Title: Routing and Rerouting in Weakly Dynamic Graphs

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Keywords: Dynamic Graph; Shortest Path Problem; Route Planning; Intelligent Traffic Management; Urban Networks.

Abstract. In this paper, we present weakly dynamic graphs, and we propose an efficient polynomial algorithm that computes in advance shortest paths for all possible configurations of a dynamic graph. No additional computation is then needed after any change in the problem because shortest paths are already known in all cases. We use these shortest paths to compute critical conditions. We apply this result to a delivery problem for trucks from one regional storehouse to several local stores when one or two possible points has a variable traversal duration. We apply this result too to the problem of rerouting delivery trucks toward their final destinations when there is a change in the traversal duration of one or two known points.

1 Introduction

Static graphs have a long history of being used to efficiently represent static problems. In these problems, all the data are known from the start. The real world is not static, however, and the solutions to static problems may not always be used [13], [2]. Some data may change, or be unknown in advance. In territorial systems, for example, the traversal duration of a location may depend on traffic density, the presence or not of traffic jams, work in progress, etc. that are all time dependent and usually hard to predict. Thus several approaches have been proposed to study parametric graphs [1] and dynamic graphs [6].

Fully dynamic algorithms, for example, are applied to problems that can be solved in polynomial time. They start with a computed optimal solution, and then try to maintain them when changes occur in the problem. They often propose sophisticated data structures to reach this goal [8], [11].

When the delay between a change and the moment a new solution is needed is very small, or when the problem itself is NP-hard, faster algorithms are needed These reoptimizing algorithms usually start from an initial solution that is not optimal but is expected to be of good quality, if possible. As soon a change is detected, they compute a new solution, trying to do it faster that classical algorithms. Or they compute a new solution as fast as the classical algorithms but this resulting solution is better than the ones found by classical algorithms. These algorithms include meta-heuristics such as ants colony algorithms [5], or swarm algorithms [3].

Another approach used is probabilistic. Probabilities are associated to some variables in the graph, such as the value of a weight, or the presence of a vertex or of a constraint, for example. The algorithms used in these problems usually compute a solution then do some robustness analysis in the probability space [9]. Or they do a quick re-optimization of the solution once the parameters of the problem are perfectly known [4], [10].

In this paper, we study weakly dynamic directed acyclic graphs. In these weakly dynamic graphs with positive weighted arcs, one or two arcs are known to be non stable. That is, the weight of each of these non stable arcs may change at any time. All other arcs have stable weights that never change. We are interested in the "One-to-All" shortest path problem (SPP), that is, in finding what are the shortest paths from one vertex to all other vertices of this graph. This must be done considering the weights of the non stable arcs. Preliminary results on weakly dynamic graphs with only one variable arc or edge were presented in [12], [7]. Here, we propose an efficient algorithm that pre-computes alternative shortest paths for all possible values of the variable weights. It also computes very small sets of very simple critical conditions. When the non stable weights change, the shortest paths for these new values may then directly and immediately be deduced from the critical conditions and may immediately be used without any further recomputation. The proposed algorithm can be used to build alternative routing tables in a computer network, or for the routing or re-routing of a truck in a truck delivery problem.

2 Problem statement

We consider a directed acyclic graph (DAG) G=(V,E). *V* is the set of vertices, *E* is the set of arcs, and to each arc is associated a positive weight. Almost all the arcs are stable and their weights never change. However two known arcs are not stable and their weights may change at any time. The first variable arc is denoted (x^1_1, x^1_2), and its variable weight is x^1. The second variable arc is denoted (x^2_1, x^2_2), and its variable weight is x^2. We call this kind of DAG a weakly dynamic graph with two known variable arcs.

