a cell, randomly assign the color of one of its end point nodes. b) For edges among nodes in different cells, give the color of the node with higher y-axis coordinate value. For the resulting edge coloring to be valid, the original node assignment should have the following properties: a) For each cell, a pool of $\Delta + 1$ different colors is available where nodes inside a cell are assigned one randomly. b) For each cell, its pool of colors is reused at cells which are apart for the distance of $3.5 \times R$ and their centers are parallel to x-axis or are at the distance of $2R$ and their centers are parallel to y-axis. c) For two cells which have the same channel reuse and are apart for distance of $2R$, if there exists nodes exactly at the corner of cells (figure 5 shows such example), these nodes should be assigned different colors from the pool of $\Delta + 1$ colors (we assume $\Delta \geq 2$ for this argument be valid. Otherwise, equation 2 gives a smaller upper bound). This channel reuse is depicted through colored cells in figure 5. It is easy to check that with such node assignment, the resultant coloring is a valid D1EC. We have shown the number of cells in figure 5 that will use different channel pools. From this figure it is evident that such coloring algorithm needs at most $18 \times (\Delta + 1)$ colors.□

Proof of Theorem 3: The proof is based on two steps: First, we construct a basic cell topology, such that its replication generates the infinite topology. Second, we find a valid D1EC for the cell with the following properties: a) permits replication by matching the boundary colors, and b) the resultant edge coloring after replication is a valid D1EC. The basic cells with the above properties for regular grids are plotted in figure 6.□

Proof of Theorem 4: The main property of the algorithm that we use in the proof of performance approximation, is based on visiting the nodes instead of edges. More precisely, since D1C-CA algorithm visits nodes in a given order and ***greedily assigns first color that is possible to assign to all edges connect to it***, we have the following property:

Suppose that a node u is selected and we want to color all uncolored edges connected to it. In order to have a valid

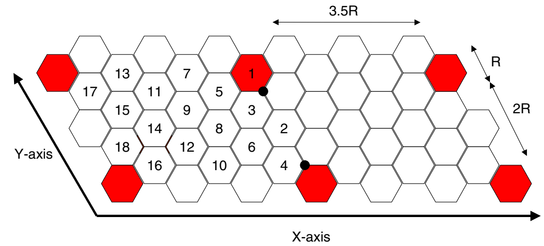
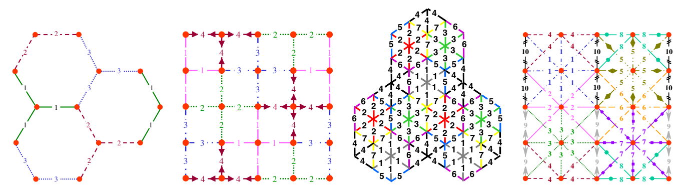


Fig. 5. D1EC of a geometric graph.



(b) (c) (d)

Fig. 6. Basic cell coloring of (a) hexagonal G_3; (b) square G_4; (c) triangular G_6; (d) octagonal G_8 grid topologies

distance-1 channel assignment, no other edge at distance-1 should have the same color. This proposition is true ***if*** no other node at distance $3R$ from u has the same color. So our greedy algorithm will not use more colors than the maximum number of nodes present in a circle of radius $3R$.

To find a tight upper bound for this problem, we use the maximal independent sets. An independent set is a set of vertices in a graph G, such that no two vertices of which are adjacent. A maximal independent set is an independent set such that adding any other node to the set forces the set to contain an edge. As a result the maximum number of nodes is equal to the size of maximal independent set times $\Delta + 1$. Since in the geometric graph any two node in the maximal independent set are more than R apart from each other, the problem is equivalent to packing circles of radius $\frac{R}{2}$ in the big circle of radius $3R$. However as the center of these small circles can be on the boundary of our big circle, an upper bound can be achieved by increasing the radius of the original circle to $3.5R$ and packing circles of radius $\frac{R}{2}$ in it. This is a well known problem and is addressed in [11]. From [11], we know that this ratio is equal to 39. Hence the number of colors used by our algorithm is upper bounded by $39 \times (\Delta + 1)$.

On the other hand our next lemma proves that K_{D1EC} for a clique of degree Δ is $\Delta - 1$, and hence the constant approximation ratio is achieved.

Lemma 2: For a clique of maximum degree Δ ($\Delta \geq 2$), K_{D1EC} is equal to $\Delta - 1$.

Proof For $\Delta = 2$ one color is sufficient. Hence we assume $\Delta \geq 3$. In a clique, a color can be *most* used if all edges connected to a node are colored the same. Choose a node and color all the edges connected to it with the least available color. Now, eliminate this node and all the edges connected to it. For the resulting graph, if $\Delta = 2$, all remained edges can be colored with one color, else repeat the procedure. Since at each step the maximum number of edges are colored, the algorithm colors the clique optimally. Since this algorithm repeats until a triangle is reached, $\Delta - 1$ colors are used. □