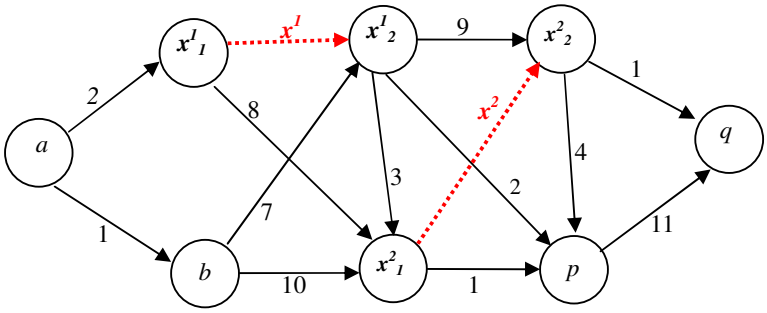

Fig. 1 – *Example of weakly dynamic graph with two variable arcs (in red on the graph).*

We now want to find a set of shortest paths between a given vertex and each other vertex of a graph G.

3 Main results

3.1 The proposed algorithm.

Some notes:

- The length of a path is the sum of the weights of its arcs.
- Shortest paths that do not include any variable arc may be computed with Dijkstra algorithm.

For each target vertex, our algorithm computes a set of a maximum of four alternative paths. So the proposed algorithm works in four steps:

1. it computes the shortest paths that do not include any variable arc, from the starting vertex to all other vertices,
2. for each starting vertex x^r_1 of a variable arc (x^r_1, x^r_2) that was reached during step 1, it computes again shortest paths that do not include any variable arc, from the other vertex x^r_2 of the variable arc to all other vertices,
3. for each starting vertex x^s_1 of a variable arc (x^s_1, x^s_2) that was reached during step 2, it computes again shortest paths that do not include any variable arc, from the other vertex x^s_2 of the variable arc to all other vertices,
4. for each vertex, it finally compares up to 4 values :
    a. the constant length of the shortest path that does not include any variable arc,
    b. the variable length of the shortest path that includes only the first variable arc,
    c. the variable length of the shortest path that includes only the second variable arc,
    d. the variable length of the shortest path that includes the two variable arc.

When comparing the four lengths, some of them have the variable values of the variable arcs as parameters. Thus, for a given vertex, it may happen that one path from the starting vertex may be better than another path for some particular values of the variable weights, and may be worse for some other values of the variable weights.

3.2 Example : We now apply this algorithm on the graph of Fig. 1.

Step1: Here we use Dijkstra's algorithm to computed shortest paths that do not include any variable edge from *a* (origin) to all other vertices, the short distance will be noted $ds^0(a, j)$:

Vertex j	***a*** (origine)	b	x^1_1	x^1_2	x^2_1	x^2_2	p	q
$ds^0(a, j)$	0	1	2	8	10	17	10	18

Step2: Computes the shortest paths that do not include any variable arc, from the starting vertex x^r_1, x^r_2 of a variable arc to all other vertices :

Vertex j	***a***	b	x^1_1	x^1_2	$\boldsymbol{x^2_1}$	$\boldsymbol{x^2_2}$	p	q
$ds^1(x^1_2, j)$: *shortest paths from the starting vertex of variable edge* x^1_2 *to all other*	-	-	-	0	3	9	2	10
$ds^1(x^2_2, j)$: *shortest paths from the starting vertex of variable edge* x^2_2 *to all other*	-	-	-	-	-	-	4	1
$ds^0(i, x^1_1) + x^1 + ds^1(x^1_2, j)$	-	-	-	$2+x^1$	$2+ x^1+3$	$2+ x^1+9$	$2+ x^1+2$	$2+ x^1+10$
$ds^0(i, x^2_1) + x^2 + ds^1(x^2_2, j)$	-	-	-	-	-	$10+ x^2$	$10+ x^2+4$	$10+ x^2+1$

Step3: Compute $ds^0(i, x^1_1) + x^1 + ds^1(x^1_2, x^2_1) + x^2 + ds^2(x^2_2, j)$ *for all vertex j :*

Vertex j	***a***	b	x^1_1	x^1_2	x^2_1	x^2_2	p	q
$ds^0(i, x^1_1) + x^1 + ds^1(x^1_2, x^2_1) + x^2 + ds^2(x^2_2, j)$	-	-	-	-	-	$2+x^1+3 + x^2$	$2+x^1+3+x^2 +4$	$2+x^1+3+ x^2 +1$

Step4: Compare the 4 values computed in the last steps to compute the shortest paths from vertex ***a*** to all other vertices *j* in function of the variables edges x^1 *and* x^2 :

Vertex j	***a***	b	x^1_1	x^1_2	x^2_1	x^2_2	p	q
Compute the shortest paths form a to all other vertices j in function of the variables edges x^1 *and* x^2	0	1	2	*Min (8, 2+*x^1*)*	*Min (10, 5+*x^1*)*	*Min (17, 11+*x^1*, 10+* x^2*, 5+*x^1*+* x^2*)*	*Min (10, 4+*x^1*, 14+* x^2*, 9+* x^1*+* x^2*)*	*Min (18 , 12+*x^1*, 11+* x^2*, 6+* x^1*+* x^2*)*

3.3 Some proprieties of the algorithm

We call *critical conditions* of a given vertex, the set of length functions associated to the four paths to this given vertex computed by the algorithm. Because the functions of this set are constant, or very simple linear functions, they can be computed and compared very easily. Thus for each target vertex, a set of a maximum of four alternative paths can be stored along with the associated set of critical conditions. As soon as any variable weight changes, the critical conditions of the target vertex just need to be recomputed and compared. Then the new shortest path may be choosen among the alternative paths stored for this vertex. No recomputation of shortest paths is needed, no data beside the current values of the variable arcs need to be exchanged, and all decisions may be taken locally. The complexity of the algorithm itself is $O(n^2)$.

4 Application to solving a truck delivery problem

A fleet of trucks in an area must be managed. Each truck has to do one unique trip from the regional warehouse to one local delivery point.

Two particular direct roads between some locations in the area are known to have independent variable traversal duration because of traffic jams, work in progress, or a lift bridge, for example. The traversal duration of each of these roads may change one or more times during the trip. The position of the truck is known at all time, thanks to a GPS.

How to direct, and if necessary redirect, one truck, so that it follows the shortest path to its destination according to its current location and the current conditions? This problem can be represented by a weakly dynamic undirected graph.

Our algorithm can be used to efficiently compute the critical conditions between all locations and the destination, and the corresponding alternative paths from each location to the destination.

We will use again the example of Fig. 1 to illustrate this problem, with vertex a being staring location, and vertex q being the destination in our problem.

To do this, we first build a new problem by reversing the arcs of the graph:

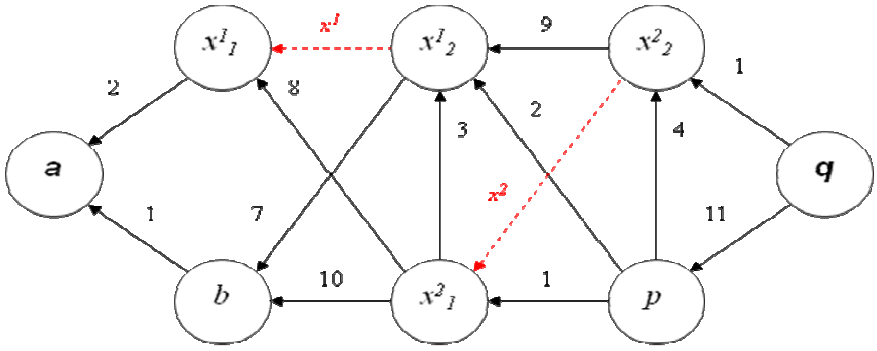

Fig2. – *Reversing the graph corresponding to the graph of the Fig.1*

Next we use the algorithm to compute shortest to go to the city q :

Table 1: Parameterized Shortest distance to go to the city q

Vertex	a	b	x^1_1	x^1_2	x^2_1	x^2_2	p	q
Critical conditions of alternative paths from vertex q	Min(18, $12+x^1$, $11+x^2$, $6+x^1+x^2$)	Min(17, $11+x^2$)	Min(20, $10+x^1$, $9+x^2$)	Min(10, $4+x^2$)	Min(12, $1+x^2$)	1	11	0

Next we use the algorithm to compute the alternative paths from vertex q to all other vertices in this new reversed problem. These computed alternative paths in the reversed problem are also, if reversed, alternative paths to vertex q in the original problem, with the same critical conditions. Thus give for all the possible values of variable arcs *x, y* a set of routing parameterized tables to go to the city *q*:

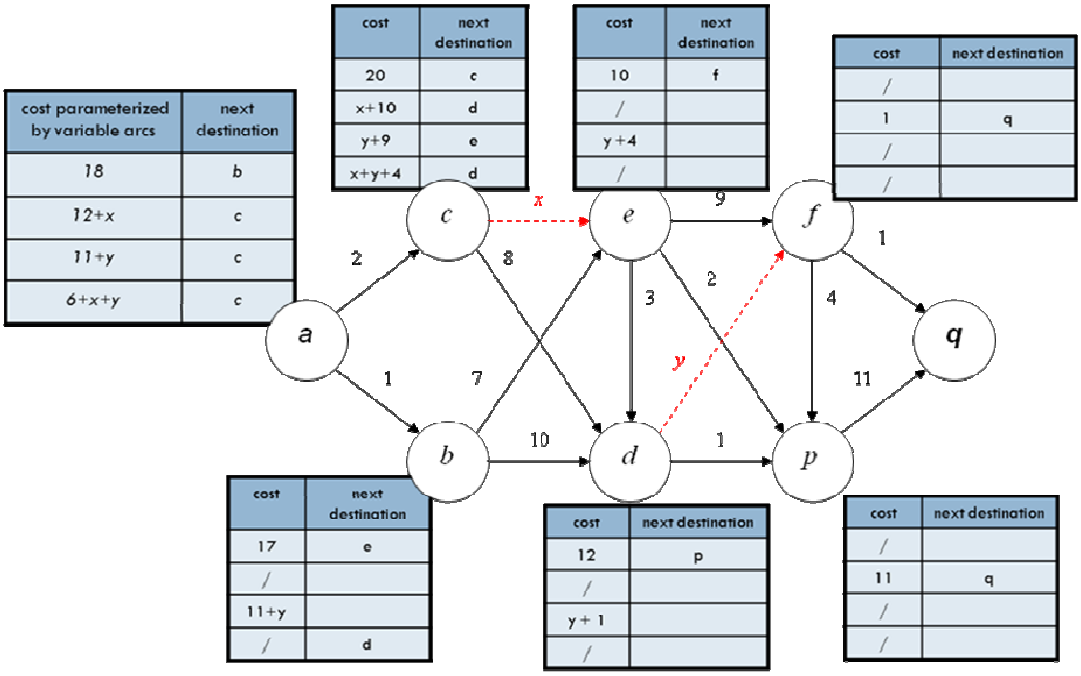

Fig. 2 – *Routing tables parameterized by the variable arcs x, y to go to the city q*

Before starting its trip, the truck receives the computed parameterized routing tables of all vertices. Each time the truck is at a vertex, it consults the current values *x* and *y* of the variable arcs. It then compares in O(1) the critical conditions for this vertex. Using the routing table, it decides what is the next vertex it must now go to.

5 Conclusions

In this paper, we proposed an efficient polynomial algorithm that computes in advance alternative shortest paths for all possible values of 2 variables. This algorithm can be used for routing and rerouting messages in a weakly dynamic network. It can also be used for routing or rerouting trucks in a transportation problem in a low dynamic network. Once all the routing tables are pre-computed, deciding which neighbour to use at each step of a routing problem is in O(1), even if the weights change several times and for any values. No recomputation of shortest paths is needed. It is more effective than the methods used in traditional dynamical routing algorithms (RIP like). In the future, we intend to work on extending this result to non

directed graphs, and to the computation of longest paths for scheduling problem (potential task graphs).

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